Article ID: 1003 - 6326(2003)06 - 1348 - 06

Analysis of localized shear deformation of ductile metal based on gradient-dependent plasticity ①

WANG Xue-bin(王学滨), DAI Shu-hong(代树红), HAI Long(海 龙), PAN Yi-shan(潘一山)
(Department of Mechanics and Engineering Sciences,
Liaoning Technical University, Fuxin 123000, China)

Abstract: Shear localization in linear strain softening heterogeneous material under simple shear was investigated analytically. The closed-form solutions obtained based on gradient plasticity theory considering interactions and interplaying among microstructures due to heterogeneity of metal material show that in the normal direction of shear band, elastic shear displacement is linear; while plastic and total shear displacement are non-linear. Elastic shear strain in the band is uniform and the non-uniformity of total shear displacement stems from localized plastic shear displacement. In the center of the band, plastic and total shear displacement all reach their maximum values. In strain-softening process, elastic displacement decreases as flow shear stress decreases. Contrarily, plastic and total shear displacement increase and manifest shear localization occurs progressively. Under the same shear stress level, plastic and total shear displacement increase as strain softening modulus and elastic shear modulus decrease. The present analytical solutions were compared with many experimental results and the agreement is good.

Key words: gradient-dependent plasticity; microstructure; shear band; localization; ductile metal; shear deformation
CLC number: TG 113 **Document code:** A

1 INTRODUCTION

Failure process of materials is particularly complex, which is a problem involved in multi-scale and many disciplines. Though scientists from many countries have contributed some important results for the problem in recent years, further investigations by mechanical scientists, physicists and material scientists are necessary to obtain a full understanding of the failure mechanisms.

Especially in last 20 years, as a mechanism of progressive failure, the problem of localization has attracted topic interest. As a consequence of softening, damage and accordingly large deformation tend to accumulate within the narrow bands, the so-called shear band. As a precursor to the final rupture, localization can be observed in a wide class of ductile metal materials.

A few investigators studied the characteristics of shear localization. What is the reason for that? One is that spurious mesh sensitivity can not be overcome in numerical simulation due to the fact that classical constitutive equations do not contain any physical property with the dimension of length. The other is that in the context of classical continuum theories at a given point the physical state of a body is completely determined by the state of the material. So, it is not possible to predict the thickness of shear band, localized deformation, non-uniform displacement and velocity in shear band. Moreover, in classical plastic theory slip line or surface has zero thickness, which is not in agreement with many experimental observations.

Motivated mainly by difficulties above, a number of modifications and generalization from the standard continuum description have been proposed. One of the approaches is gradient continuum that incorporates the second order gradient of plastic strain in the yield function[1-5].

The paper is organized as follows. Firstly, two different mechanisms for shear localization are discussed. Secondly, the gradient-dependent plasticity is briefly introduced and analysis of shear localization is carried out. Moreover, the relation between microstructures and strain gradient or characteristic scale of ductile metal materials is discussed. At last, present analytical results obtained are discussed and are compared with experimental results.

2 ANALYSIS OF SHEAR DEFORMATION

2.1 Two different mechanisms for shear localization

Because of diversity of materials, geometry, scales of observation and loading conditions, a number

① **Foundation item:** Project(50309004) supported by the National Natural Science Foundation of China
Received date: 2002 - 12 - 02; **Accepted date:** 2003 - 03 - 24
Correspondence: WANG Xue-bin, PhD; E-mail: wxbbb@263.net

of mechanisms of localization have been suggested. Shawki et al[6] have classified the mechanisms into two main categories: one is mechanism that is associated with localization during the quasi-static, isothermal deformation of rate-independent materials; the other one is concerned with localization during the dynamic, adiabatic deformation of rate-sensitive materials. The present mechanism of localization belongs to the first category.

2.2 Introduction to gradient-dependent plasticity and its progress

Based on the early non-local elasticity models proposed by Eringen et al[3], the definition of non-local plastic shear strain $\bar{\gamma}^p$ can be expressed as the weighted average of its local counterpart γ^p over a surrounding length L:

$$\bar{\gamma}^p = \frac{1}{L}\int_L g(\xi)\gamma^p(y+\xi)\mathrm{d}\xi \tag{1}$$

where y denotes coordinate, ξ is a point of interest and $g(\xi)$ represents the Gaussian distribution weight function.

$g(\xi)$ function can be expressed as follows:

$$g(\xi) = \exp(-\xi^2/4l^2) \tag{2}$$

where l is the internal length parameter.

This integral in Eqn. (1) can be evaluated by expanding $\gamma^p(y+\xi)$ into a Taylor series around the point $\xi = 0$

$$\gamma^p(y+\xi) = \gamma^p(y) + \frac{\mathrm{d}\gamma^p(y)}{\mathrm{d}y}\xi + \frac{1}{2!}\frac{\mathrm{d}^2\gamma^p(y)}{\mathrm{d}y^2}\xi^2 + \cdots \tag{3}$$

The combination of Eqn. (3) and (1) yields the following definition of the non-local plastic strain:

$$\bar{\gamma}^p = \frac{1}{L}\int_L g(\xi)\gamma^p(y)\mathrm{d}\xi + \frac{1}{L}\int_L g(\xi)\frac{\mathrm{d}\gamma^p(y)}{\mathrm{d}y}\xi\mathrm{d}\xi + \frac{1}{2!L}\int_L g(\xi)\frac{\mathrm{d}^2\gamma^p(y)}{\mathrm{d}y^2}\xi^2\mathrm{d}\xi + \cdots \tag{4}$$

With the assumption of isotropic influence of the averaging equation, the integrals of the odd terms vanish. Truncating the Taylor series of Eqn. (4) after the quadratic term leads to the following definition of the non-local plastic shear strain:

$$\bar{\gamma}^p = \sum_{n=0,2}\int_L \frac{g(\xi)}{n!L}\cdot\frac{\mathrm{d}^n\gamma^p(y)}{\mathrm{d}y^n}\xi^n\mathrm{d}\xi \tag{5}$$

Using Eqn. (2), the following expression for non-local plastic shear strain can be derived

$$\bar{\gamma}^p = \gamma^p(y) + l^2\frac{\mathrm{d}^2\gamma^p(y)}{\mathrm{d}y^2} \tag{6}$$

The present formula of non-local plastic shear strain can be seen as a particular case of three-dimensional expression proposed by Ellen et al[2]. Substituting non-local plastic shear strain in Eqn. (6) for local plastic shear strain in classical plastic theory, we can consider the effects of microstructures in the context of classical plastic theories.

In numerical simulation aspects, many kinds of variational principles, finite element formulations and algorithms have been proposed for gradient plasticity models, recently[1-5]. However, unfortunately, in analytical aspects, only a few analytical solutions for strain localization based on gradient-dependent plasticity were proposed[1, 7-13].

2.3 Microstructures and gradient-dependent plasticity

For most of the materials the texture is heterogeneous and any microstructure will be influenced significantly by its neighborhoods. The extent of long-range interaction is governed by the characteristic length that depends on mean grain diameter. Moreover, the interactions among microstructures occur only in shear band and outside the band the effect can be ignored. Microstructures of different kinds of metals are of very importance and have been studied extensively by experiments[14-16]. In a word, the second-order strain gradient term describes the interactions and interplaying among microstructures.

2.4 Analysis of strain, displacement and velocity in shear band

2.4.1 Classical plastic theory for bi-liner constitutive relation

We suppose that localization is initiated at the peak shear stress and the shear deformation shown in Fig. 1 only occurs in the horizontal direction. The slope of shear stress—shear strain curve up to the peak can be considered approximately the same constant G. So, in the elastic regime, the shear elastic modulus G governs the relation between shear stress and elastic shear strain: $\tau = G\gamma$. The bi-linear diagram of constitutive relation is shown in Fig. 2. The absolute slope of shear stress—shear strain curve can also be considered approximately the constant λ usually referred to as strain softening modulus. For simplicity, the shear band is treated as a one-dimensional shear problem. In the conventional one-dimensional plasticity theory for linear strain softening material, the flow shear stress τ is an explicit function of τ_c, λ, G, and the accumulated plastic shear strain γ^p, i.e.

$$\tau = \tau_c - \frac{G\lambda}{G+\lambda}\gamma^p \tag{7}$$

2.4.2 Inclusion of second-order spatial derivative and analysis

Substituting Eqn. (6) for Eqn. (7), we can get the following expression:

$$\tau = \tau_c - \frac{G\lambda}{G+\lambda}\left(\gamma^p + l^2\frac{\mathrm{d}^2\gamma^p}{\mathrm{d}y^2}\right) \tag{8}$$

It is supposed that the shear band has a thickness w beyond the peak shear stress. The bounda-

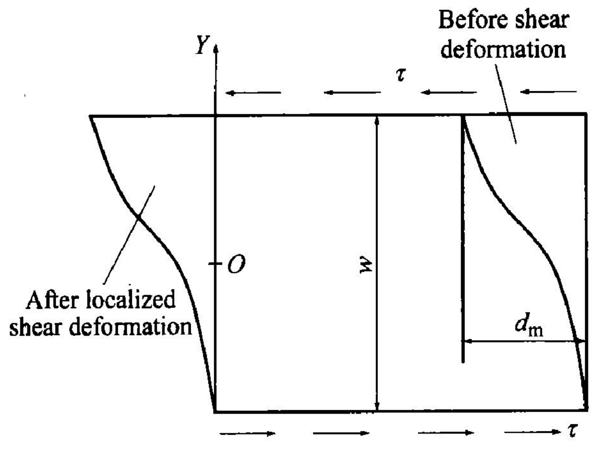

Fig. 1 Deformation of shear band after onset of localization

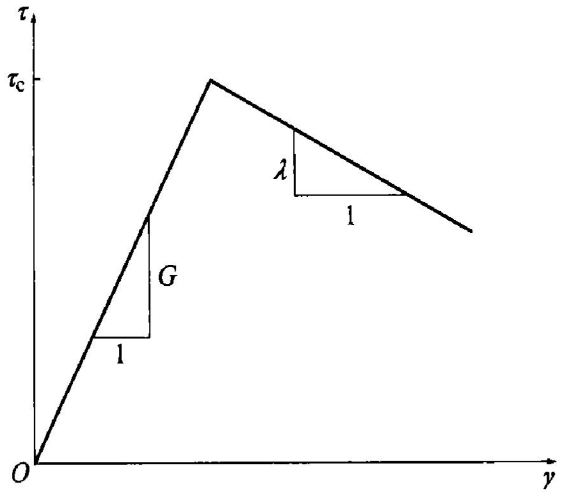

Fig. 2 Simplified shear stress vs shear strain (i. e. constitutive relation)

ry condition is

$$\gamma^p = 0 \text{ at } y = \pm w/2 \quad (9)$$

Applying the boundary condition, the results is

$$\gamma^p = \frac{\tau_c - \tau}{c}\left|1 - \cos\left(\frac{y}{l}\right)/\cos\left(\frac{w}{2l}\right)\right| \quad (10)$$

where $c = \frac{G\lambda}{G+\lambda}$. We are now interested in the aperiodic solution that results in the steepest descending branch. This solution is obtained by requiring

$$\frac{d\gamma^p}{dw} = 0 \quad (11)$$

So, we can obtain the smallest non-trivial argument of w and local plastic shear strain, respectively

$$w = 2\pi l \quad (12)$$

$$\gamma^p = \frac{\tau_c - \tau}{c}\left(1 + \cos\frac{y}{l}\right) \quad (13)$$

Total shear strain in shear band becomes $\gamma = \gamma^e + \gamma^p$, where $\gamma^e = \tau/G$, so that

$$\gamma = \frac{\tau}{G} + \frac{\tau_c - \tau}{c}\left(1 + \cos\frac{y}{l}\right) \quad (14)$$

2.4.3 Total shear strain, displacement and velocity according to complex constitutive relation

In fact, the shear constitutive relation shown in Fig. 3 is more complex than that of present analysis. But as peak shear stress is attained, we can assume that specimen unloads alone a straight line AB whose slope is referred to as shear elastic modulus G. For simplicity, in strain softening regime the shear stress—shear strain curve can be approximately considered as a line whose absolute slope is called strain-softening modulus λ. Thus, in the coordinate system $\gamma O\tau$ the constitutive relation is still bi-linear. Residual plastic strain γ^r in pre-peak stage is uniform and is equal to the length of line segment OO'. So, for complex constitutive relation beyond the peak stress, the total and plastic shear strain has the form as follows:

$$\gamma = \frac{\tau}{G} + \gamma^r + \frac{\tau_c - \tau}{c}\left(1 + \cos\frac{y}{l}\right) \quad (15)$$

$$\gamma^p = \gamma^r + \frac{\tau_c - \tau}{c}\left(1 + \cos\frac{y}{l}\right) \quad (16)$$

Integrating the total shear strain, we can get

$$d(y) = \int_0^y \gamma dy = \left(\frac{\tau}{G} + \gamma^r\right)y + \frac{\tau_c - \tau}{c}\left(y + l\sin\frac{y}{l}\right) \quad (17)$$

where $d(y)$ is total shear displacement along the shear stress direction. When $y = w/2$, the displacement $d(y)$ reaches its maximum $d_m/2$. d_m shown in Fig. 1 is the relative displacement between the upper and lower boundary of the shear band

$$d_m = \left(\frac{\tau}{G} + \gamma^r\right)w + \frac{\tau_c - \tau}{c}w \quad (18)$$

The displacement $d(y)$ can be composed of elastic and plastic shear displacement. The elastic and plastic part can be given, respectively

$$d_e(y) = \left(\frac{\tau}{G} + \gamma^r\right)y \quad (19)$$

$$d_p(y) = \frac{\tau_c - \tau}{c}\left(y + l\sin\frac{y}{l}\right) \quad (20)$$

Considering that the residual strain is a constant and differentiating Eqns. (19) and (20) with respect to time lead to

$$v_e(y) = \frac{\dot{\tau}}{G}y \quad (21)$$

$$v_p(y) = \frac{-\dot{\tau}}{c}\left(y + l\sin\frac{y}{l}\right) \quad (22)$$

where $v_e(y)$ and $v_p(y)$ are elastic and plastic shear velocity respectively, $\dot{\tau}$ is shear stress rate while in strain-softening regime its value is always negative. The total shear velocity is

$$v(y) = v_e(y) + v_p(y) \quad (23)$$

3 EXAMPLES AND DISCUSSION

For simplicity in present analysis , let the re-

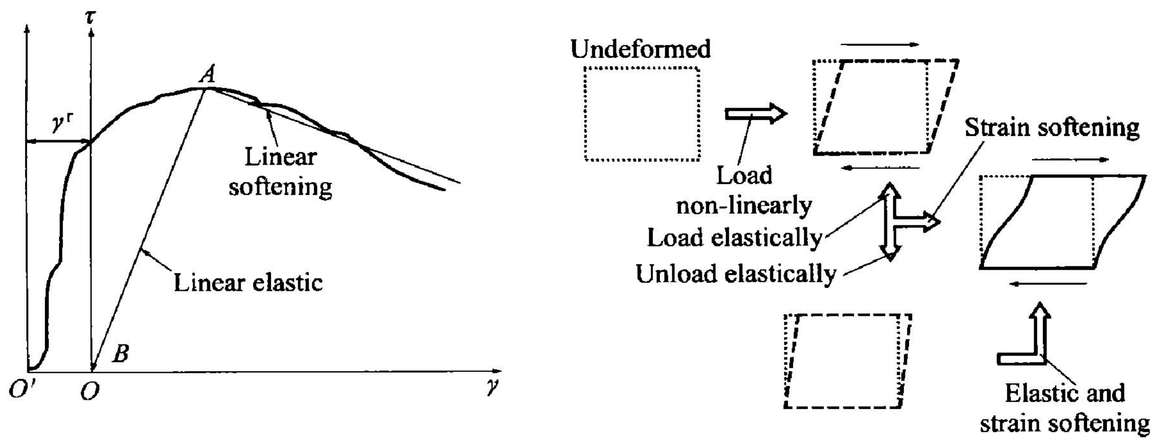

Fig. 3 Complex constitutive relation and its simplification

sidual plastic shear strain $\gamma^r = 0$.

3.1 Characteristics of shear deformation

Herein, we take parameters as follows: w = 100 μm, G = 80 × 10^9 Pa, λ = 0.2G and τ_c = 10^8 Pa. The influences of flow shear stress on shear deformation may be seen from Figs. 4 – 6. In the normal line of shear band, elastic shear displacement is linear. However, in the normal line of shear band, both plastic and total shear displacement are non-linear. In the boundary of shear band, they all attain their maximum. Due to the fact that shear strain denotes the tangent line of shear displacement, in shear band, elastic shear strain is uniform, and plastic shear strain and total shear strain are non-uniform. The reason for the non-uniformity of total shear strain stems from plastic shear strain. Elastic shear displacement is decreased with decreasing flow shear stress in strain softening regime. Contrarily, plastic shear displacement and total shear displacement have been increased. Similarly, elastic shear strain is decreased, and plastic shear strain and total shear strain are increased monotonously in shear band. In conclusion, after localization is initiated, localized deformation develops progressively and more and more non-uniform deformations are localized in shear band, as observed in many tests[17, 18].

3.2 Influences of softening modulus on shear displacement

Herein, we take parameters as follows: w = 100 μm, G = 80 × 10^9 Pa, τ = 0.7τ_c and τ_c = 10^8 Pa. The influences of flow shear stress on shear deformation may be seen from Figs. 7 and 8. For the same flow shear stress level, plastic shear displacement and total shear displacement are increased as strain-softening modulus is decreased.

3.3 Influence of shear modulus on shear displacement

In order to investigate the effect of elastic modulus on shear displacement, let w = 100 μm, λ = 16 × 10^9 Pa, τ = 0.7τ_c and τ_c = 10^8 Pa. From Figs. 9 and 10, we can see that under the same shear stress level larger plastic shear displacement and total shear displacement are caused by lowering elastic shear modulus.

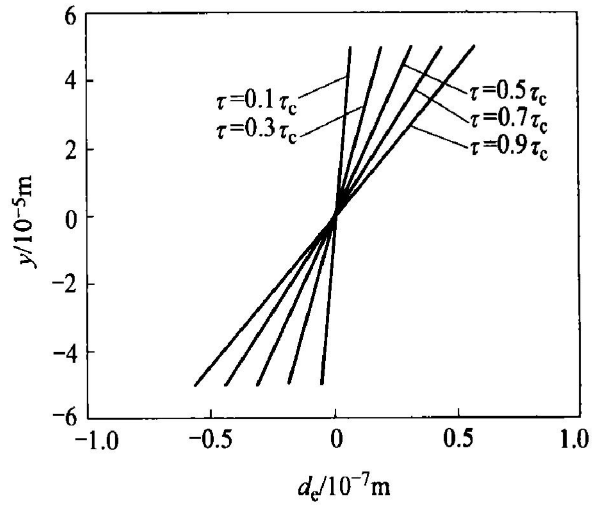

Fig. 4 Distribution of elastic shear displacement with different flow shear stresses

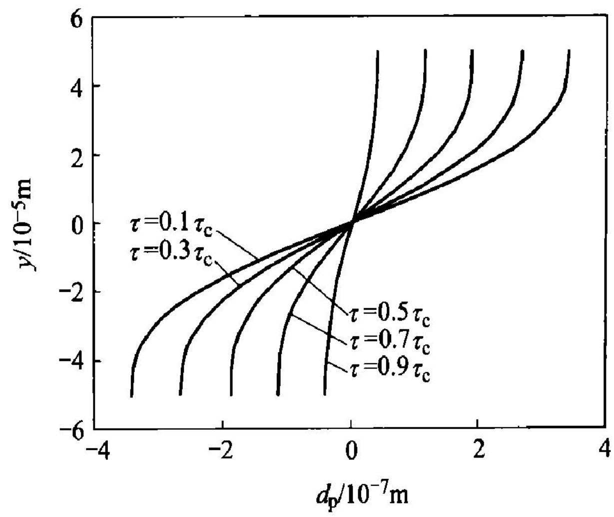

Fig. 5 Distribution of plastic shear displacement with different flow shear stresses

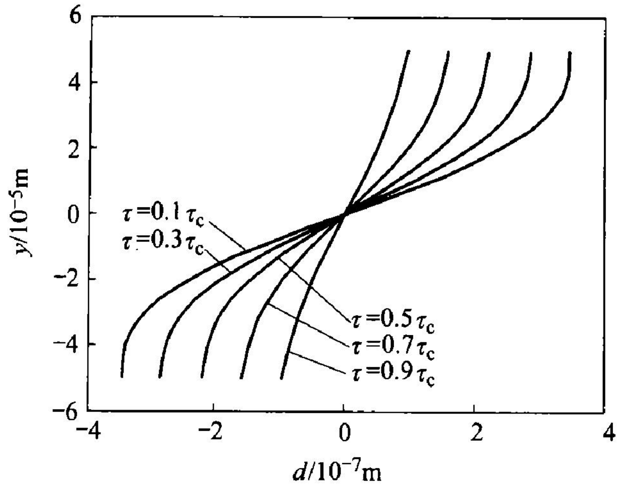

Fig. 6 Distribution of total shear displacement with different flow shear stresses

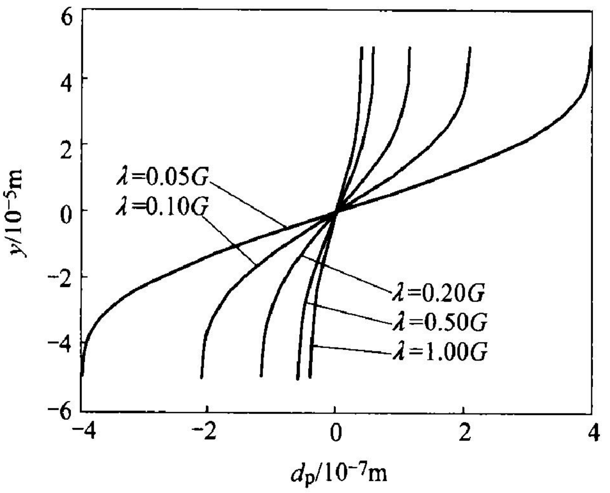

Fig. 7 Distribution of plastic shear displacement with different softening moduli

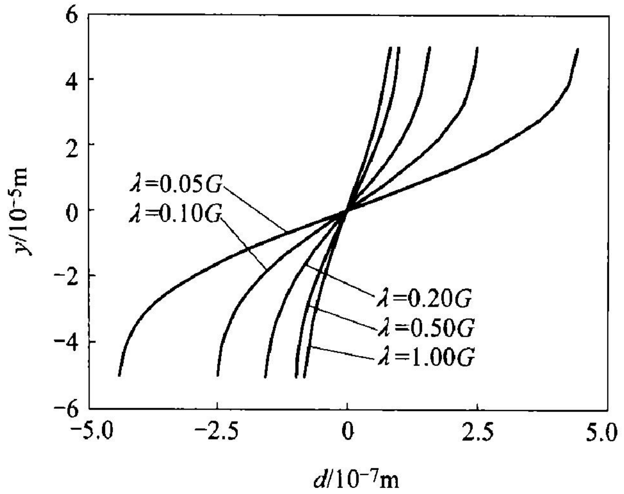

Fig. 8 Distribution of total shear displacement with different softening moduli

3.4 Experimental results for shear localization of metals

In the shearing process after localization occurs, any line in the normal line of shear band no longer remains straight line, as observed in tests[17, 18]. Compared with many experimental re-

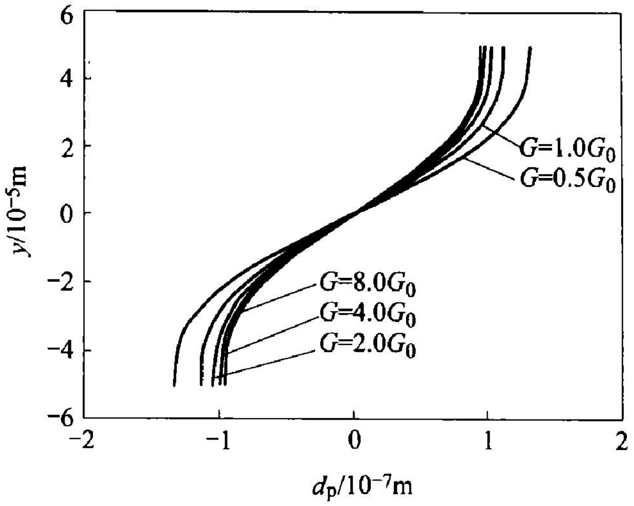

Fig. 9 Distribution of plastic shear displacement with different elastic shear moduli

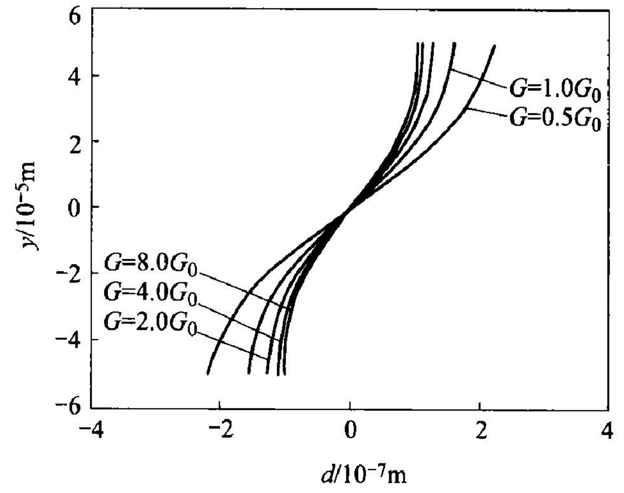

Fig. 10 Distribution of total shear displacement with different elastic shear moduli

sults for shear localization, the present analytical solutions on shear localization based on gradient-dependent plasticity considering microstructures interaction due to heterogeneous texture are able to predict the localized characteristics of shear band successfully.

4 CONCLUSIONS

After shear localization is initiated, shear band develops progressively and more and more non-uniform shear deformations are localized in shear band. Elastic shear strain in the band is uniform and the non-uniformity of total shear displacement stems from localized plastic shear displacement. Moreover, shear modulus and strain-softening modulus have a quite opposite influence on shear deformation. The present analytical solution can be used to predict the localized characteristics of shear band, as can be observed in many experimental tests for shear localization of ductile metal materials.

REFERENCES

[1] de Borst R, Muhlhaus H B. Gradient-dependent plasticity: formulation and algorithmic aspects[J]. International Journal for Numerical Methods in Engineering, 1992, 35: 521 - 539.

[2] Ellen K, Ekkehard R, de Borst R. An anisotropic gradient damage model for quasi-brittle materials[J]. Computer Methods in Applied Mechanics and Engineering, 2000, 183: 87 - 98.

[3] Askes H, Pamin J. Dispersion analysis and element-free Galerkin solutions of second- and fourth-order gradient-enhanced damage models[J]. International Journal for Numerical Methods in Engineering, 2000, 49: 811 - 832.

[4] Vardoulakis I, Aifantis E C. A gradient flow theory of plasticity for granular materials[J]. Acta Mechanica, 1991, 87: 197 - 217.

[5] Zhang H W, Schrefler B A. Gradient-dependent plasticity model and dynamic strain localization analysis of saturated and partially saturated porous media: one dimensional model[J]. Eur J Mech —A/Solids, 2000, 19: 503 - 524.

[6] Shawki T G, Clifton R J. Shear band formation in thermal viscoplastic materials[J]. Mechanics of Materials, 1989, 8: 13 - 43.

[7] WANG Xue-bin, PAN Yi-shan. Size effect and snap back based on conservation of energy and gradient-dependent plasticity[A]. Lin Yunmei et al. Second International Symposium on New Development in Rock Mechanics and Rock Engineering[C]. New Jersey: Rinton Press, 2002. 89 - 92.

[8] PAN Yi-shan, WANG Xue-bin, LI Zhong-hua. Analysis of the strain softening size effect for rock specimens based on shear strain gradient plasticity theory[J]. International Journal of Rock Mechanics and Mining Sciences, 2002, 39(6): 801 - 805.

[9] WANG Xue-bin, PAN Yi-shan, SONG Wei-yuan. The model of plastic shear strain gradient on size effect in uniaxial compression of rock specimen[J]. Chinese Journal of Geotechnical Engineering, 2001, 23(6): 711 - 713. (in Chinese)

[10] WANG Xue-bin, PAN Yi-shan. Theoretical analysis of relation between stress and strain in consideration of confining pressure and pore pressure of rock specimen [J]. Journal of Geomechanics, 2001, 7(3): 265 - 270 (in Chinese)

[11] WANG Xue-bin, PAN Yi-shan, MA Jin. Theoretical analysis of stability and unstable sliding of the system composed of shear band and elastic rock[J]. Chinese Journal of Geotechnical Engineering, 2002, 24(3): 360 - 363. (in Chinese)

[12] WANG Xue-bin, PAN Yi-shan. Analysis of size effect of inclination angle of shear band and the onset, skip and stability of shear strain localization[J]. Journal of Rock and Soil Mechanics, 2002, 23(4): 446 - 450. (in Chinese)

[13] WANG Xue-bin, PAN Yi-shan, MA Jin. A theoretical study on ductile shear band based on strain gradient plasticity theory[J]. Journal of Geomechanics, 2002, 8(1): 79 - 86. (in Chinese)

[14] LI Chen-hui, XIONG Wei-hao, YU Li-xin. Evolution of Ti(C, N)-based cermet microstructures[J]. Trans Nonferrous Met Soc China, 2002, 12(2): 214 - 218.

[15] LI Xiao-qiang, HU Lian-xi, WANG Er-de. Microstructure of TiAl alloy prepared by intense plastic deformation and subsequent reaction sintering[J]. Trans Nonferrous Met Soc China, 2002, 12(4): 621 - 625.

[16] XIA Yuan, YU Sheng-xue, YAO Mei. Effect of diffusion on coating microstructure and oxidation resistance of aluminizing steel[J]. Trans Nonferrous Met Soc China, 2001, 11(6): 817 - 822.

[17] XU Y B, BAI Y L, XUE Q. Formation, microstructure and development of localized shear deformation in low-carbon steels[J]. Acta Mater, 1996, 44: 1917 - 1927.

[18] BAI Y L, XUE Q, XU Y B. Characteristics and microstructure in the evolution of shear localization in Ti-6Al-4V alloy[J]. Mechanics of Materials, 1994, 17: 155 - 168 .

(Edited by YANG Bing)