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Yield surfaces of fcc crystals with crystallographic slip and mechanical twinning①

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Abstract: The mixed yield surfaces of fcc single crystals for slip on {111}⟨110⟩ and mechanical twinning on {111}⟨112⟩ systems have been derived when their critical resolved shear stresses are equal. It has been found that there are 259 stress states that can be classified into 21 groups according to the crystal symmetry. Each group activates 5, 6 or 8 slip or/and twinning systems depending on crystallographically non-equivalent slip or/and twinning systems groups. Among all those stress states, 3 groups activate 8 systems, including 21 stress states; 9 groups activate 5 systems, including 70 ones; the remaining 9 groups activate 5 systems, including 168 ones.

Key words: yield surfaces; fcc metals; {111}⟨110⟩ slip; {111}⟨112⟩ mechanical twinning **Document code:** A

1 INTRODUCTION

The single crystal yield surfaces (SCYS) are extremely important to the study of the yield and deformation behavior of polycrystalline materials, texture development and plastic anisotropy. The crystal plastic theory points out that activation of at least five independent shear systems (including slip and mechanical twinning) is required in order to fulfill the accommodation of deformation at the crystallite level in the case where all five independent strain components are imposed (for instance, the FC Taylor model), while the stress states which activate five independent shear systems simultaneously are the vertices which lie on the yield surfaces[1]. When less than five strain components are imposed (the RC Taylor models), the yield stress states can be derived on the basis of the knowledge of the yield vertices, they are located at the connection lines between the vertices[2]. Therefore, only the yield vertices of single crystal that can fulfill an arbitrary shape change have been investigated in this paper. Material researchers have done much investigation for different structural crystals[3~8]. But there is few investigations for the case that slip and twinning can take place simultaneously. In fact, both slip and twinning can occur for fcc metals in many cases[9~10] (such as fcc metals with low stacking fault energy, particularly, when deformed at low temperatures and high strain rates). Since the single crystal yield surfaces are the fundamental to the study of polycrystalline plastic deformation, the mixed yield surfaces of fcc single crystals for slip on the {111}⟨110⟩ and twinning on the {111}⟨112⟩ systems have been investigated on the basis of Taylor/Bishop-Hill theory when their critical resolved shear stresses are equal. One critical scientific problem for plastic deformation in fcc metals would be solved and the theory fundamental would be supplied for the further study of the plastic deformation and the development of deformation textures.

2 SINGLE CRYSTAL YIELD SURFACES

2.1 Schmid law and stress, strain vectors

Each slip system of single crystal can be characterized by the normal to the slip plane $\boldsymbol{n}$ and the slip direction unit vector $\boldsymbol{b}$, respectively, and each twinning system by the normal to the twinning plane and twinning direction as well. Here $\boldsymbol{n}$ and $\boldsymbol{b}$ are assumed to be normalized. For the rate-insensitive materials, if the stress state is specified by the tensor σ_{ij}, the yield surfaces can be decided by the Schmid law

$$m^s_{ij}\sigma_{ij} \leqslant \tau_c \qquad (i, j = 1, 2, 3) \tag{1}$$

where m^s_{ij} are the generalized Schmid factors for different shear systems, $m^s_{ij} = (b^s_i n^s_j + b^s_j n^s_i)/2$. The equality sign in equation (1) holds for every active shear system and the inequality for inactive systems. The strain increment $d\varepsilon$ associated with the microscopic shear that takes place in the system is decided by

$$d\varepsilon_{ij} = m^s_{ij} d\gamma^s \tag{2}$$

For convenience, they are often represented by vector. The vector representations introduced by Kocks[2] in five- or six-dimensional stress space are used in this paper.

Using the concept of stress vector, the Schmid law can be expressed by

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$$m^s_k \sigma_k \leqslant \tau^s_c \qquad (k = 1, 5 \text{ or } 6) \qquad (1(a))$$

The corresponding strain increment is given by

$$d\varepsilon_k = m^s_k d\gamma^s \qquad (2(a))$$

For the five- or six-dimensional stress vectors, the m^s_k are the different linear combinations of the m^s_{ij}.

2.2 Symmetry properties of SCYS[7]

According to the crystal symmetry property, if $\boldsymbol{H}$ is the matrix representation of an allowed symmetry operation, when it is applied to the shear system s characterized by b^s and $\boldsymbol{n}^s$, then $H_{ki}b^s_i$ and $H_{lj}b^s_j$ can obtain the shear system s' of the same type. The corresponding Schmid factors associated with shear system s' are transformed by

$$m^{s'}_{kl} = H_{ki}H_{lj}m^s_{ij} \qquad (i, j, k, l = 1, 3) \quad (3)$$

Assuming that the stress tensor σv is the yield vertex associated with five independent shear systems s_n ($n = 1, 5$), then according to the definition of a vertex, there is

$$m^{s_n}_{ij}\sigma^v_{ij} = \tau^{s_n}_c \qquad (4)$$

The CRSS τ^{s_n} is assumed to be the same for all shear systems of the same type, from equation (3) and the orthogonality property of the symmetry operation matrix, then

$$m^{s'_n}_{kl}\sigma^{v'}_{kl} = \tau^{s'_n}_c \qquad (n = 1, 5) \qquad (5)$$

Where $\sigma v'$ is the equivalent yield vertex of σv, the shear system s' associated with $\sigma v'$ is just the transformation of the shear system s associated with σv. $\sigma v'$ is the transform of the stress tensor σv

$$\sigma^{v'}_{kl} = H_{ki}H_{lj}\sigma^v_{l} \qquad (6)$$

The above properties allow us to describe SCYS using a number of basic yield vertices while the rest yield vertices can be derived from them by applying the crystal symmetry operations. The minimum group of vertices is called the irreducible set of vertices, the chosen criterion is that at least one shear system associated with the vertex must belong to the standard stereographic triangle of cubic crystal. The calculation time can be reduced considerably using the crystal symmetry properties, furthermore, the shear systems of the same type can be obtained conveniently.

2.3 Shear systems and Schmid factor vectors

Table 1 and Table 2 list all the {111} ⟨110⟩ slip systems and {111} ⟨112⟩ twinning systems and the generalized Schmid vector components m^s_{ks} and m^s_{kt} in the cubic crystallographic axes. They are given in two different vector representations. The second column labeled Neg in Table 1 represents the opposite slip direction, for which all the components of $\boldsymbol{m}$ must be replaced by their negatives. Assuming that twinning can not happen in the anti-twinning directions, so there is not the Neg column in Table 2. They can be looked upon as unit strain vectors in stress space in terms of equation (2(a)).

Assuming that the critical shear stresses are τ_{cs} for all the {111} ⟨110⟩ slip systems and τ_{ct} for all the {111} ⟨110⟩ twinning systems. For convenience, the stress vector σ is normalized as $\boldsymbol{M}$ with respect to the CRSS of the {111} ⟨110⟩ slip system τ_{cs}. At the same time, the CRSS ratio for slip on {111} ⟨110⟩ and twinning on the {111} ⟨112⟩ is denoted ζ, for active systems

$$\left.\begin{array}{ll} m^s_{ks} M_k = 1 & \text{for slip system (s)} \\ m^s_{kt} M_k = \xi & \text{for twinning system (s)} \end{array}\right\} \qquad (7)$$

The above equations define a series of hyperplanes in five- or six-dimensional stress space, the M_k and m^s_k components are all calculated with respect to the cubic crystallographic axes.

Table 1 {111} ⟨110⟩ slip systems and generalized Schmid vectors m^s_{ks}

Number of slip systems		Slip systems						$m^s_{ks} = 1/\sqrt{6}\times$								Number of slip systems	
								5D space:			$m53$	$m54$	$m55$	$m51$	$m52$		
Pos	Neg	$n = 1/\sqrt{3}\times$			$b = 1/\sqrt{2}\times$			$m61$	$m62$	$m63$	$m64$	$m65$	$m66$	6D space		Pos	Neg
1	13	1	1	1	0	1	−1	0	1	−1	0	−1	1	−1	−1	s1	s13
2	14	1	1	1	−1	0	1	−1	0	1	1	0	−1	−1	1	s2	s14
3	15	1	1	1	1	−1	0	1	−1	0	−1	1	0	2	0	s3	s15
4	16	−1	−1	1	0	−1	−1	0	1	−1	0	1	1	−1	−1	s4	s16
5	17	−1	−1	1	1	0	1	−1	0	1	−1	0	−1	−1	1	s5	s17
6	18	−1	−1	1	−1	1	0	1	−1	0	1	−1	0	2	0	s6	s18
7	19	−1	1	1	0	1	−1	0	1	−1	0	1	−1	−1	−1	s7	s19
8	20	−1	1	1	1	0	1	−1	0	1	1	0	1	−1	1	s8	s20
9	21	−1	1	1	−1	−1	0	1	−1	0	−1	−1	0	2	0	s9	s21
10	22	1	−1	1	0	−1	−1	0	1	−1	0	−1	−1	−1	−1	s10	s22
11	23	1	−1	1	−1	0	1	−1	0	1	−1	0	1	−1	1	s11	s23
12	24	1	−1	1	1	1	0	1	−1	0	1	1	0	2	0	s12	s24

Table 2 $\{111\}\langle 112\rangle$ twinning systems and generalized Schmid vectors $m_{\mathrm{kt}}^{\mathrm{s}}$

Number of twinning systems	Twinning systems						$m_{\mathrm{ks}}^{\mathrm{s}} = 1/\sqrt{18}\times$								Notations of twinning systems
							5D space			$m53$	$m54$	$m55$	$m51$	$m52$	
	$n = 1/\sqrt{3}\times$			$b = 1/\sqrt{6}\times$			$m61$	$m62$	$m63$	$m64$	$m65$	$m66$	6D space		
25	1	1	1	-2	1	1	-2	1	1	2	-1	-1	-3	1	t1
26	1	1	1	1	-2	1	1	-2	1	-1	2	-1	3	1	t2
27	1	1	1	1	1	-2	1	1	-2	-1	-1	2	0	-2	t3
28	-1	-1	1	2	-1	1	-2	1	1	-2	1	-1	-3	1	t4
29	-1	-1	1	-1	2	1	1	-2	1	1	-2	-1	3	1	t5
30	-1	-1	1	-1	-1	-2	1	1	-2	1	1	2	0	-2	t6
31	-1	1	1	2	1	1	-2	1	1	2	1	1	-3	1	t7
32	-1	1	1	-1	-2	1	1	-2	1	-1	-2	1	3	1	t8
33	-1	1	1	-1	1	-2	1	1	-2	-1	1	-2	0	-2	t9
34	1	-1	1	-2	-1	1	-2	1	1	-2	-1	1	-3	1	t10
35	1	-1	1	1	2	1	1	-2	1	1	2	1	3	1	t11
36	1	-1	1	1	-1	-2	1	1	-2	1	-1	-2	0	-2	t12

3 YIELD VERTICES FOR SLIP ON $\{111\}\langle 110\rangle$ AND TWINNING ON $\{111\}\langle 112\rangle$ SYSTEMS WITH $\xi = 1$

Now we calculate the yield vertices when their CRSS ($\xi = 1$) are equal.

All the resolved shear stresses on slip and twinning systems can be obtained in terms of the $m_{\mathrm{ks}}^{\mathrm{s}}$ and $m_{\mathrm{kt}}^{\mathrm{s}}$ in Table 1 and Table 2. Since slip is reversible and twinning is unidirectional, there are 36 slip and twinning systems, that is, there are 36 equations altogether. According to the Schmid law, if the resolved shear stress on slip system s $\tau^{\mathrm{s}} = \tau_{\mathrm{cs}}$, crystal yields and slip occurs; if the resolved shear stress on twinning system t $\tau^{\mathrm{t}} = \tau_{\mathrm{ct}}$, crystal yields and twinning occurs. In order to accommodate an arbitrary shape change, solving the simultaneous equations, equation (7) should be fulfilled on five or more than five shear systems (5 independent active systems are needed, including slip and twinning). Meantime, the following conditions should be satisfied:

$$\left.\begin{array}{l} m_{\mathrm{ks}}^{\mathrm{s}} M_{\mathrm{k}} < 1 \text{ for slip system (s)} \\ m_{\mathrm{kt}}^{\mathrm{s}} M_{\mathrm{k}} < \xi \text{ for twinning system (s)} \end{array}\right\} \qquad (8)$$

on the remaining inactive systems

For slip on $\{111\}\langle 110\rangle$ and twinning on $\{111\}\langle 112\rangle$ systems, the 21 groups of basic yield vertices can be obtained by solving the simultaneous equations discussed above. There are 259 stress states considering the crystal symmetry:

——3 groups of them activate 8 systems, there are 21 vertices and each of them activate 8, 6 or 4 $\{111\}\langle 110\rangle$ slip systems and 0, 2, 4 $\{111\}\langle 112\rangle$ twinning system (s), the group of them activating 8 slip systems is the particular case of Bishop - Hill yield vertices.

——9 groups of them activate 6 systems, there are 70 vertices; and 8 groups activate 4 $\{111\}\langle 110\rangle$ slip systems and 2 $\{111\}\langle 112\rangle$ twinning systems (66 vertices), the remaining group of them activating 6 slip systems is the particular case of Bishop-Hill yield vertices.

——The remainder 9 groups activate 5 systems simultaneously. There are 168 vertices, 3 groups of them activate 4 $\{111\}\langle 110\rangle$ slip systems and 1 $\{111\}\langle 112\rangle$ twinning system (60 vertices), 3 groups of them activate 3 $\{111\}\langle 110\rangle$ slip systems and 2 $\{111\}\langle 112\rangle$ twinning systems (72 vertices), the remaining 3 groups activate 2 $\{111\}\langle 110\rangle$ slip systems and 3 $\{111\}\langle 112\rangle$ twinning systems (36 vertices).

Table 3 gives the 21 group of basic yield vertices, the number of sets of 5 independent shear systems and the number of equivalent yield vertices according to the crystal symmetry associated with each particular group. All the other yield stresses can be found out according to Table 3 and the appendix in reference [8]. From the known yield vertices, the resolved shear stress on every system can be worked out as in Table 3 in terms of the $m_{\mathrm{ks}}^{\mathrm{s}}$ and $m_{\mathrm{kt}}^{\mathrm{s}}$ in Table 1 and Table 2.

The results showed that most of the resolved shear stresses on the inactive system are not zero, this is quite different from $\{111\}\langle 110\rangle$ slip systems. For slip on $\{111\}\langle 110\rangle$ systems, all the resolved shear stresses of the yield vertices on inactive systems are zero. In particular, for twinning systems, the resolved shear stress of yield vertices on anti-twinning directions can be more than τ_{ct} as in Table 3. These stress states can be validity when the direction of twinning is considered. It also should be pointed out that the number of yield vertices for which there is active system ambiguity (i. e. more than 5 active systems) are 91. The first group and the forth group in Table 3 are the vertices of Bishop-Hill yield surfaces, but the numbers of the equivalent yield vertices are only the half of the latter. Furthermore, in Table 3, the yield vertices of the 4th and 5th, 6th and 7th

Table 3 Yield vertices, values of resolved shear stress on inactive systems, the numbers of sets of 5 independent active slip and twinning systems and active systems

Groups				$\overline{M} = \sqrt{6}/90 \times$					Resolved shear stress for all {111}<110> slip and {111}<112} twinning systems											
NO	M_{61}	M_{62}	M_{63}	M_{64}	M_{65}	M_{66}			1	2	3	4	5	6	7	8	9	10	11	12
				M_{53}	M_{54}	M_{55}	M_{51}	M_{52}	-13	-14	-15	-16	-17	-18	-19	-20	-21	-22	-23	-24
1	-30.0	60.0	-30.0	0	0	0	-45.0	-45.0	90.0	0	-90.0	90.0	0	-90.0	90.0	0	-90.0	90.0	0	-90.0
2	-8.0	49.0	-41.0	32.9	0	0	-28.5	-61.5	90.0	0	-90.0	90.0	-65.9	-24.1	90.0	0	-90.0	90.0	-65.9	-24.1
3	13.9	38.0	-52.0	0	0	0	-12.1	-77.9	90.0	-65.9	-24.1	90.0	-65.9	-24.1	90.0	-65.9	-24.1	90.0	-65.9	-24.1
4	0	0	0	45.0	-45.0	45.0	0	0	90.0	0	-90.0	0	-90.0	90.0	-90.0	90.0	0	0	0	0
5	-12.1	12.1	0	0	0	77.9	-12.1	0	90.0	-65.9	-24.1	90.0	-65.9	-24.1	-65.9	90.0	-24.1	-65.9	90.0	-24.1
6	12.1	-12.1	0	0	0	77.9	12.1	0	-90.0	65.9	24.1	-90.0	65.9	24.1	65.9	-90.0	24.1	65.9	-90.0	24.1
7	-32.9	32.9	0	0	0	57.1	-32.9	0	90.0	-24.1	-65.9	90.0	-24.1	-65.9	-24.1	90.0	-65.9	-24.1	90.0	-65.9
8	32.9	-32.9	0	0	0	-57.1	32.9	0	-90.0	24.1	65.9	-90.0	24.1	65.9	24.1	-90.0	65.9	24.1	-90.0	65.9
9	-19.0	38.0	-19.0	32.9	0	32.9	-28.5	-28.5	90.0	0	-90.0	90.0	-65.9	-24.1	24.1	65.9	-90.0	24.1	0	-24.1
10	0	0	0	12.1	-77.9	12.1	0	0	90.0	0	-90.0	-65.9	-24.1	90.0	-90.0	24.1	65.9	65.9	0	-65.9
11	22.0	-11.0	-11.0	12.1	-45.0	45.0	16.5	-16.5	90.0	-65.9	-24.1	0	-90.0	90.0	-90.0	24.1	65.9	0	0	0
12	-2.0	4.0	-2.0	0	-84.0	0	-3.0	-3.0	90.0	0	-90.0	-77.9	0	77.9	-77.9	0	77.9	90.0	0	-90.0
13	11.0	11.0	-22.0	32.9	-57.1	0	0	-32.9	90.0	0	-90.0	-24.1	-65.9	90.0	-24.1	0	24.1	90.0	-65.9	-24.1
14	4.0	4.0	-8.0	12.1	-77.9	0	0	-12.1	90.0	0	-90.0	-65.9	-24.1	90.0	-65.9	0	65.9	90.0	-24.1	-65.9
15	13.0	13.0	-26.0	45.0	-45.0	6.0	0	-39.0	90.0	0	-90.0	0	-90	90.0	-12.1	12.1	0	77.9	-77.9	0
16	-19.0	26.0	-7.0	20.9	0	57.1	-22.5	-10.4	90.0	-24.1	-65.9	90.0	-65.9	-24.1	-24.1	90.0	-65.9	-24.1	48.2	-24.1
17	35.0	2.0	-37.0	12.1	-45.0	6.0	16.5	-55.4	90.0	-65.9	-24.1	0	-90.0	90.0	-12.1	-53.8	65.9	77.9	-77.9	0
18	4.0	4.0	-8.0	-32.9	-57.1	20.9	0	-12.1	90.0	-65.9	-24.1	-24.1	0	24.1	-65.9	-24.1	90.0	48.2	41.8	-90.0
19	24.0	24.0	-47.9	12.1	-12.1	6.0	0	-71.9	90.0	-65.9	-24.1	65.9	-90.0	24.1	53.8	-53.8	0	77.9	-77.9	0
20	-4.0	2.0	2.0	-26.9	-45.0	45.0	-3.0	3.0	90.0	-65.9	-24.1	0	-12.1	12.1	-90.0	24.1	65.9	0	77.9	-77.9
21	-2.0	4.0	-2.0	-32.9	-51.0	32.9	-3.0	-3.0	90.0	-65.9	-24.1	-12.1	0	12.1	-77.9	0	77.9	24.1	65.9	-90.0

continued by Table 3: (note: only one effective number is given in Table 3)

	Resolved shear stress for all {111}⟨110⟩ slip and {111}⟨112⟩ twinning system												Number of active systems	Number of sets of 5 independent active systems	Number of equivalent vertices	Active systems
No	25	26	27	28	29	30	31	32	33	34	35	36				
1	52.0	-103.9	52.0	52.0	-103.9	52.0	52.0	-103.9	52.0	52.0	-103.9	52.0	8	32	3	1 4 7 10 15 18 21 24
2	52.0	-103.9	52.0	-24.1	-65.9	90.0	52.0	-103.9	52.0	-24.1	-65.9	90.0	8	40	12	1 4 7 10 15 21 30 36
3	-24.1	-65.9	90.0	-24.1	-65.9	90.0	-24.1	-65.9	90.0	-24.1	-65.9	90.0	8	44	6	1 4 7 10 27 30 33 36
4	52.0	-103.9	52.0	-103.9	52.0	52.0	52.0	52.0	-103.9	0	0	0	6	6	4	1 6 8 15 17 19
5	-24.1	-65.9	90.0	-24.1	-65.9	90.0	65.9	24.1	-90.0	65.9	24.1	-90.0	6	6	6	1 4 8 11 27 30
6	24.1	65.9	-90.0	24.1	65.9	-90.0	-65.9	-24.1	90.0	-65.9	-24.1	90.0	6	6	6	13 16 20 23 33 36
7	24.1	-90.0	65.9	24.1	-90.0	65.9	90.0	-24.1	-65.9	90.0	-24.1	-65.9	6	6	6	1 4 8 11 31 34
8	-24.1	90.0	-65.9	-24.1	90.0	-65.9	-90.0	24.1	65.9	-90.0	24.1	65.9	6	6	6	13 16 20 23 26 29
9	52.0	-103.9	52.0	-24.1	-65.9	90.0	90.0	-65.9	-24.1	13.9	-27.8	13.9	6	4	12	1 4 15 21 30 31
10	52.0	-103.9	52.0	-65.9	90.0	-24.1	-24.1	90.0	-65.9	38.0	-76.1	38.0	6	6	12	1 6 15 19 29 32
11	-24.1	-65.9	90.0	-103.9	52.0	52.0	-24.1	90.0	-65.9	0	0	0	6	6	12	1 6 17 19 27 32
12	52.0	-103.9	52.0	-45.0	90.0	-45.0	-45.0	90.0	-45.0	52.0	-103.9	52.0	6	6	6	1 10 15 24 29 32
13	52.0	-103.9	52.0	-90.0	65.9	24.1	-13.9	27.8	-13.9	-24.1	-65.9	90.0	5	1	24	1 6 10 15 36
14	52.0	-103.9	52.0	-65.9	90.0	-24.1	-38.0	76.1	-38.0	24.1	-90.0	65.9	5	1	24	1 6 10 15 29
15	52.0	-103.9	52.0	-103.9	52.0	52.0	7.0	7.0	-13.9	-45.0	-45.0	90.0	5	1	12	1 6 15 17 36
16	24.1	-90.0	65.9	-24.1	-65.9	90.0	90.0	-24.1	-65.9	41.8	0	-41.8	5	1	24	1 4 8 30 31
17	-24.1	-65.9	90.0	-103.9	52.0	52.0	-69.1	45.0	24.1	-45.0	-45.0	90.0	5	1	24	1 6 17 27 36
18	-24.1	-65.9	90.0	-13.9	27.8	-13.9	-65.9	90.0	-24.1	76.1	-79.8	3.7	5	1	24	1 9 24 27 32
19	-24.1	-65.9	90.0	-65.9	-24.1	90.0	-31.1	-31.1	62.2	-45.0	-45.0	90.0	5	1	12	1 17 27 30 36
20	-24.1	-65.9	90.0	-13.9	7.0	7.0	-24.1	90.0	-65.9	90.0	-45.0	-45.0	5	1	12	1 19 27 32 34
21	-24.1	-65.9	90.0	-7.0	13.9	-7.0	-45.0	90.0	-45.0	90.0	-65.9	-24.1	5	1	12	1 24 27 32 34

groups are equal but the sign is opposite. However, they can not be classified into the same group, it is because that the twinning yield surfaces are unidirectional (that is not reversible), while the slip yield surfaces are symmetry about the original (this accounts to that the crystal symmetry includes inversion center for slip, but not for twinning). In addition, not every symmetry operation can obtain a new vertex when a different symmetry operation is applied to the basic yield vertex, some of them may be redundant, so the number of the equivalent yield vertices for different groups may be different. Besides, by duality, the obtained yield vertices in this paper are identical to those of bcc metals for slip on {110} ⟨111⟩ and twinning on {112} ⟨111⟩ systems.

Using the calculated stress states in this paper, for an arbitrary deformation mode of an arbitrarily orientated crystal, one can calculate its taylor factor and predict the behavior of plastic deformation by invoking the principle of the maximum work. In addition, the corresponding active systems are also decided, hence the change of the crystal orientation. The calculated yield vertices of single crystal for slip and twinning establish a foundation for further investigation of plastic deformation of polycrystalline materials, the formation and development of deformation textures and the prediction of macroscopic mechanical properties.

4 CONCLUSIONS

1) The mixed yield surfaces of fcc single crystals for slip on {111} ⟨110⟩ and twinning on {111} ⟨112⟩ systems have been derived when their critical resolved shear stresses (CRSS) are equal, it is found that there are 21 groups of the basic yield stress states.

2) 259 stress states can be obtained from the 21 groups of basic yield vertices according to the crystal symmetry. 3 group of them activate 8 systems, there are 21 stress states; 9 groups activate 6 systems, there are 70 ones; the remainder 9 groups activate 5 systems, there are 168 ones altogether.

3) The fraction of ambiguous vertices (91/259 = 35.1%) is reduced considerably for mixed slip and twinning compared with the pure {111} ⟨110⟩slip.

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