



Equivalent strain hardening exponent of anisotropic materials based on spherical indentation response

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Received 4 August 2017; accepted 17 July 2018

Abstract: Uniaxial strain hardening exponent is not suitable for describing the strain hardening behaviors of the anisotropic materials, especially when material deforms in the multi-axial stress states. In this work, a novel method was proposed to estimate the equivalent strain hardening exponent of anisotropic materials based on an equivalent energy method. By performing extensive finite element (FE) simulations of the spherical indentation on anisotropic materials, dimensionless function was proposed to correlate the strain hardening exponent of anisotropic materials with the indentation imprint parameters. And then, a mathematic expression on the strain hardening exponent of anisotropic materials with the indentation imprint was established to estimate the equivalent strain hardening exponent of anisotropic materials by directly solving this dimensionless function. Additionally, Meyer equation was modified to determine the yield stress of anisotropic materials. The effectiveness and reliability of the new method were verified by the numerical examples and by its application on the TC1M engineering material.

Key words: equivalent strain hardening exponent; anisotropic material; spherical indentation; residual imprint

1 Introduction

Anisotropic plastic properties are widely observed in the industrial materials, especially in the aviation, such as anisotropic alloys, composite materials and coatings [1]. Uniaxial test is one of the classical methods to measure anisotropic properties of materials. However, this testing method is not applicable for the specimens with finite volumes. Nano-indentation, as an alternative approach, has received great attention for its obvious advantages, e.g. easy sample preparation [2]. Nano-indentation has shown the ability to nondestructively measure the local properties of the exceedingly small sub-volumes of specimens [3,4], such as elastic modulus [5], residual stress [6,7], fracture toughness [8,9], as well as the strain hardening exponent in crystal plasticity [10].

TABOR [11] proposed the representative stress and strain method firstly to extract the stress–strain curves from indentation test. MOUSSA et al [12] proposed a method for the determination of strain hardening law of materials using the load–displacement curve of a spherical indentation test. Although some theories and

methods have been well established, techniques in the relevant fields are still in the developing stages [13–17].

Strain hardening exponent is an important parameter, which essentially represents the ability of material to resist the further strain hardening in metal forming process. For the anisotropic materials, Hill'48 yield criterion was widely used, for its simple form. However, the strain hardening of the Hill'48 yield criterion is usually assumed as isotropic hardening, using the Hollomon hardening law. Besides, in some conventional FE softwares (e.g. ABAQUS), although the yield stresses along different orthogonal directions are different, the strain hardening of materials was assumed to be isotropic. In fact, the uniaxial strain hardening exponent is suitable for isotropic materials or the anisotropic materials along different uniaxial directions. When dealing with multi-axial stress problems, uniaxial exponent cannot take other directions into consideration. Therefore, an equivalent strain hardening exponent is needed to describe the strain hardening behaviors of the anisotropic materials.

In practice, it is quite difficult to compute the residual imprint in indentation test because the shape cannot be captured accurately [18]. The development of

FEM (finite element method) provides a very versatile approach to explore the relationship between the residual imprint and the mechanical properties. It is able to avoid many uncertainties in indentation experiments, e.g. the existence of measuring error [19]. Furthermore, it gives more complete and accurate predictions than the plastic deformation theory [20], such as the complicated multi-axial stress strain states and the pile-up effect [21].

In this work, a new method is suggested to estimate the equivalent hardening exponent n by only using the residual imprint of spherical indentation. The advantage of this method is that it does not need to know the entire indentation loading history [22]. The residual imprint profile contains the essential deformation characteristics of a material, and it is closely related to the material hardening exponent [21]. When a rigid spherical indenter is penetrated into the surface of in-plane anisotropic specimens, the residual imprint is different among orthogonal directions. Inspired by that, dimensionless function between mechanical properties parameters and residual imprint is established [2]. Because it only considers in-plane anisotropy and uses dimensionless function, this approach is quite simple. Furthermore, the effectiveness of the method is verified by the numerical experiment and the effectiveness of the equivalent strain hardening exponent is verified by the experimental results of TC1M anisotropic materials.

2 Numerical approach

2.1 Material model

The anisotropic theory used in the indentation simulation is introduced. There are many criteria to describe the plastic deformation behaviors of anisotropic materials. Here, the Hill'48 criterion [23] is considered, for its simple form. Hill's theory is expressed as Eq. (1):

$$F(\sigma_y - \sigma_z)^2 + G(\sigma_x - \sigma_y)^2 + H(\sigma_x - \sigma_z)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = 1 \quad (1)$$

where F , G , H , L , M and N are anisotropic parameters, σ_x , σ_y , σ_z , τ_{xy} , τ_{yz} and τ_{zx} are the normal stresses and shear stresses along three axes. The material coordinate is shown in Fig. 1.

Figure 1 describes the material coordinate used in the present work, xy plane is the in-plane that characterizes the anisotropic properties. Three normal stresses and three shear yield stresses along the orthogonal axes (x , y , z), are defined as τ_{xy} , τ_{yz} , τ_{zx} and σ_{yx} , σ_{yy} , σ_{yz} , respectively.

Since the materials exhibit in-plane anisotropic properties, their stress strain curve is expressed as

$$\sigma_y = \sigma_{yy} = \sigma_{yz} < \sigma_{yx} \quad (2)$$

where σ_y is the yield strength.

$$\tau_{xy} = \tau_{yz} = \tau_{zx} = \frac{\sigma_y}{\sqrt{3}} \quad (3)$$

And m is defined as the ratio σ_{yx}/σ_{yy} , that is,

$$\sigma_{yx} = m\sigma_{yy} \quad (4)$$

The present work considers the materials which obey Holloman hardening law, as shown in the following equations:

$$\sigma = E\varepsilon \quad (\sigma < \sigma_y) \quad (5)$$

$$\sigma = K\varepsilon^n \quad (\sigma > \sigma_y) \quad (6)$$

n is the strain hardening exponent, E is the elastic modulus and K is the work-hardening strength.

$$K = \sigma_y \left(\frac{E}{\sigma_y} \right)^n \quad (7)$$

Holloman hardening law is able to describe the hardening behavior of most materials [24–26]. Figure 2 shows the stress–strain curves of anisotropic materials that obey the Holloman hardening law. When E is known in advance, two parameters need to be identified, which are σ_y and n . In the present work, we focus on the strain hardening exponent n , independently, and we also extend

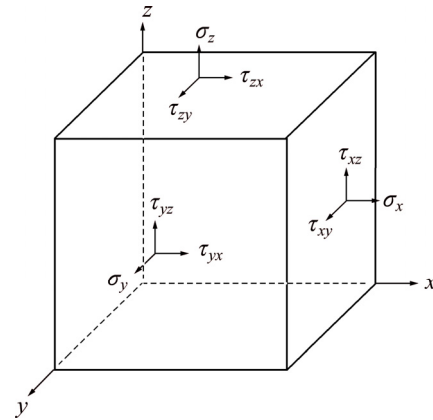


Fig. 1 Material coordinate system

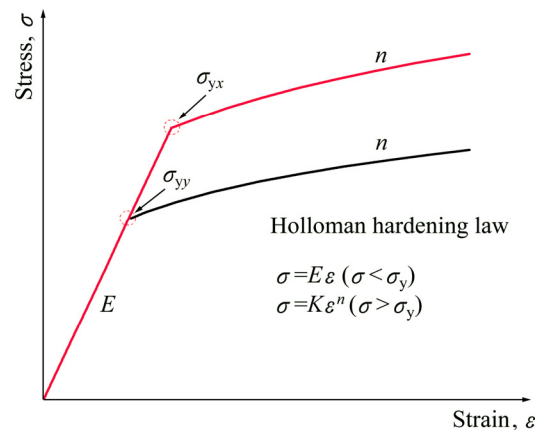


Fig. 2 Stress–strain curves of anisotropic materials

the approach to estimate the yield stress in anisotropic materials.

2.2 Finite element model in spherical indentation of anisotropic materials

The one-quarter finite element (FE) model of spherical indentation test is built with ABAQUS software [27], as shown in Fig. 3. The longitudinal direction is along 1-direction, transverse direction is along 2-direction, and 3-direction is the indenter loading–unloading direction.

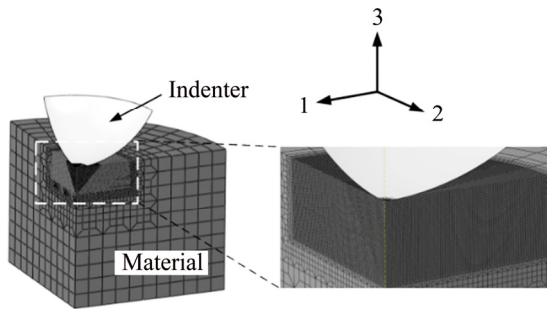


Fig. 3 FE model used in indentation simulation

This model comprises 17200 elements, and refined meshes are created around the local contact region between indenter and specimen, as shown in Fig. 3. The specimen is modeled using C3D8R element type. Contact friction coefficient between the surfaces of these two bodies is defined to be 0.1 [28]. Because the contact friction coefficient between diamond and metals is around this value, and it is a minor factor in indentation [29,30]. The displacement of bottom nodes of specimen is fixed, and the axisymmetric boundary constraints are applied on the center axis of specimen. The surfaces of the specimens are assumed as a planar plate and the indenter load can only move vertically to penetrate the specimen, and the ratios between maximum indentation depth and indenter radius h_m/R are fixed to be 0.1.

The elastic–plastic properties used for the extensive FE simulations are listed in Table 1. Poisson ratio ν was fixed at 0.3 because its influence on the indentation response was small [31–34]. The ratio range of

$75 < E/\sigma_y < 1000$, contains most metallic materials. The ratio m varies from 1.0 (isotropic material) to 2.0. In addition, the hardening exponent n varies from 0 to 0.5. Totally 180 cases were calculated to obtain the data of residual imprint. Figure 4 shows the residual imprint of the specimen after indenter withdrawal.

2.3 Effect of anisotropic plasticity on residual imprint

Figure 5 shows the indentation response of in-plane anisotropic materials, of which $E=100$ GPa, $\sigma_y=400$ MPa and $n=0.1$, anisotropic parameter m varies from 1.0, 1.2, 1.5, 1.8 to 2.0. The maximum load increases as m increases, but the differences are small. This indicates that it is difficult to evaluate the anisotropy of in-plane anisotropic materials using solely the load–displacement curves. So, we suggested here to use the residual imprint as useful indentation information. When the spherical indenter penetrates into the specimen, the specimen will deform as the indenter moves, and an imprint will be left after indenter withdrawal. Figure 6 shows the simulated residual imprints of specimens with different m values. The maximum pile-up value varies obviously as m value changes. The pile-up height in the low yield stress direction becomes larger than that in the high yield stress direction, which is coincident with the results of YONEZU et al [2]. This indicates that, the shape of the residual imprint relies on the plastic anisotropic properties. Consequently, the relationship between residual imprint and the hardening exponent n should be investigated.

The schematic diagram of the residual imprint is shown in Fig. 7. a is defined as the contact radius under the fully unloaded state. The contact radii along different directions are equal for isotropic materials while the contact radii are different for anisotropic materials. Different contact radii are influenced by the pile-up or the sinking-in effect. Actually, these differences are caused by the plastic anisotropy, such as the yield stress and strain hardening exponent in different directions. So, it is reasonable to connect the residual imprint with plastic properties.

Further, the influence of strain hardening exponent on the shape of the residual imprint was investigated.

Table 1 Elastic–plastic properties used in FE simulations

Specimen No.	E/GPa	σ_y/GPa	n	$m=\sigma_{yx}/\sigma_{yy}$	E/σ_y
1	60	0.1	0, 0.1, 0.2, 0.3, 0.4, 0.5	1.0, 1.2, 1.5, 1.8, 2.0	600
2	60	0.4	0, 0.1, 0.2, 0.3, 0.4, 0.5	1.0, 1.2, 1.5, 1.8, 2.0	150
3	60	0.8	0, 0.1, 0.2, 0.3, 0.4, 0.5	1.0, 1.2, 1.5, 1.8, 2.0	75
4	100	0.1	0, 0.1, 0.2, 0.3, 0.4, 0.5	1.0, 1.2, 1.5, 1.8, 2.0	1000
5	100	0.4	0, 0.1, 0.2, 0.3, 0.4, 0.5	1.0, 1.2, 1.5, 1.8, 2.0	250
6	100	0.8	0, 0.1, 0.2, 0.3, 0.4, 0.5	1.0, 1.2, 1.5, 1.8, 2.0	125

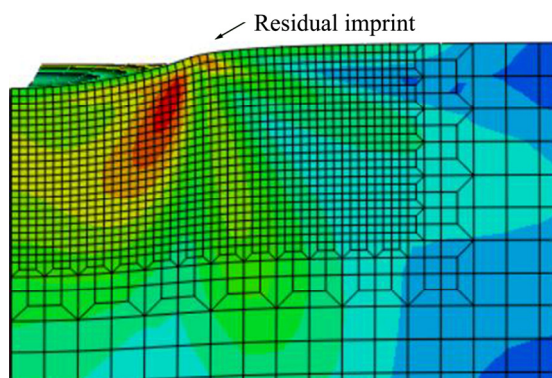


Fig. 4 Residual imprint after spherical indentation

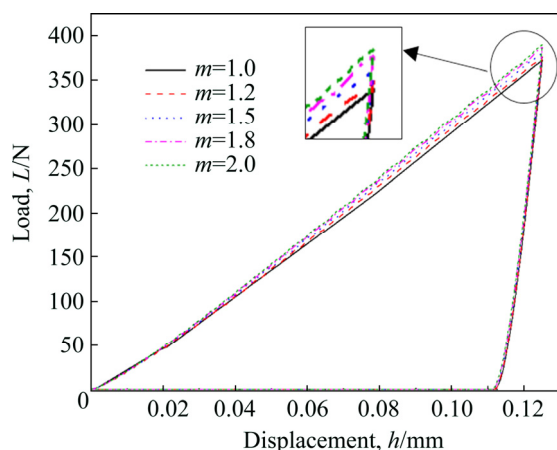


Fig. 5 Indentation curves of five materials

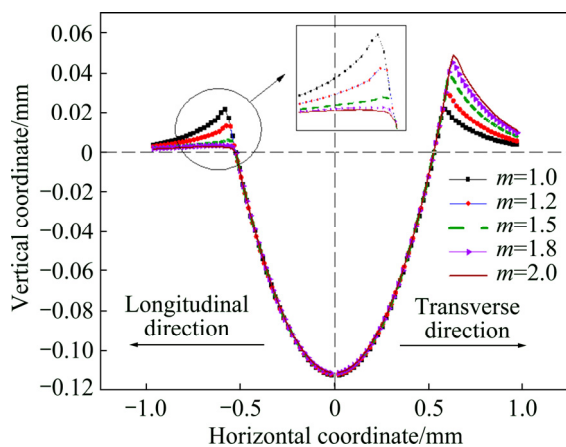


Fig. 6 Residual imprints of specimens with different m values after indenter withdrawal

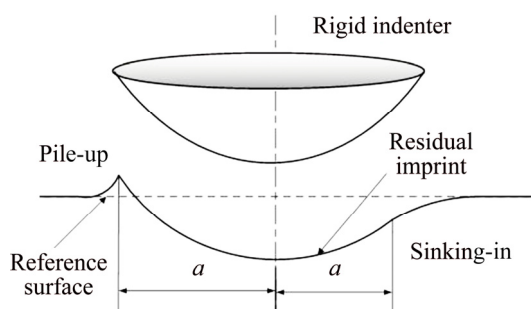


Fig. 7 Schematic diagram of residual imprint in spherical indentation

Figure 8 shows the changes of the residual imprints at different n and m values. The height of pile-up and the contact radius decrease as n increases and then the mathematic relationship between contact radius a and hardening exponent n is explored.

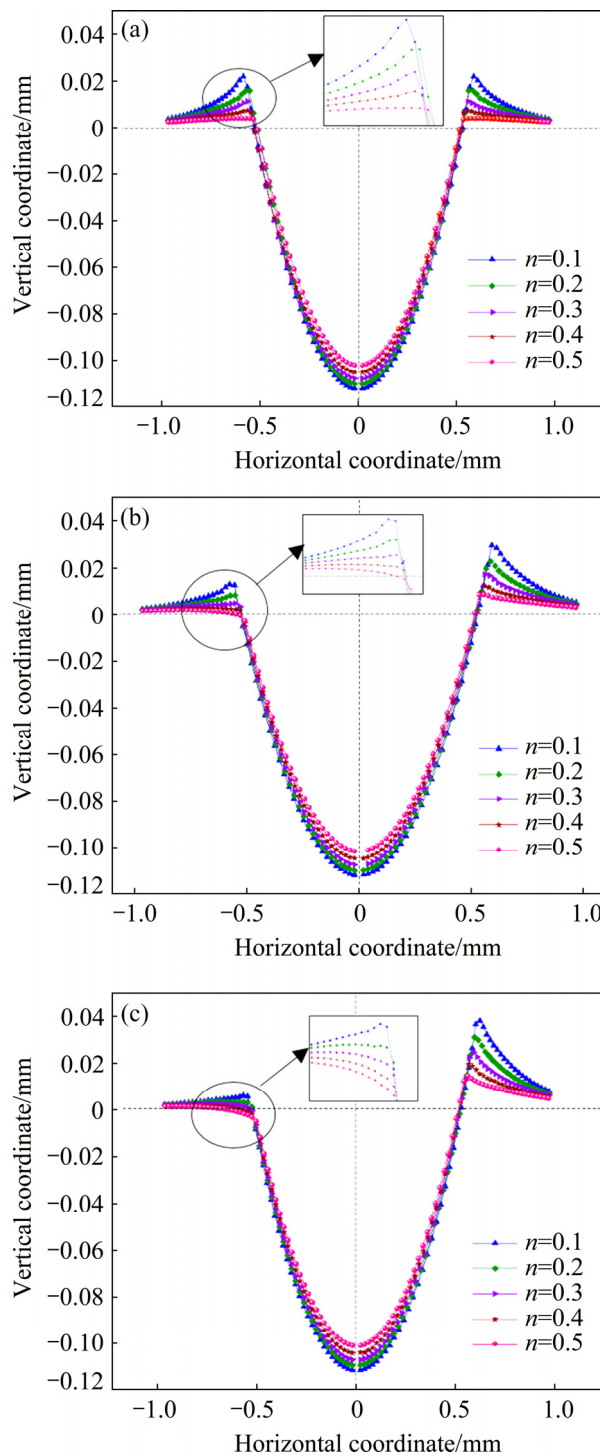


Fig. 8 Residual imprints for materials with three m values: (a) $m=1.0$; (b) $m=1.2$; (c) $m=1.5$

The equivalent indentation work was used to characterize the anisotropic parameters of materials, of which the elastic deformation was assumed in the

unloaded stage. The load–depth curve in indentation test is shown in Fig. 9, where h_m is the maximum indentation depth, F_m is the maximum indentation force. The area surrounded by loading curve and x -axis is defined as the total work W_t , the area surrounded by unloading curve and x -axis is the elastic recover work W_e , and the difference of W_t and W_e is plastic work W_p .

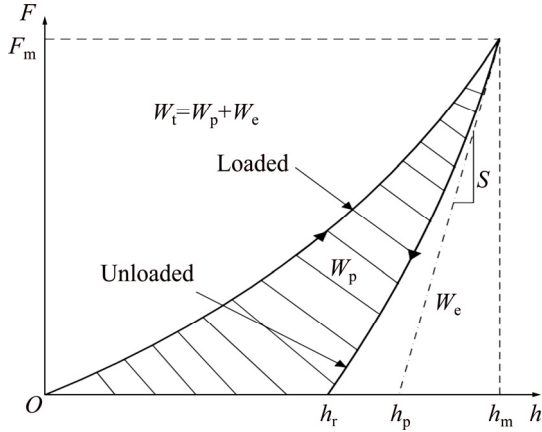


Fig. 9 Schematic diagram of relation curve between indentation force and depth (h_r is residual displacement, h_p is plastic displacement)

Important displacement variables include the maximum displacement h_m , the contact depth h_c , the elastic displacement h_e , and the residual displacement h_r . In the indentation test, the materials are assumed that only elastic deformation happens in the unloading stage. The profile of the imprint is shown in Fig. 10. According to Hertz's contact theory [35], for rigid spherical indenter and flat non-rigid specimen, the indentation depths can be described as

$$h_c = h_m - \frac{h_e}{2} = h_m - \frac{h_m - h_r}{2} = \frac{h_m + h_r}{2} \quad (8)$$

In current literature, the relationship between indentation work and depth in the dimensionless form is described as [36]

$$\frac{h_r}{h_m} = \frac{W_p}{W_t} \quad (9)$$

According to the indentation geometry, the relationship between contact depth h and contact radius can be described as

$$h = R - \sqrt{R^2 - a^2} \quad (10)$$

where R is the radius of rigid indenter.

So Eq. (9) can be rewritten as

$$\frac{W_p}{W_t} = \frac{h_r}{h_m} = \frac{R - \sqrt{R^2 - a^2}}{R - \sqrt{R^2 - a_m^2}} \quad (11)$$

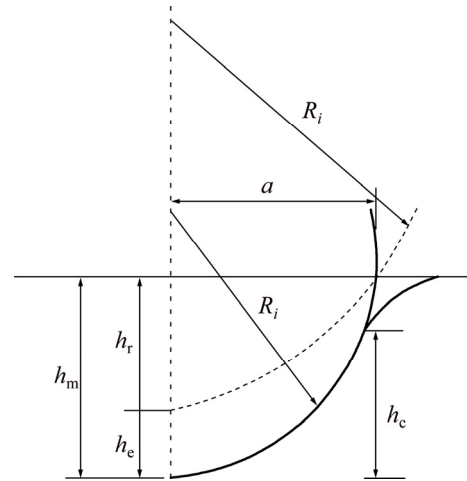


Fig. 10 Schematic diagram of geometrical relationships among contact radius and characteristic depths

Anisotropic materials are also assumed to obey Eq. (11), but a^2 should be written as $\bar{a}^2$. As for orthogonal anisotropic materials, there are two different directions. And the projection of the residual imprint is ellipse, which has two contact radii. $\bar{a}^2$ is defined and described as

$$\bar{a}^2 = a_x a_y \quad (12)$$

When the indentation depth is fixed, the maximum contact radius and the indenter radius are constant parameters. The equivalent expression can be described as

$$a^2 = \bar{a}^2 = a_x a_y \quad (13)$$

$$\pi a^2 = \pi \bar{a}^2 = \pi a_x a_y \quad (14)$$

Equation (14) can be regarded as the same proportion of the normal imprint for isotropic and anisotropic materials. Here, $\pi a_x a_y$ represents the area of the ellipse in Fig. 11.

As for anisotropic materials, the nominal imprint after indenter withdrawal is ellipse while it is a circle for isotropic materials. For further illustration, the contact radii in orthogonal direction are different, $a_x < a_y$, because the yield stress in x direction (longitudinal direction) is higher than that in y direction (transverse direction). As assumed early, hardening exponent is an isotropic parameter, the ellipse anisotropic imprint will be replaced by an isotropic imprint, which can be regarded as the isotropic response under the same load. Here, equivalent circular imprint is proposed to replace the real imprint such as shown in Fig. 11. The proportion of real imprint is equal to the equivalent imprint and the radius of the equivalent imprint is defined as $\bar{a}$, which is regarded as the representative radius of the two axis (x -axis and y -axis) of the ellipse.

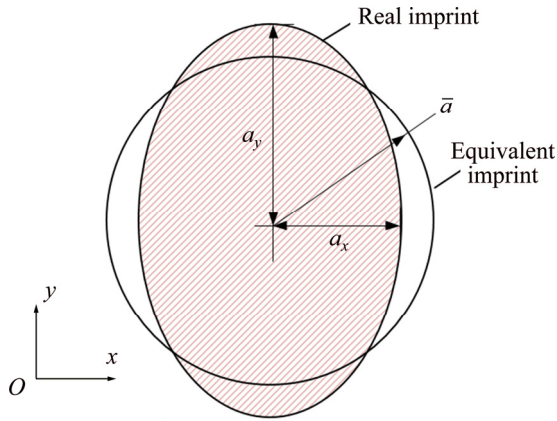


Fig. 11 Schematic diagram of equivalent imprint

3 Dimensionless analysis and numerical computation

In this section, the method is proposed to evaluate the in-plane strain hardening exponent n based on dimensional analysis and numerical computation. Dimensionless function is explored which can correlate the exponent n with the impression geometry. Π theorem [37] is a useful tool to establish functional relation between independent parameters. Contact radius can be expressed by the following Π function:

$$\begin{cases} a_x = \Pi_1(E, E_i, \nu, \nu_i, R, m, n, \sigma_y, h) \\ a_y = \Pi_2(E, E_i, \nu, \nu_i, R, m, n, \sigma_y, h) \end{cases} \quad (15)$$

where subscript i refers to the corresponding indenter, and ν is the Poisson ratio.

The reduced elastic modulus E^* can be expressed as

$$\frac{1}{E^*} = \frac{1-\nu^2}{E} + \frac{1-\nu_i^2}{E_i} \quad (16)$$

Indenter is regarded as rigid body, and the elastic modulus of indenter is infinite. So, Eq. (16) can be rewritten as

$$\frac{1}{E^*} \approx \frac{1-\nu^2}{E} \quad (17)$$

When R , h and ν are fixed, Π_1 and Π_2 functions can be simplified to Π_3 and Π_4 functions, as shown in the following equation:

$$\begin{cases} a_x = \Pi_3(E, m, n, \sigma_y, h, R) \\ a_y = \Pi_4(E, m, n, \sigma_y, h, R) \end{cases} \quad (18)$$

As stated in Section 2.3, the residual imprint strongly depends on the strain hardening exponent n , and the function is established based on this relation. Contact radius contains all the characters, such as load depth, and pile-up or sinking-in effects. The values of the contact radius are different by changing the material characters,

such as yield stress σ_y , anisotropic parameter m , elastic modulus E and strain hardening exponent n .

In order to establish the relationship between the imprint and material properties, indentation depth is fixed as $h_m/R=0.1$. By using the Π theorem, the parameter $\bar{a}/R$ can be expressed as

$$\frac{\bar{a}}{R} = \Pi_5\left(\frac{E}{\sigma_y}, m, n\right) \quad (19)$$

In Eq. (19), Π_5 reveals that the value of $\bar{a}/R$ depends on the plastic properties ($E/\sigma_y, m, n$).

TABOR [11] suggested the following relationship between flow stress and strain in uniaxial tension and the parameters in spherical indentation:

$$\sigma_t = \frac{1}{\psi} \frac{F}{\pi a^2} \quad (20)$$

$$\varepsilon_t = 0.2 \frac{a}{R} \quad (21)$$

where σ_t and ε_t are flow stress and strain in uniaxial tension, respectively, ψ is the plastic constraint factor (taken as 3 in Tabor's work). Inserting Eqs. (20) and (21) into Eq. (6) gives

$$\frac{F}{\pi a^2} = 3\sigma_y (0.2 \frac{a}{R})^n \quad (22)$$

When the test condition (load depth) is fixed, the indentation load F is a constant. The relationship between $\ln(\bar{a}/R)$ and hardening exponent n at different plastic strains is shown in Fig. 12. From the results in all the FEM data, Fig. 12 shows a close relationship between $\ln(\bar{a}/R)$ and hardening exponent n , in spite of the small deviation in anisotropic parameters m from 1.0 to 2.0. For simple mathematical expressions, FEM data are fitted as

$$\ln(\bar{a}/R) = An^2 + Bn + C \quad (23)$$

where A , B and C are regression coefficients.

In Fig. 12, the curves are similar under different strains. $\ln(\bar{a}/R)$ is linearly related to the strain hardening exponent n . The values of coefficients A , B and C used in Eq. (23) are listed in Table 2.

Coefficients A , B and C are strongly dependent on the strain value. Their relationships are fitted by Eq. (24), (25) and (26), respectively:

$$A = 3.36 \times 10^{-17} \left(\frac{E}{\sigma_y}\right)^{5.071} + 0.1199 \quad (24)$$

$$B = -1.632 \times 10^{-16} \left(\frac{E}{\sigma_y}\right)^{5.071} - 0.3396 \quad (25)$$

$$C = -1.461 \left(\frac{E}{\sigma_y}\right)^{-0.3548} - 0.5932 \quad (26)$$

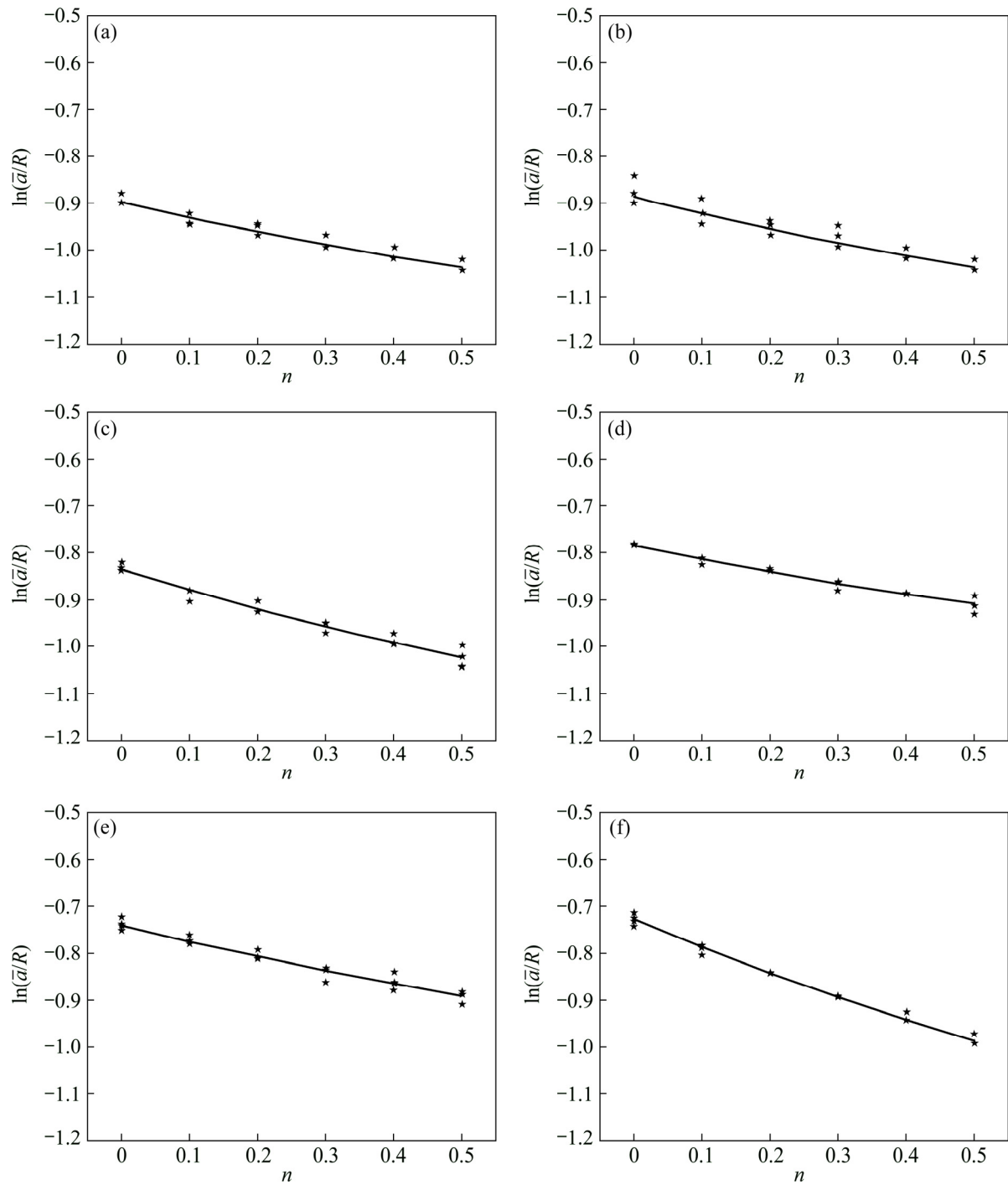


Fig. 12 Dimensionless function of $\bar{I}_5$ at $h_m/R=0.1$ and different E/σ_y values: (a) $E/\sigma_y=75$; (b) $E/\sigma_y=125$; (c) $E/\sigma_y=150$; (d) $E/\sigma_y=250$; (e) $E/\sigma_y=600$; (f) $E/\sigma_y=1000$

Table 2 Coefficients A , B and C used in Eq. (23)

E/σ_y	A	B	C
75	0.1241	-0.3389	-0.8977
125	0.1331	-0.367	-0.8863
150	0.1525	-0.4521	-0.8351
250	0.1202	-0.3069	-0.7836
600	0.1024	-0.3518	-0.7395
1000	0.1766	-0.6067	-0.7258

4 Results and discussion

4.1 Numerical verification

In this section, the new approach will be employed to estimate the strain hardening exponent of anisotropic materials using several numerical cases, and results are shown in Table 3. The effectiveness of the approach is verified by comparing the “input” hardening exponent in

the FE simulation with those identified by the established numerical method.

Table 3 Representative numerical cases for effectiveness verification of strain hardening exponent of anisotropic materials

Numerical case No.	Input material property				Identified n	Difference of n
	E/GPa	σ_y/GPa	m	n		
Case 1	60	0.1	1.2	0.40	0.3829	-0.0172
Case 2	60	0.4	1.0	0.50	0.5810	0.081
Case 3	100	0.4	1.5	0.30	0.2845	-0.0155
Case 4	60	0.8	2.0	0.30	0.2701	-0.0299
Case 5	100	0.8	1.5	0.20	0.2065	0.0065
Case 6	100	0.4	1.4	0.10	0.1170	0.017
Case 7	60	0.1	1.6	0.35	0.3486	-0.0014
Case 8	80	0.3	1.7	0.25	0.3023	0.0523

It can be seen from Table 3 that the strain hardening exponent identified by the present method shows good agreement with the FE “input” hardening exponent values, and the maximum difference is 0.081. This indicates that the new approach is reliable and has satisfactory accuracy.

4.2 Modified Mayer’s relationship for estimation of yield strength of anisotropic materials

In Section 3, we have established the method to directly estimate the strain hardening exponent of anisotropic materials, but the yield stress is still unknown. In this section, the modified Mayer’s relationship will be proposed to estimate the yield strength of anisotropic materials. In the spherical indentation, the relationship between the indentation contact force and the depth was obtained [38]:

$$\frac{F}{d^2} = A_1 \left(\frac{d}{D} \right)^{k-2} \quad (27)$$

where d is the contact diameter, A_1 is a regression

coefficient, D is the diameter of the rigid indenter, and k is the fitting exponent. However, this equation is only suitable for the isotropic materials. Here, the equation for the anisotropic material is rewritten, as shown in the following forms:

$$\begin{cases} \frac{F}{d_x^2} = A_{1x} \left(\frac{d_x}{D} \right)^{k_x-2} \\ \frac{F}{d_y^2} = A_{1y} \left(\frac{d_y}{D} \right)^{k_y-2} \end{cases} \quad (28)$$

GEORGE et al [39] proposed the following equation to correlate the constant A_1 with yield stress

$$\sigma_y = cA_1 \quad (29)$$

where c is a constant that varies with the specific material class. In this study, the c value of Ti alloys was known to be 0.3 [40]. Similarly, Eq. (29) can also be extended to Eq. (30):

$$\begin{cases} \sigma_{yx} = cA_{1x} \\ \sigma_{yy} = cA_{1y} \end{cases} \quad (30)$$

Equations (24)–(26) require a single yield stress to calculate the fitting coefficients A , B and C . While for the anisotropic materials considered in the present study, there are two yield stress values along different orthogonal directions, and they can be obtained from Eq. (30). Here, we defined the equivalent yield stress $\bar{\sigma}_y$, for the anisotropic materials, by using the concept of equivalent contact diameter $\bar{d}$, which are written as follows:

$$\bar{d} = 2\bar{a} \quad (31)$$

$$\bar{\sigma}_y = c\bar{A}_1 \quad (32)$$

Table 4 shows the comparison of the equivalent yield stress estimated by using spherical indentation with those from the tensile tests. From Table 4, it can be seen that good agreement can be found, indicating that the modified Mayer’s relation in the present study is effective.

Table 4 Equivalent yield stress estimated from spherical indentation and those from tensile tests

Material	Input yield stress/MPa		Identified yield stress/MPa		Equivalent yield stress/MPa
	x -direction	y -direction	x -direction	y -direction	
Ti–5Al–2.5Sn	885.57	737.98	948.6	794.1	872.1
Ti–6Al–4V	830.00	691.67	996.9	876.6	934.5
Ti–6Al–6V–2Sn	1009.23	841.03	1007.4	917.4	996.0
TC1M	740.00	597.00	713.0	605.0	652.5

4.3 Application of equivalent strain hardening exponent in multi-axial stress fields

In this section, the effectiveness of the equivalent strain hardening exponent is verified by its application in the TC1M anisotropic material. The multi-axial stress strain field is calculated by separately using the uniaxial direction strain hardening exponent and equivalent strain hardening exponent. The material parameters of TC1M titanium alloys in five different directions are listed in Table 5.

The equivalent strain hardening exponent of TC1M material calculated by Eq. (25) is 0.078. Figure 13 shows the comparison of FE simulation results using different

directions of the strain hardening exponent. The stress–strain curves are extracted from the representative point beneath the indenter. This representative data acquisition point is defined as the optimal data acquisition point,

Table 5 Tensile experimental data of TC1M titanium alloys

Direction/(°)	σ_y /MPa	n	K /MPa
0	605	0.118	1123
22.5	577	0.103	986
45.0	583	0.078	854
67.5	658	0.062	864
90.0	740	0.06	965

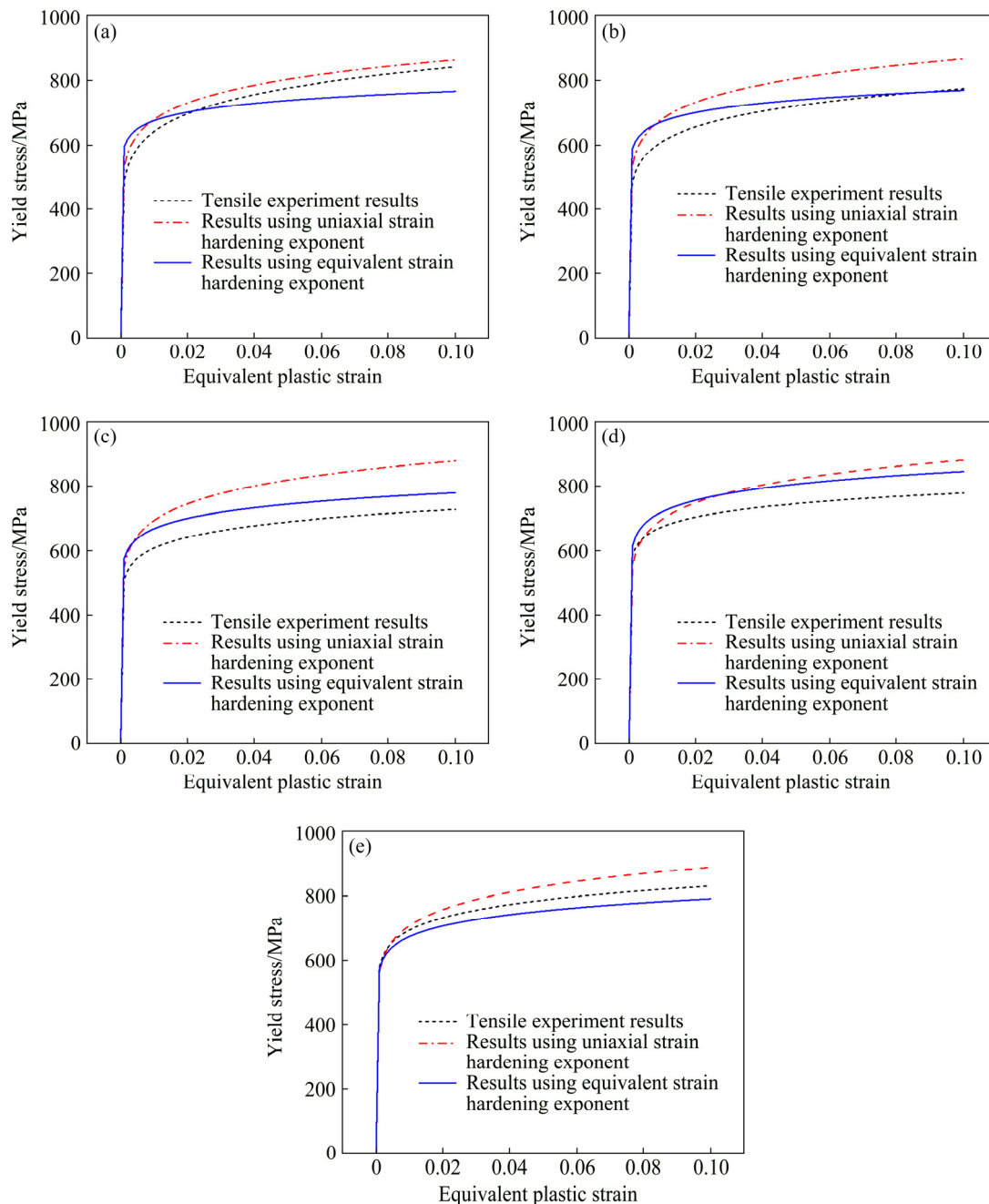


Fig. 13 Comparison of stress–strain curves of TC1M between uniaxial tests and those calculated by using equivalent strain hardening exponent, respectively, in different directions: (a) 0°; (b) 22.5°; (c) 45.0°; (d) 67.5°; (e) 90.0°

$r/a=0.8$ and $l/D=10\%$, at which the frictional effect can be ignored and the maximum strain for a given indentation depth can be obtained [41].

Figure 13 shows three stress–strain curves that are separately calculated by using the uniaxial strain hardening exponent along different reference directions (the longitudinal direction is regarded as the reference direction in this work), the equivalent strain hardening exponent, and the tensile experimental data. It can be seen from Fig. 13 that, the stress–strain curves calculated by equivalent strain hardening exponent show better agreement than those calculated by uniaxial strain hardening exponent in the four directions (22.5° , 45.0° , 67.5° and 90.0° (except 0° direction)). This is because the uniaxial strain hardening exponent can only give relatively accurate results in its own direction, while it gives coarse results in other directions. For the anisotropic materials, the equivalent strain hardening exponent is calculated by the parameters in different directions, thus it takes into account the strain hardening in all the directions. This explains why the equivalent strain hardening exponent gives more accurate results.

5 Conclusions

(1) Equivalent strain hardening exponent is suitable for describing the anisotropic materials in the multi-axial stress states.

(2) The established dimensionless function effectively correlates the equivalent strain hardening exponent with the residual contact radius of the anisotropic materials. The effectiveness of the proposed approach is numerically verified by its application on a series of numerical cases.

(3) The application of the equivalent strain hardening exponent is verified by its application in the TC1M engineering material.

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基于球形压痕响应的各向异性材料等效应变硬化指数

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摘 要: 针对各向异性材料, 讨论单一方向的应变硬化指数对复杂应力问题求解的局限性, 提出了一种基于等效能量法计算各向异性材料等效应变硬化指数的新方法。首先, 设置一系列有限元仿真实验, 得出残余压痕参数与等效应变硬化指数的无量纲函数关系式; 其次, 使用此无量纲函数建立等效应变硬化指数与残余压痕的数学表达式; 最后, 通过对比数值试验结果验证该方法的有效性。另外, 为了确定各向异性材料的屈服强度, 对其 Meyer 关系进行扩展。通过数值实例及对 TC1M 钛合金的实际应用验证了该新方法的有效性和可靠性。

关键词: 等效应变硬化指数; 各向异性材料; 球形压痕; 残余压痕

(Edited by Wei-ping CHEN)