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Symmetry and conservation laws for optimum design of open pit mines①

SHI Zhong-min(石忠民)

School of Resource and Environment Engineering,

Wuhan University of Technology, Wuhan 430070, P. R. China

Abstract: The classical physics theory respectively obeys the three famous conservation laws referred to as charge conjugation, parity and time reverse, and the open pit block model is equal to a Newtonian mechanics system. Consequently, there would exist some correspondent symmetry principles and conservation laws within the 3D fixed block model of the deposit and the theory for the optimum design of the open pit mine. Reversing a series of relevant fundamental concepts, several conservation laws, which the theory for the optimum design of open pit mines should obey, as block weight conjugation, block model parity and combined symmetry of the both, were expounded. From the symmetry principle, the theoretic significance for a series of the current optimum techniques was discussed and explained, and a kind of conjugate heuristics which can check the error of itself was presented and demonstrated. Thus it is shown that the symmetry principle lays the foundations and opens up the prospects for the further research with mine design and scheduling problem.

Key words: open pit mine; optimum design; block model; symmetry; conservation **Document code: A**

1 INTRODUCTION

It is well known that all of the classical physics theories and most of the modern ones respectively obey the three symmetry referred to as C (charge), P (parity) and T (time), and some combined symmetry such as CP and CPT[1,2]. Lerchs and Grossmann have used a free mechanical system to illustrate the optimum design for the ultimate pit contour of the block model[3]. If introducing some concepts of physics such as mass and acceleration into the block model of the deposit, the evolutive process of the mechanical system can be still used to analogize the form mode of the optimum intermediate pit contours (nested pits). To sum up, it is deduced that the pitting model of the deposit, which is equivalent to the free mechanics system, has some specified symmetry. On the other hand, the theory for the optimum design of the open pit mines, similar to the theory of Newtonian mechanics, would obey some correspondent conservation laws. The same with the case of the current optimization techniques, in all symmetry principles and conservation laws, the predominant type of ore body model used is the regular 3D (or 2D) fixed block model referred by Kim[4]. Consequently, the subsequent discussion on the symmetry and the conservation laws is confined to this type of model. Without loss in generality, the 2D block model is used in the concrete.

2 FUNDAMENTAL CONCEPTS AND REVERSE

The theory for optimum design of the open pit mines includes a correspondent concept system[5], within the system each fundamental concept can be homologised with a reverse concept.

The 2D fixed block model as an example is assumed to be the one with $m(=5)$ layers and $n(=9)$ columns, as shown in Fig. 1.

$i \downarrow$ \ $j \rightarrow$	1	2	3	4	5	6	7	8	9
1	-1	-1	-2	-1	-1	-1	-1	-1	0
2		-1	-1	-2	1	1.8	4.6	-2	
3			1	5	3	-1	1.6		
4				0.5	0.8	1.1			
5					-1				

Fig. 1 2D fixed block model or original system (B^o, A^o, W^o)

2.1 Block and its reverse block

A basic unit of the ore body model is referred to as a block, which is marked by b_{ij} (see the grid shown in Fig. 1). Each block b_{ij} is assigned a real number w_{ij} as its weight (see the number in the grid shown in Fig. 1). The blocks can be classified as either real blocks (if $w_{ij} \neq 0$) and imaginary blocks (if $w_{ij} = 0$), the real blocks can be further classified as positive blocks (if $w_{ij} > 0$) and negative blocks (if $w_{ij} < 0$).

In order to facilitate distinguishing between a block and its reverse, the original block and its weight are respectively marked by b^o_{ij} and w^o_{ij}. Let $w^r_{ij} = - w^o_{ij}$, then a block b^r_{ij} with weight w^r_{ij} is referred to as the reverse block of the b^o_{ij}. Of course, the reverse block of a positive block is the negative

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block which still is in the same place, the reverse is true. The reverse block of an imaginary block is the block itself. For example, if the weights of all the blocks shown in Fig. 1 are multiplied by -1, then an ore body model composed of the reverse blocks shown in Fig. 2 will be obtained. Let $B^o = \{b^o_{ij}\}$ and $B^r = \{b^r_{ij}\}$, correspondingly, $W^o = \{w^o_{ij}\}$ and $W^r = \{w^r_{ij}\}$.

$i\downarrow$ \ $j\rightarrow$	1	2	3	4	5	6	7	8	9
1	1	1	2	1	1	1	1	1	0
2		1	1	2	-1	-1.8	-4.6	2	
3			-1	-5	-3	1	-1.6		
4				-0.5	-0.8	-1.1			
5					1				

Fig. 2 Ore body model composed of reverse blocks or system(B^r, A^o, W^r)

2.2 Supporting set and reverse one

Assuming Γ^o_{ij} is the supporting set of block b^o_{ij}, then the reverse supporting set of b^o_{ij} is defined as $\Gamma^r_{ij} = \{b_{rs} | b_{ij} \in \Gamma^o_{rs}\}$, Γ^r_{ij} represents a set of block which are directly covered by b^o_{ij}. In other words, if the mining of block b_{rs} is dependent upon the removal of block b_{ij}, then $b_{ij} \in \Gamma^o_{rs}$, and $b_{rs} \in \Gamma^r_{ij}$. For the simplest case, $\Gamma^o_{ij} = \{b^o_{i-1,k} | i \neq 1, k = j-1, i, j+1\}$, and $\Gamma^r_{ij} = \{b^o_{i+1,k} | i \neq m, k = j-1, j, j+1\}$. Let $A^o = \{\Gamma^o_{ij}\}$ and $A^r = \{\Gamma^r_{ij}\}$.

2.3 Pitting model of deposit and its image

The system (B^o, A^o, W^o) is referred to as a pitting model of the deposit. Let $\overline{b^o_{ij}} = b^o_{m+1-i,n+1-j}$, and $\overline{B^o} = \{\overline{b^o_{ij}}\}$, correspondingly, $\overline{w^o_{ij}} = w^o_{m+1-i,n+1-j}$ and $\overline{W^o} = \{\overline{w^o_{ij}}\}$. The system $(\overline{B^o}, \overline{A^r}, \overline{W^o})$ is referred to as an image of system (B^o, A^o, W^o). In other words, system (B^o, A^o, W^o) can be transformed into its image by rotating it for 180°, a supporting set in the image is substantially a reverse one of the original system. The images of the pitting models shown in Fig. 1 and Fig. 2 are respectively shown in Fig. 3 and Fig. 4.

2.4 Reverse closure and its weight

Within system (B^o, A^o, W^o), a reverse closure is a set of blocks C^r_l such that if a block b^o_{ij} belongs to C^r_l then its reverse supporting set Γ^r_{ij} must also belong to C^r_l. The weight w^r_l of a reverse closure C^r_l is the sum of the weight of the total blocks belonging to C^r_l, namely $w^r_l = \sum_{b^o_{ij} \in C^r_l} w^o_{ij}$.

$\bar{i}\downarrow$ \ $\bar{j}\rightarrow$	1	2	3	4	5	6	7	8	9
1					-1				
2				1.1	0.8	0.5			
3			1.6	-1	3	5	1		
4		-2	4.6	1.8	1	-2	-1	-1	
5	0	-1	-1	-1	-1	-1	-2	-1	-1

Fig. 3 Image of system $(\overline{B^o}, \overline{A^o}, \overline{W^o})$ or system (B^o, A^r, W^o)

$\bar{i}\downarrow$ \ $\bar{j}\rightarrow$	1	2	3	4	5	6	7	8	9
1					1				
2				-1.1	-0.8	-0.5			
3			-1.6	1	-3	-5	-1		
4		2	-4.6	-1.8	-1	2	1	1	
5	0	1	1	1	1	1	2	1	1

Fig. 4 Image of system $(\overline{B^r}, \overline{A^o}, \overline{W^r})$ or system (B^r, A^r, W^r)

2.5 Strong reverse closure

There exists a reverse closure C^r_l whose weight $w^r_l < 0$, for any $C^r_s \subset C^r_l$, if $w^r_s \geqslant w^r_l$, then C^r_l is referred to as a strong reverse closure.

2.6 Maximum reverse closure

Within system (B^o, A^o, W^o), a maximum reverse closure is a reverse closure with minimum weight or, in other words, with negative weight whose absolute value is maximum, namely, the set of all the blocks is not mined by open cast.

Similar to the reverse of the closure and its deriv concepts, the reverse of the increment-closure and its deriv concepts may be defined, it is not necessary to enumerate them one by one here but to point out that the images of closures or increment-closures are reverse ones and the reverse is true.

3 SYMMETRY AND CONSERVATION LAWS

In physics, symmetry generally means in form some specific nature of objects studied such that there is something permanent on putting the objects in certain operation. Each and every symmetry will directly result in conservation. The object studied for the optimum design of the open pit mines is system (B^o, A^o, W^o), within this system, the nature of something can be generalized by a set of fundamental theorems[5].

The symmetry of system (B^o, A^o, W^o) and the conservation laws of the fundamental theorems may be described as follows.

Conservation law 1 (block weight symmetry): For the original blocks $\{b^o_{ij}\}$ and its reverse blocks $\{b^r_{ij}\}$ the fundamental theorems are the same.

Conservation law 2 (block model parity): To the pitting model of a deposit (B^o, A^o, W^o) and its image (B^o, $\overline{A^r}$, W^o) the fundamental theorems are constant.

Conservation law 3 (combined symmetry of block weight and model): If each block b^o_{ij} is replaced by its reverse block b^r_{ij}, for the image (B^r, $\overline{A^r}$, W^r) of the model (B^r, A^o, W^r) which consists of the reverse blocks and the original pitting model (B^o, A^o, W^o) the fundamental theorems are all suitable.

The three conservation laws above are hereafter referred to as simply W symmetry, M symmetry, and WM symmetry respectively. The maximum closure and the maximum reverse closure, are mutually replaced in W symmetry, all put upside down in M symmetry, and mutually transformed in WM symmetry. Hence, all of the optimizing techniques for the maximum closure are universally applicable to the maximum reverse closure.

From the three conservation laws, the following inferences of the fundamental theorems may be obtained.

Inference 1: The combination set of a strong reverse closure and its strong reverse increment-closure is still a strong reverse closure.

Inference 2: Within system (B^o, A^o, W^o) the maximum reverse closure is a strong reverse closure.

Inference 3: The sufficient and necessary condition for a strong reverse closure being a maximum one is that there does not exist any strong reverse increment-closure of the strong reverse closure.

On the fundamental theorems and their inferences the following theorem is obtained.

Theorem (maximum division theorem): Within system (B^o, A^o, W^o), there exists a division $C^o_{\max}$ and $C^r_{\max}$, if $C^o_{\max}$ is the maximum closure, then $C^r_{\max}$ must be the maximum reverse closure; the reverse is true.

4 DERIV TECHNIQUES

The symmetry for the block model of the deposit becomes rarely known, but some current optimizing techniques have related to this principle. On the other side, the symmetry principle will certainly derive a great many of new applied techniques.

Within the various dynamic programming algorithms for optimum design of the open pit mines, such as Wilke and Wright's dynamic cone method[6], both the supporting set and the reverse supporting set must be employed.

Gershon's heuristic method for optimum production scheduling has utilised the concept of a reverse closure to calculate a block's positional weight, thereby further determining when that block should be mined[7].

The "Pre-Pass" approach described by Chen[8] is essentially a method to determine a subset of the maximum closure or the maximum reverse closure of the pitting model in advance.

Within numerical calculuses, a heuristic method is normally accompanied by a method of the error estimate. By WM symmetry, a kind of conjugate heuristics can be proposed, which can be used not only as a method of calculating error but also as a pre-pass approach. The heuristics is divided into two steps as follows.

Step 1: Search for an approximate maximum closure $C^o_{\max'}$ of system (B^o, A^o, W^o) using one of the design heuristics.

Step 2: Search for an approximate maximum reverse closure, $C^r_{\max'}$, of system (B^o, A^o, W^o), or an appoximate maximum closure of system (B^r, $\overline{A^r}$, W^r) using the same one.

Within system (B^o, A^o, W^o), all blocks would be divided by the conjugate heuristics into four sections as follows:

$$C^{o\setminus r} = \{b^o_{ij} \mid b^o_{ij} \in C^o_{\max'} \setminus C^r_{\max'}\}$$
$$C^{r\setminus o} = \{b^o_{ij} \mid b^o_{ij} \in C^r_{\max'} \setminus C^o_{\max'}\}$$
$$C^{o\cap r} = \{b^o_{ij} \mid b^o_{ij} \in C^o_{\max'} \cap C^r_{\max'}\}$$
$$C^{\overline{o\cup r}} = \{b^o_{ij} \mid b^o_{ij} \overline{\in} C^o_{\max'} \cup C^r_{\max'}\}$$

Generally, it is considered that $C^{o\setminus r} \subseteq C^o_{\max}$, and $C^{r\setminus o} \subseteq C^r_{\max}$. $C^{o\cap r}$ and $C^{\overline{o\cup r}}$ are referred respectively to as mistaken zone and blind zone for the heuristics, the error range of the solution of the heuristics is the combination set $C^{o\cap r} \cup C^{\overline{o\cup r}}$ of both. On the other side, $C^{o\setminus r} \cup C^{r\setminus o}$ would be regarded as the solution of the pre-pass approach, furthermore, determining a division C^o and C^r of $C^{o\cap r} \cup C^{\overline{o\cup r}}$ by a rigorous graph theory algorithm, results in $C^o \cup C^{o\setminus r} = C^o_{\max}$, and $C^r \cup C^{r\setminus o} = C^r_{\max}$.

As an example, a conjugate moving cone algorithm is applied to solve the pitting model of the deposit shown in Fig. 1, the solution produced by Lemieux's sequence of testing for positive cones is[9] shown in Fig. 5. In this example, $C^o = C^{\overline{o\cup r}}$ and $C^r = C^{o\cap r}$. The mistaken zone and the blind zone of the moving cone algorithm are repectively caused by the help compensation increment-closures and the common compensation ones[5].

In a sense, the theory for the optimum design of

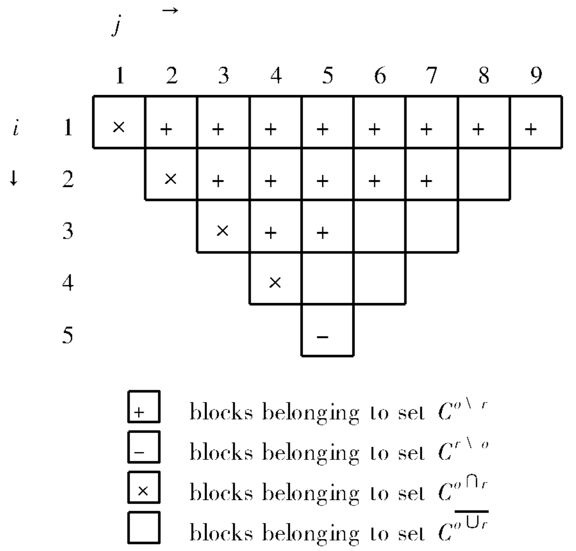

Fig. 5 Solution of conjugate moving cone algorithm

the open pit mines seem to be still the time reverse conservation. As for the searching sequence, Lemieux's negative moving cone technique is a reverse of the positive one[9], and Wang-Serim's heuristics[10] is a reverse of the nested Lerchs-Crossmann algorithm[11].

In general, the symmetry and the conservation laws can be applied in system (B^o, A^o, W^o) to decrease calculating quantity of the rigorous algorithms, and increase calculating precision of the heuristic algorithms.

5 CONCLUSIONS

The three important principles of the pit block model, namely, the symmetric principle which is being discussed, the fundemental principle which has been discussed, and the decoupling principle which will be discussed, have constructed a total theoretical frame for the optimum design and scheduling of the open pit mines. The frame not only can support the various strategies and techniques which have been reviewed by Kim and Thomas for determining the current pit design and scheduling[4, 12], but also will be replenished by many more that follow.

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