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Nonclassical constitutive model involving void evolution of casting magnesium alloy

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Abstract: The void evolution equation and the elastoplastic constitutive model of casting magnesium alloy were investigated. The void evolution equation consists of the void growth and the void nucleation equations. The void growth equation was obtained based on the continuous supposition of the material matrix, and the void nucleation equation was derived by assuming that the void nucleation follows a normal distribution. A softening function related to the void evolution was given. After the softening function was embedded to a nonclassical elastoplastic constitutive equation, a constitutive model involving void evolution was obtained. The numerical algorithm and the finite element procedure related to the constitutive model were developed and applied to the analysis of the distributions of the stress and the porosity of the notched cylindrical specimens of casting magnesium alloy ZL305. The computed results show satisfactory agreement with the experimental data.

Key words: casting magnesium alloy; void volume fraction; void evolution equation; constitutive model; elastoplasticity

1 Introduction

Due to low density and excellent machinability in terms of metal removal rates and low tools wear, casting magnesium alloys are predestined for light-mass constructions of components in automotive industry. For example, steering wheels, door structures and oil sumps. However, because casting alloys contract when solidified, they inevitably contain a certain amount of microscopic voids[1]. As the microscopic voids grow during a thermomechanical loading process, the walls or ligaments between the voids thin down and ductile fracture may occur due to the coalescence of the voids (Fig.1). Constitutive models for such kind of materials should, therefore, take into account the effects of the voids. RICE and TRACEY[2] have put forward in their pioneering works the exponential dependence of void growth-rate on the triaxiality ratio of remote stresses in a rigid-plastic material. GURSON[3] has investigated two different deformation modes of a representative volume element for a porous material with cylindrical and spherical voids. BAASER and GROSS[4] have analyzed the evolution of microvoids in a ductile material in a crack tip loaded by a remote K_{I} field. HORSTEMEYER et al[5] have investigated the internal state variable rate equations in continuum framework to model void nucleation and growth in a casting Al-Si-Mg magnesium alloy. In this work, the evolution equation of the voids of casting magnesium alloys was presented, which consists of the growth and nucleation equations of the voids, and then a softening function involved the void evolution was given. The softening function was embedded to a nonclassical elastoplastic constitutive equation, a constitutive model of casting magnesium alloy was obtained, which can reflect the effect of the void evolution on the elastoplastic behavior of the material. The corresponding numerical algorithm and finite element procedure were developed and applied to the analysis of the distributions of the stress and the porosity of notched cylindrical specimens of casting magnesium alloy ZL305.

2 Void evolution equation

An important microstructural characteristic of casting magnesium alloy materials is that the materials include many microscopic voids that are formed during

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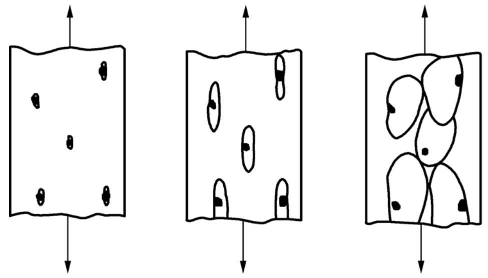

Fig.1 Schematic diagram of void nucleation, growth and coalescence

the processing course of the materials (A representative volume element of the materials can be seen in Fig.2). These microscopic voids will grow and new microvoids will nucleate when the material endures a larger outer load and an elastoplastic deformation occurs. The growth and nucleation of these microscopic voids are main modality of the evolution of the voids and may make the distinct effect on the behavior of the materials. The evolution equation of the microscopic voids, including the growth and nucleation of voids, can be written as

$$\dot{f} = \dot{f}_{\mathrm{growth}} + \dot{f}_{\mathrm{nucleation}} \tag{1}$$

The void growth equation related to void growth can be obtained based on the conception of void volume fraction[2−4] and the continuous supposition of the material matrix[2, 6−7]. The void volume fraction is defined as

$$f = (V - V_{\mathrm{M}}) / V \tag{2}$$

where V and V_{M} are the volumes of the element and matrix of the material, respectively (Fig.2). The void growth equation can be obtained from the supposition of matrix continuum and can be written as[3−4]

$$\dot{f}_{\mathrm{growth}} = V_{\mathrm{M}} \cdot \dot{V} / V^2 = (1-f)\dot{\varepsilon}_{kk}^{\mathrm{p}} \tag{3}$$

where $\dot{\varepsilon}_{kk}^{\mathrm{p}}$ is the plastic volume strain rate. It can be

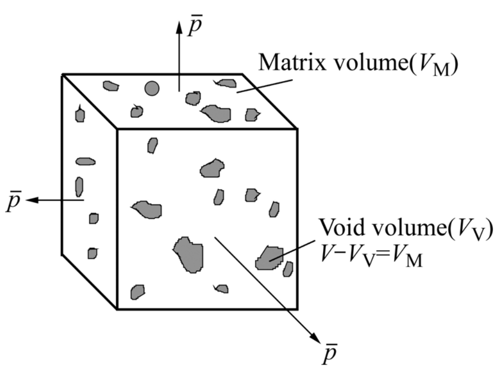

Fig.2 Schematic diagram of material element with voids

known from Eqn.(3) that the void growth rate is related to the current void volume fraction and the plastic volume strain rate.

For getting the void nucleation equation it can be assumed that the void nucleation follows a normal distribution and is controlled by intrinsic time measure[8]. From the assumption and similar to the work of CHU and LENEEDMAN[9], a void nucleation equation can be given by

$$\dot{f}_{\mathrm{nucleation}} = \frac{f_{\mathrm{N}}}{S_{\mathrm{N}}\sqrt{2\pi}} \exp[-\frac{1}{2}(\frac{Z - Z_{\mathrm{N}}}{Z_{\mathrm{N}}})^2]\dot{\zeta} \tag{4}$$

where f_{N} is the volume fraction of void nucleating part, S_{N} is the corresponding standard deviation, Z, Z_{N} and ζ are intrinsic, mean intrinsic times and intrinsic time measure, respectively[8−9].

3 Softening function and constitutive model

The evolution of voids in casting magnesium alloys will affect the elastoplastic behaviors of the materials. Generally, the evolution of voids will result in material softening, so a softening function related to the void evolution can be given:

$$w(f) = 1 - \beta f^{\gamma} \tag{5}$$

where β and γ are the material parameters, which are related to the material properties and loading condition. It is well known that most casting magnesium alloy materials do not distinctly exhibit yield, which can be described with a nonclassical elastoplastic theory[10]. The theory does not need yield idea and can well describe the elastoplastic behavior of casting magnesium alloys. The constitutive equation with mixed isotropic and kinematics hardening characteristics of the theory can be written as[10]

$$s_{ij} = s_{ij}^0(1 + k\zeta)\frac{\mathrm{d}\varepsilon_{ij}}{\mathrm{d}\zeta} + \int_0^{\zeta}\sum_{r=1}^{3} C_r \exp[-\alpha_r(\zeta - \zeta')]\frac{\partial \varepsilon_{ij}}{\partial \zeta'}\mathrm{d}\zeta' \tag{6}$$

The first term on the right hand side denotes the isotropic hardening and the second term describes the kinematics hardening, s_{ij} and s_{ij}^0 are the current and initial deviatoric stresses, respectively, C_r, α_r(r=1, 2, 3) and k are material constants, which can be determined from the σ−ε^p curve of the material under simple tensile loading[10]. Because the material parameter C_r in Eqn.(6) takes into account the material's elastoplastic behavior and relates to the void evolution, it can be supposed that the parameter will be degraded by the following relation:

$$\overline{C}_r = w(f)C_r \tag{7}$$

Replacing C_r in Eqn.(6) with $\overline{C}_r$, the nonclassical constitutive model of casting magnesium alloys involving void evolution is obtained as

$$s_{ij}=s_{ij}^0(1+k\zeta)\frac{\mathrm{d}\varepsilon_{ij}}{\mathrm{d}\zeta}+$$
$$\int_0^{\zeta}\sum_{r=1}^{3}w(f)C_r\exp[-\alpha_r(\zeta-\zeta')]\frac{\partial\varepsilon_{ij}}{\partial\zeta'}\mathrm{d}\zeta' \qquad (8)$$

It can be seen explicitly in Eqn.(8) the effect of the void evolution on the stress response. Taking tensile loading as an example, the increase in the porosity f results in a reduction in the deviatoric stress s_{ij}. If f=0, $w(f)$=1, it can be easily shown that Eqn.(8) returns to the ordinary form of nonclassical elastoplastic constitutive equation. Furthermore, the Chaboche' s constitutive law for back stress is also the special case provided $w(f)$=1 and $f(\zeta)$ is constant.

4 Experimental verifications

Based on the obtained constitutive model, the corresponding numerical algorithm and finite element procedure were developed, and they were applied to the analysis of the distributions of the stress and porosity of notched cylindrical specimens of casting magnesium alloy ZL305. The geometry of the cylindrical specimens was 200 mm in length and 12 mm in the diameter of the working section. The deep of the notch was 1 mm. The upper and right quarter of the specimen was taken for the analysis due to the symmetry of the problems. The eight-node isoparametric element with 2×2 Gaussian points was adopted. The axial displacement was prescribed at the end of the specimen with the incremental step 0.02 mm, and no radial constraint was applied to the surface of the specimen.

The experiment was carried out on an Instron 1251 servo-hydraulic universal testing system. The distribution of the voids was observed by using a photointerpreter after polishing samples. Since the void distribution is stochastic, a quantitative metallographic method was adopted. Fig.3 shows the stress distribution along the radius in the smallest section of the specimens considering void and without considering void obtained by computation. It can be seen from Fig.3 that the stress of without considering void is larger than that of considering void. It is also observed that the stress reaches the maximum in the region near the root of the notch.

Fig.4 shows the distribution of the porosity along the radius in the smallest section. It can be seen that the porosity reaches the maximum in the region near the root of the notch and the computed results agree reasonably with the experimental results.

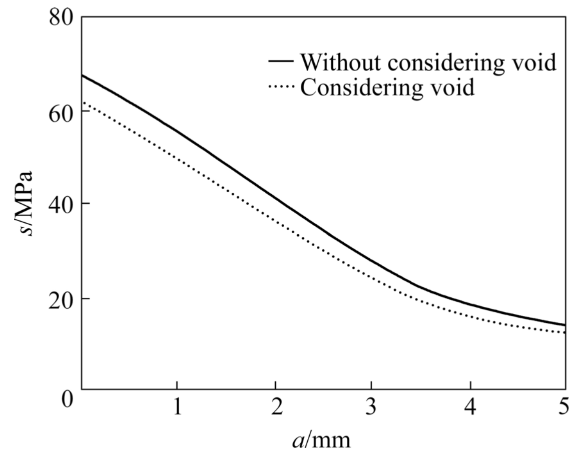

Fig.3 Stress distribution along section radius

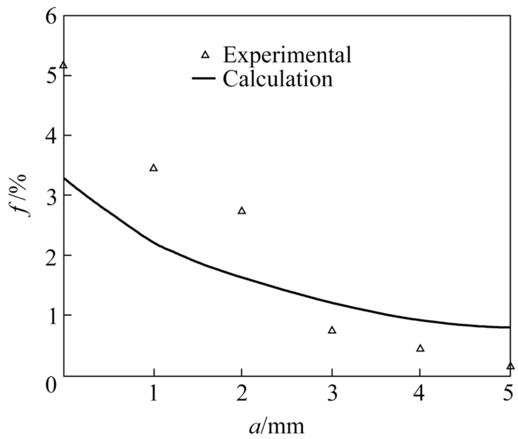

Fig.4 Porosity distribution along section radius

5 Conclusions

1) The evolution equation of void, involving both the growth and nucleation of voids, was obtained from the suppositions of the matrix continuum and the normal distribution of the void nucleation.

2) After a softening function related to the void evolution equation was embedded to a nonclassical elastoplastic constitutive equation, a constitutive model involving void evolution was obtained.

3) The corresponding numerical algorithm and the finite element approach were developed and applied to the computations of the distributions of the stress and porosity of the notched cylindrical specimens of casting magnesium alloy ZL305, the computed and experimental results showed reasonable agreement.

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