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Calculating method of circle radius using genetic algorithms①

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Abstract: Some improved methods of the traditional genetic algorithms were proposed and their application in calculating circle radius was discussed. Simulation shows that real number encoding, simulated annealing and saving the best individual into the next generation can effectively improve the accuracy and speed up the searching for global optimization. It is also shown that improved genetic algorithms can effectively calculate the circle radius and be realized easily using computer.

Key words: genetic algorithms; circle radius; calculating method; error analysis **Document code:** A

1 INTRODUCTION

The measuring data processing of circle radius must reflect the real status of the measured outline. At present, the methods of data processing of circle radius only calculate circle radius. But the measured points of real outline perhaps do not lie on a circumference, so merely calculating circle radius can not show the real status of the measured outline. If the minimal area theorem is adopted to calculate the circle radius and circle error, the real status of measured outline can be obtained. Circle radius of measured outline is assessed based on the minimal area theorem, whose essentials are searching the minimal radius odds of two concentric circles containing measured outline. Concentric circles containing measured outline have infinite pairs, but there is only one pair whose radius odds is minimal. So this assessment method is the optimization problem, the following is its mathematical mode description,

$$e = \min | r_2 - r_1 | \tag{1}$$

$$r = (r_1 + r_2)/2 \tag{2}$$

where r_2, r_1 are radii of concentric circles containing measured outline; e is their radius odds; r is the theoretical radius of the measured outline, whose real radius is $r \pm e/2$. If genetic algorithms are adopted to process this problem, the real radius of measured outline can be obtained and the calculation can be easily realized by computer.

2 GENETIC ALGORITHMS AND ITS IMPROVEMENT

The GA (Genetic Algorithms), a global search method based on natural selection and genetics, was first proposed by Holland[1]. GA searches in the space of problem solution and enables population of feasible solution evolving into global optimization solution. GA is different from traditional optimization methods. GA merely uses fitness function according to object function to work; it does not need other qualification or aid information, and uses random conversion role to work. Using the GA to solve problem, we must encode for the problem and form individual, different individuals form population. Every individual has a fitness value according to the fitness function. Then we enable population to evolve a new generation population. Evolving like this until satisfactory solution is obtained[2~5]. But the traditional GA has some defects[6,7] as follows: precocity and stagnancy, blandness of individuals mutation bringing on losing of genetics, the precision limited by length of binary encode. For overcoming the defects of traditional GA, improving the convergence speed and performance of searching for optimization solution, some improvements of the GA have been proposed in this paper.

Following are some improving contents.

1) Adopting real number encoding can enable the search optimization area to the whole area of the feasible solution, thus, the precision of search optimization is not limited by the length of encoding.

2) Draw out the fitness function using simulated annealing theory for reference:

$$f_i^t = \frac{e^{f_i^t/T}}{\sum_{j=1}^{n} f_j^t}, \qquad T = T_0 \cdot 0.99^t \tag{3}$$

where T is temperature, T_0 is initial temperature, and t is number of evolve generation.

3) Saving the best individual into the next generation can effectively avoid precocity and stagnancy.

3 CALCULATING CIRCLE RADIUS BY IMPROVING GA

Assuming the measured points of real outline is $p\{(x_j, y_j) \mid j = 1, 2, \cdots, m\}$, and the distribution

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of measured points exceeds the half of circumference, the maximum and minimal values of the coordinate of measured points can be obtained as

$x_{max} = \max\{x_j\}$, $x_{min} = \min\{x_j\}$, $y_{max} = \max\{y_j\}$, $y_{min} = \min\{y_j\}$, where $j = 1, 2, \cdots, m$. Thus the central area of concentric circles containing measured outline is $x_{min} \leqslant x_{co} \leqslant x_{max}$, $y_{min} \leqslant y_{co} \leqslant y_{max}$.

Exchange (x_{co}, y_{co}) to (x_c, y_c), and let $0 \leqslant x_c \leqslant 1$, $0 \leqslant y_c \leqslant 1$, then

$$x_{co} = x_{min} + x_c \cdot (x_{max} - x_{min}) \tag{4}$$

$$y_{co} = y_{min} + y_c \cdot (y_{max} - y_{min}) \tag{5}$$

After such process, we can adopt improved GA to calculate the circle radius.

STEP 1 Determination of the number of generation t_{max}, cross probability p_c and mutation probability p_m

In this paper, their values are selected as: $t_{max} = 1\,000$, $p_c = 0.8$, and $p_m = 0.02$.

STEP 2 Initialization of population

Adopting real number encoding makes searching area of (x_c, y_c) to the whole area of feasible solution, thus confirming n individuals in area of feasible solution to form initial population:

$$A(t) = \{x^t_{ci}, y^t_{ci} \mid i = 1, 2, \cdots, m\}$$

STEP 3 Calculating the fitness of every individual

Object function $g(\cdot)$ is the radius odds of concentric circles containing measured outline,

$$g(x^t_{ci}, y^t_{ci}) = r^t_{maxi} - r^t_{mini} \tag{6}$$

where $i = 1, 2, \cdots, n$.

r^t_{maxi} and r^t_{mini} are the maximum and minimum of distance from measured points $p\{(x_j, y_j) \mid j = 1, 2, \cdots, m\}$ to the concentric circles centric $(x_{min} + x^t_{ci} \cdot (x_{max} - x_{min}), y_{min} + y^t_{ci} \cdot (y_{max} - y_{min}))$ of individual (x^t_{ci}, y^t_{ci}). But the value direction of the object function is different from the fitness function, the larger the value of the object function, the less the fitness of individual. So we must build up the relation between the object function $g(\cdot)$ and the fitness function $f(\cdot)$:

$$f(x^t_{ci}, y^t_{ci}) = g^t_{max} - g(x^t_{ci}, y^t_{ci}) \tag{7}$$

where g^t_{max} is the maximum of object function $g(\cdot)$ of current generation population.

For avoiding the problems of precocity and stagnancy, draw out the fitness function using simulated annealing theory:

$$\left.\begin{aligned} f(x^t_{ci}, y^t_{ci}) &= \frac{e^{f(x^t_{ci}, y^t_{ci})/T}}{\sum_{j=1}^{n} f(x^t_{cj}, y^t_{cj})} \\ T &= T_0 \cdot 0.99^t \end{aligned}\right\} \tag{8}$$

STEP 4 Reproduction operation

According to the fitness of individuals, their proportions in the pool of reproduction are determined:

$$p^t_i = f(x^t_{ci}, y^t_{ci}) / \sum_{j=1}^{n} f(x^t_{cj}, y^t_{cj}) \tag{9}$$

After reproduction, an individual (x^t_{ci}, y^t_{ci}) copy number may be

$$n^t_i = p^t_i \cdot n \tag{10}$$

If $n^t_i \geqslant 1$, the number of individual (x^t_{ci}, y^t_{ci}) in the pool of reproduction is integer of n^t_i. If $n^t_i < 1$, according to the taxis of fitness , the number of individual (x^t_{ci}, y^t_{ci}) in the pool of reproduction is 1 until the number of individuals is n.

After reproduction, n new individuals in the reproduction pool are formed:

$$A_1(t) = \{x^t_{c1i}, y^t_{c1i}) \mid i = 1, 2, \cdots, n\} \tag{11}$$

STEP 5 Cross operation

If decimal digit is l, then the encoding length of individual (x^t_{ci}, y^t_{ci}) is $2l$. The process of cross operation is as follows: choosing two individuals independently with the same probability, randomly choosing cross positions among $1 \sim 2l - 1$, then exchanging the code string of two individuals, generating two new individuals. Fig. 1 shows the process of cross operation.

Before cross

String 1 (0.1537 | 86, 0.127563)
String 2 (0.1349 | 35, 0.579324)

Position of cross

Cross operation → After cross

(0.153735, 0.579324)
New string 1

→ (0.134986, 0.127564)
New string 2

Fig. 1 Process of cross operation

Repeating this process until transitional population is formed:

$$A_2(t) = \{(x^t_{c2i}, y^t_{c2i}) \mid i = 1, 2, \cdots, n\} \tag{12}$$

STEP 6 Mutation operation

According to probability mutation p_m, changing every genetic (i. e. the value of decimal position) of every individual. The changing method is as follows: the value v of chosed genetic is replaced by integer among 0 to 9 (except v). After reproduction, cross and mutation operation, the new generation population is formed:

$$A(t+1) = \{(x^{t+1}_{ci}, y^{t+1}_{ci}) \mid i = 1, 2, \cdots, n\}$$

The value of object function and fitness of every individual in the new generation population are calculated as follows:

$$g(x^{t+1}_{ci}, y^{t+1}_{ci}) = r^{t+1}_{maxi} - r^{t+1}_{mini} \tag{13}$$

$$f(x^{t+1}_{ci}, y^{t+1}_{ci}) = g^{t+1}_{maxi} - g(x^{t+1}_{ci}, y^{t+1}_{ci}) \tag{14}$$

where $i = 1, 2, \cdots, n$; r^t_{maxi} and r^t_{mini} are the maximum and minimum of distance from measured points $p\{(x_j, y_j) \mid j = 1, 2, \cdots, m\}$ to the concentric circles centric $(x_{min} + x^{t+1}_{ci} \cdot (x_{max} - x_{min}), y_{min} + y^{t+1}_{ci} \cdot (y_{max} - y_{min}))$ of individual $(x^{t+1}_{ci}, y^{t+1}_{ci})$; g^{t+1}_{max} is the maximum of object function in new generation population.

Drawing out the fitness function using simulated annealing theory for reference:

$$\left.\begin{aligned} f(x_{ci}^{t+1}, y_{ci}^{t+1}) &= \frac{e^{f(x_{ci}^{t+1}, y_{ci}^{t+1})/T}}{\sum_{j=1}^{n} f(x_{ci}^{t+1}, y_{ci}^{t+1})} \\ T &= T_0 \cdot 0.99^{t+1} \end{aligned}\right\} \quad (15)$$

Saving the best individual into the next generation and replacing the worst individual, then finding out value of the object function corresponding to the individual $(x_{ck}^{t+1}, y_{ck}^{t+1})$ of maximum fitness:

$$e = g(x_{ck}^{t+1}, y_{ck}^{t+1}) \quad (16)$$

At last searching out the maximum and minimum distances $r_{\max k}^{t+1}$ and $r_{\min k}^{t+1}$ from the measured points $p\{(x_j, y_j) \mid j = 1, 2, \cdots, m\}$ to the individual $(x_{ck}^{t+1}, y_{ck}^{t+1})$.

Here we can obtain the radius of the measured outline.

$$r^{t+1} = \left(\frac{r_{\max k}^{t+1} + r_{\min k}^{t+1}}{2}\right)^{+e/2}_{-e/2} \quad (17)$$

STEP 7 If $t < t_{\max}$ then go to STEP 4, else stopping process, here r_{t+1} is the global optimization solution, i.e. the calculating value of measured radius satisfies the minimal area theorem.

4 SIMULATION EXAMPLE

We have adopted the improved GA to largely imitatively calculate circle radius of measured outline. Table 1 is an example of simulation. The calculating result is $r = 50.020309^{+0.230597}_{-0.230597}$ mm. Simulation result indicates that adopting the improved GA to calculate circle radius of the measured outline can obtain global optimization solution, which accords with real situation of measured outline.

5 CONCLUSIONS

Adopting the improved GA to calculate circle radius of the measured outline, the precision is very

Table 1 Data of measured points (mm)

No.	x	y	No.	x	y
1	50.5000	0	7	-50.0080	0.0013
2	43.3099	25.0049	8	-43.3278	-25.0148
3	25.0000	43.3014	9	-25.0001	-43.3012
4	0.0006	50.0000	10	-0.0019	-50.0000
5	-25.0093	43.3185	11	25.0108	-43.3360
6	-43.3015	25.0006	12	43.3019	-25.0002

high and can obtain any given value. It accords with the minimal area theorem and theoretically infinitely approaches the real value. This method is simple and fast, and can be easily realized with computer, thus it is very fit for data processing of 3D measurement tool[8].

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