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Dynamic discrete element method and its application in rock mass engineering①

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[Abstract] Based on conventional discrete element method, the idea and calculating method of dynamic discrete element (DDEM) was proposed, and a relevant program was developed. The application of the method is presented in rock mechanics and engineering, which indicates that the method can be widely used in dynamic response and stability analysis of jointed rock mass under dynamic load.

[Key words] dynamic response; discrete element method; rock mass engineering
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1 INTRODUCTION

Rock engineering is generally stable when in normal condition. They may destabilize when under the action of intensive disturbance, for example, landslide and debris flow as well as rock structure failure and destabilization.

At present, the method and theory assessing structure stability on the ground is rather mature and scientific, and relevant design standards or criteria are available. However, there is still in lack of a better understanding of structures (such as underground excavation, steep and high slope) constructed in jointed rock mass when subjected to dynamic load. Such rock structures are often in unfavorable condition due to actions of geostress, ground water and intensive disturbance. Dynamic response in jointed rock mass has its unique feature and the dynamic stability of large rock structure directly influences the safe use and construction of the engineering. So it is important to study dynamic response and stability of jointed rock-mass when subjected to intensive disturbance.

Almost all the discrete element programs are based on Cundall's work, that is, explicit integral is used to solve the motion equation and obtain system response for each block. By introducing damp to prevent non-physical oscillation, elastic-plastic deformation in the blocks may be computed by inner finite difference grid. In such a dynamic way a quasi-static solution of a system can be obtained. So discrete element method (DEM) itself is suitable to solve dynamic problems, Cundall[1], Stephansson[2] and Bardet[3] have successfully applied discrete element method in solving dynamic problems.

2 DYNAMIC DISCRETE ELEMENT METHOD

The method Cundall used in the discrete element system takes time step as variable to calculate the velocity, displacement, contact force and stress for each block. By introducing damp, quasi-static solution of the system may thus be obtained. Such calculation method and idea are also suitable in dynamic problems. Then, by introducing dynamic condition and relevant calculation, it is easy to apply discrete element principle to dynamic problem.

2.1 Dynamic relationship between force and displacement

In discrete element, the interaction among blocks is shown in Fig.1, composed by shear and normal spring as well as dashpot. At the contacting point C, the relative velocity u_n and u_s may be obtained according to the rigid displacement and rotational velocity.

Suppose the present physical time is t, then at time interval Δt, the displacement at contact point is derived as

$$\Delta u_n = u_n \Delta t \tag{1}$$

$$\Delta u_s = u_s \Delta t \tag{2}$$

The increment ΔF_n and ΔF_s of the contact force F at the relevant point are

$$\Delta F_n = \Delta u_n k_n \tag{3}$$

$$\Delta F_s = \Delta u_s k_s \tag{4}$$

where k_n and k_s are stiffness of shear spring and normal spring respectively, illustrating the elastic

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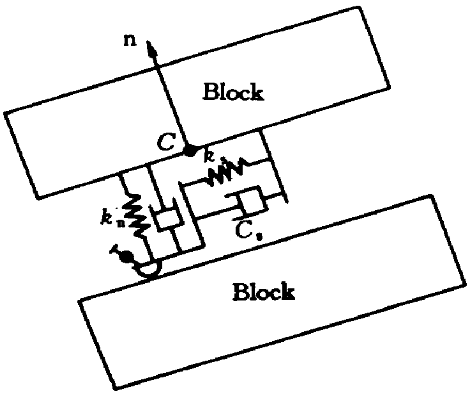

Fig. 1 Dynamic contact between blocks

behavior of the block.

It is thus easy to get the force $F(t + \Delta t)$ at contact point P at time $t + \Delta t$:

$$F_n(t + \Delta t) = F_n(t) + \Delta F_n(t) \tag{5}$$

$$F_s(t + \Delta t) = F_s(t) + \Delta F_s(t) \tag{6}$$

Block contact model is same as Cundall's. Tensile stress is not allowed at block points, i.e. $F_n > 0$. When normal force decreases to zero, separation or opening will occur when normal force decreases to zero.

Shear force F_s is nonlinear and related with stress path and loading/unloading history. Shear slip may be judged by Mohr-Coulomb criteria, i.e.

$$F_s \leqslant f_n \mathrm{tg}\,\varphi + C \tag{7}$$

where C and φ are cohesion and fictional angle at the contact point.

The viscous force F_d is characterized by the normal and shear dashpot. According to Newton's motion law, F_d is dependent on the relative velocity at the contact point u and damping feature:

$$F_{dn} = C_n u_n \tag{8}$$

$$F_{ds} = C_s u_s \tag{9}$$

Damp coefficients C_n and C_s in the above formula may be derived from stiffness damp ratio coefficient β:

$$C_n = \beta k_n \tag{10}$$

$$C_s = \beta k_s \tag{11}$$

The contact force and viscous force at contact point are thus obtained.

2.2 Dynamic motion equation

According to Newton's motion law, the motion equations of block can be expressed as

$$m\ddot{u}_x + \alpha_m m\dot{u}_x = \sum_{i=1}^{n_c}(F_x^i + D_x^i) \tag{12}$$

$$m\ddot{u}_y + \alpha_m m\dot{u}_y = \sum_{i=1}^{n_c}(F_y^i + D_y^i) \tag{13}$$

$$I\ddot{\theta} + \alpha_m I\dot{\theta} = \sum_{i=1}^{n_c} M_i \tag{14}$$

where n_c is contact number of the adjacent blocks; m, I are mass inertia moments of block; u_x, $\dot{u}_x$, $\ddot{u}_x$ are horizontal displacement, velocity and acceleration of block, respectively; u_y, $\dot{u}_y$, $\ddot{u}_y$ are vertical displacement, velocity and acceleration of block, respectively; θ_y, $\dot{\theta}_y$, $\ddot{\theta}_y$, are rotational angle, angular velocity and acceleration of block, respectively; α_m is mass damp ratio coefficient; g is gravitational acceleration; F_x^i, F_y^i are horizontal and vertical component of contact force at contact i, respectively; F_{dx}^i, F_{dy}^i are horizontal and vertical component of viscous force at contact i; M^i is moment of contact force and viscous force to centroid at contact i.

2.3 Solving motion equation

The above three motion equations are nonlinear differential equations of time t, which can only be solved numerically. Explicit Newmark approach may be adopted in solving the equations. All the equations can be written in the following form:

$$\ddot{x} + \alpha x = \frac{1}{m}F(x, \dot{x}) \tag{15}$$

where x, $\dot{x}$, $\ddot{x}$ are generalized displacement, velocity and acceleration, respectively; m is generalized mass; α is generalized damp coefficient; $F(x, \dot{x})$ is nonlinear force as the function of x and $\dot{x}$.

Let a_n, V_n and F_n are acceleration, velocity and force at time t_n. The initial time $t_0 = 0$, time step is Δt, then $t_n = t_0 + n\Delta t$, Eqn.(15) can be rewritten as

$$a_n + \alpha V_n = \frac{1}{m}F_n \tag{16}$$

In this way explicit Newmark integral method can be applied with the following process:

Assume the velocity at time $t_{m+1/2} = (n + 1/2)\cdot\Delta t$ and $t_{n-1/2} = (n - 1/2)\Delta t$ are as follows respectively

$$V_{n+\frac{1}{2}} = V_n + \frac{1}{2}a_n\Delta t \tag{17}$$

$$V_n = V_{n-\frac{1}{2}} + \frac{1}{2}a_n\Delta t \tag{18}$$

Then the acceleration at time t_n can be derived from Eqn.(17) and Eqn.(18):

$$a_n = \frac{V_{n+\frac{1}{2}} - V_{n-\frac{1}{2}}}{\Delta t} \tag{19}$$

Similarly, acceleration at time t_n:

$$V_n = \frac{V_{n+\frac{1}{2}} - V_{n-\frac{1}{2}}}{2} \tag{20}$$

The velocity at the middle time step $t_{n+\frac{1}{2}}$ can be derived from Eqns.(16),(19) and (20):

$$V_{n+\frac{1}{2}} = \frac{1}{1 + \frac{\alpha\Delta t}{2}}\cdot\left[\left(1 - \frac{1}{2}\alpha\Delta t\right)V_{n-\frac{1}{2}} + \frac{1}{m}F_n\Delta t\right] \tag{21}$$

Then block position at time t_{n+1} is determined as

$$d_{n+1} = d_n + V_{n+\frac{1}{2}} \Delta t \quad (22)$$

The above-mentioned approach is repeated for each block and Eqn.(15) may be solved in such a way, among which the velocity at time $t_{n+\frac{1}{2}}$ is an important middle variable by which the new block position and present acceleration can be easily derived.

Three main loops are conducted in turn based on the above-mentioned explicit integral approach, i.e. all the blocks → all the contacts → all the voids among blocks. Before introducing dynamic load, such cyclic computing is repeated.

It should be noted the chief disadvantage of the numerical approach is its conditional convergence. Only when the time step is smaller than a limited value can a steady and convergent solution be obtained, which physically means that a disturbance at a point of a system only influences the adjacent element within very shot time interval, and will not contribute to the other points of the system.

For such a nonlinear problem, Cundall gives a formula estimating limited time step:

$$\Delta t \leqslant \frac{1}{10} T_{\min} \quad (23)$$

$$T_{\min} = 2\pi \min_{1 \leqslant i \leqslant n} \sqrt{\frac{m_i}{k_i}} \quad (24)$$

where $T_{\min}$ is the minimum natural period of any element; m_i is block mass, k_i is block stiffness; and n is the number of blocks.

3 MODELLING OF BLOCK DEFORMATION

When under condition of low stress level, deformation and failure of jointed rock mass are dominantly controlled by joints and thus block deformation itself may be ignored in general. However when in the condition of high stress level(in the case of large buried depth or high tectonic stress), especially with the acting of dynamic load, the deformation of block should not be ignored.

Cundall suggested that the deformation of joints be modelled by discontinuous element, whereas the elastic and plastic deformation modelled by continuum model. This is the deformable block model that can be divided into fully deformable model and simply deformable model[4].

Fully deformable model is developed on the basis of rigid block model, the block is divided by finite difference grid and triangle element with constant strain. Each block is characterized with liner deformation in order to keep block edge straight during deformation.

The displacement and strain of the triangular element is solved by adopting Gauss's theory and Wilkins' finite difference typical of large deformation. The solving process is the same as that in FLAC program[5].

For dynamic problem, the deformable model above is also applicable. So the main difference between Cundall's conventional discrete element and dynamic DEM(DDEM) lies in the contact relationship and solution of the nonlinear differential equations. Therefore many techniques of conventional DEM may be used in dynamic problems.

4 COUPLING METHOD OF DYNAMIC DISCRETE ELEMENT AND DDEM CODE

Coupling of DDEM with boundary element is similar with Lorig's approach except coupling with dynamic boundary element. Nardini and Brebbia[6] have given the fundamental equation of dynamic boundary element, adopting Betti-Reyleigh's dynamic equivalent law and solving the equation numerically with boundary discretization. Based on the idea and algorithm of DDEM mentioned above, a generalized DDEM code is developed by modifying the conventional DEM, which consists of preprocessing block, analyzer and post-processing block[7~9]. The preprocessing block is responsible for block system generation, boundary and material property processing, which is similar to the conventional DEM. Moreover, the block is also capable of dynamic load processing including frequency and spectrum analysis of seismic wave and explosive impact wave. The processed results can be directly inputted to the DDEM model. When dynamic load is not considered in the model, the DDEM code becomes a standard discrete element code.

When DDEM code is used to dynamic problem, the processing steps should be: 1) generating joint pattern based on field investigation; 2) restricting the boundary; 3) assigning material property; 4) making excavations; 5) introducing dynamic disturbance by applying vertical and(or) horizontal stress wave to the model from the boundaries. The whole model is thus like lying on a vibrating plate, which is consistent with the actual condition in rock mass. Acceleration, velocity and force are transmitted to the whole blocky system through springs, dashpots and frictional units with time. DDEM calculates the acceleration, velocity, displacement, force and stress as the function of time for each block. In such a way, the system dynamic response can be modelled for jointed rock mass. The DDEM code has been verified and used extensively in Refs.[7~10].

5 DYNAMIC RESPONSE ANALYSIS OF UNDERGROUND EXCAVATION UNDER SEISMIC LOAD

By utilizing the above-mentioned approach and program, a deeply buried underground tunnel is analyzed in dynamic condition.

5.1 Model description

The rock mass lies in uniform stress field with the same vertical and horizontal stress 24 MPa equivalent to a depth of 800 meters. For the model is in infinite medium, all the boundaries of the model are set as non-reflecting or viscous. There exist two dip reversed joints with dip angle of 45° and 15° respectively, and joint spacing 5 m in the model. The tunnel is excavated in the shape of straight wall and half circular arch, the sidewall is 7.5 m high and 10 m wide, the radius of the half circular arch is 10 m. The intact rock block behaves elastically with the following properties: density of 3.0×10^3 kg/m^3; elastic module of 75×10^3 MPa and Poisson's ratio of 0.1. The joints agrees with Mohr-Coulomb criteria, stiffness $k_s = k_n = 2 \times 10^4$ MPa/m; frictional angle 30°; cohesion $C = 0$.

Seismic effect is represented by a vertically applied sinusoidal stress wave at the top of the model, the seismic stress is superimposing the existed stress field. The effective stress is half of the amplitude of the stress wave (6.25 MPa). The frequency of the sinusoidal stress wave is $0.01\ s^{-1}$, approximately equivalent to a earthquake with intensity of VI degrees.

5.2 Results analysis

1) Before the chamber is excavated, uniform compressive stress is applied to the model in both vertical and horizontal direction in order to form the initial stress field. After the chamber is excavated, the whole system reaches equilibrium by sufficient iteration. Under the action of initial stress field, displacement occurs around the excavation where main displacement appears at the crown with maximum elastic displacement of 7.85 mm. At the two bottom corners there exist concentrating stress about twice of the initial stress. Tensile stress also arises in small region, however for both the magnitude and extent are limited, the surrounding rock and the joints will not fail. So the excavation is stable in the initial state.

2) Stability of chamber under seismic load

The model is disturbed by an earthquake with intensity VI degrees. By the seismic action, tensile stress zone emerges around the excavation with maximum magnitude of 6.3 MPa, and maximum shear stress as high as 25 MPa sufficient to make some blocks slip along joints.

The vertical displacement at point P (center point of the crown) increases with the physical time (see Fig.2), indicating that the point moves considerably. The displacements at other tracked points also have such features, demonstrating that the chamber is unstable under such a dynamic load and will fail progressively if there are no proper support and reinforcement.

The displacement vector of the surrounding rock is shown in Fig.3. The maximum displacement occurs at point P with magnitude of 24.8 mm. Some blocks cut by the unfavorable joints already slip along the joints. For the total effective stress is not so large, the failure form is progressively creeping but it may be ruptured finally.

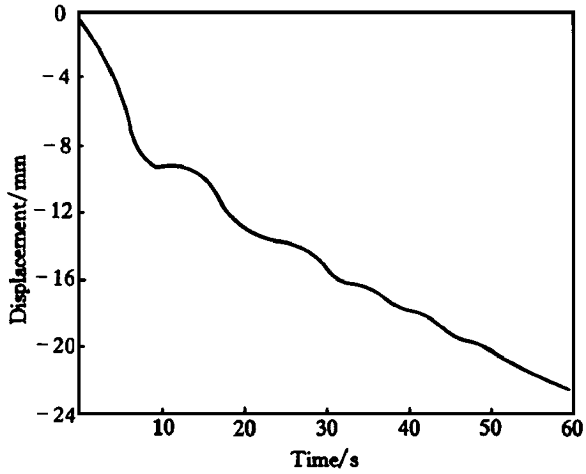

Fig.2 Variation of displacement at point P

Fig.3 Displacement vector diagram

6 CONCLUSIONS

From the analysis and computing results, it is shown that dynamic discrete element method is suitable for dynamic analysis of jointed rock mass. The results can reasonably interpret the dynamic response and failure mechanism of underground excavation in jointed rock mass. The dynamic response of underground excavation is different from seismic. The amplitude, duration and frequency have remarkable effects on rock mass failure and stability. When the magnitude is small, the disturbance has less influence on chamber stability due to small effective stress acting on the model. However when the magnitude is large enough, it will directly control the stability or failure pattern of an underground excavation. In the example above-mentioned, the characteristics of the rock mass is controlled by two sets of joints, therefore the failure of the chamber is dominantly the failure of the joints, i.e. bed separation and shear slipping along joints or rotating in some intensive failure case. DDEM provides an effective way in dynamic analysis for jointed rock mass, and it can be used in dynamic response and stability analysis of underground chamber, slope and mining stope. When coupled with boundary element, DDEM will has a wider application in rock mass dynamics.

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