

## FEM and FVM compound numerical simulation of aluminum extrusion processes<sup>①</sup>

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**Abstract:** The finite element method (FEM) and the finite volume method (FVM) numerical simulation methods have been widely used in forging industries to improve the quality of products and reduce the costs. Because of very concentrative large deformation during the aluminum extrusion processes, it is very difficult to simulate the whole forming process only by using either FEM or FVM. In order to solve this problem, an FEM and FVM compound simulation method was proposed. The theoretical equations of the compound simulation method were given and the key techniques were studied. Then, the configuration of the compound simulation system was established. The tube extrusion process was simulated successfully so as to prove the validity of this approach for aluminum extrusion processes.

**Key words:** FEM; FVM; compound simulation; aluminum extrusion

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### 1 INTRODUCTION

The finite element method (FEM) is widely used in the simulation of metal forming processes since it can describe the deformation behaviors of materials in details, especially for free surface tracking. But the mesh system is easy to be distorted frequently during the deformation process due to Lagrange coordinates are adapted in FEM. Therefore, the rezoning has to be done in order to simulate the large deformation processes. However, the rezoning will decrease the precision of simulation results and even lead the simulation process to be terminated.

By using the FVM, the rezoning is not needed because the calculations are performed in a fixed finite volume mesh system and materials are simplified to flow through this frame. But the free surface of deformation materials is hard to be described precisely in the FVM simulation. So, a long time computation is still needed for its integral calculus.

Aluminum extrusion is an extremely concentrative large deformation process. It is difficult to simulate this kind of process by only using either FEM or FVM alone. So, it is necessary to find a combined solution which possesses the both advantages.

### 2 PLASTIC FEM THEORY

#### 2.1 Basic equations

The rigid visco-plastic formulation has been used extensively for the prediction of material response during metal forming operations<sup>[1-5]</sup>. The basic equa-

tions that have to be satisfied during a forming process are

1) Equilibrium condition

$$\sigma_{ij,j} = 0$$

2) Strain rate definition

$$\dot{\epsilon}_{ij} = \frac{1}{2}(\dot{v}_{i,j} + \dot{v}_{j,i})$$

3) Constitutive relation

$$\sigma'_{ij} = \frac{2\sigma}{3\dot{\epsilon}}\dot{\epsilon}_{ij}$$

4) Incompressibility condition

$$\dot{\epsilon}_{kk} = 0$$

5) Boundary conditions

$$\sigma_{ij}n_j = \bar{F}_i \quad \text{on} \quad S_f$$

$$v_i = \bar{v}_i \quad \text{on} \quad S_v$$

The field equations given above can be solved by a variational principle expressed as<sup>[6]</sup>

$$\delta\Phi = \int_V \bar{\sigma}\delta\dot{\epsilon} dV + \int_V k\dot{\epsilon}_{kk}\delta\dot{\epsilon}_{kk} dV + \int_{S_f} F_i\delta v_i dS = 0 \quad (1)$$

where  $V$  is the volume of the work-piece;  $S_f$  is the force surface;  $k$  is a large positive constant to penalize the volume change. The variational function can be converted to non-linear algebraic equations by utilizing the FEM discretization procedure. The solution of non-linear simulation equations can be obtained by Newton-Raphson method.

#### 2.2 Heat transfer formation

Temperature plays an important role in forming operations since the material properties are highly de-

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pendent on temperature. So the heat resulting from deformation and friction must be considered. The energy balance equation can be expressed as<sup>[7]</sup>

$$\int_V k T_{,i} \delta T_{,i} dv + \int_V \rho \bar{T} \delta T dv - \int_V \alpha \bar{\sigma} \dot{\epsilon} \delta T dv - \int_S q_n \delta T ds = 0 \quad (2)$$

where  $\rho$  is the material density,  $\alpha$  is the fraction of mechanical energy converted into heat energy and is assumed to be 0.9 in this paper;  $q_n$  is the heat flux normal to the boundary surface;  $c$  is the specific heat capacity. Eqn. (2) can be converted to a matrix equation by utilizing the FEM discretization procedure:

$$CT + K \dot{T} = Q \quad (3)$$

The first-order differential equation in Eqn. (3) is rewritten by using a two-points recurrence relation as

$$\frac{C}{\Delta t} (T_n - T_{n-1}) + \beta K T_n = \frac{Q}{\Delta t} \quad (4)$$

where  $\beta = \frac{C}{C + \Delta t K}$ ;  $\beta$  is a positive constant between 0 and 1.

### 3 PLASTIC FVM THEORY

#### 3.1 Mechanical formulations

The finite volume method is cell-centered discretized. The governing equations are given from the conservation of mass, momentum and energy, and the constitutive relation<sup>[8]</sup>.

1) Mass conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_i)}{\partial x_i} = 0 \quad (5)$$

2) Momentum conservation

$$\frac{\partial (\rho v_i)}{\partial t} + \frac{\partial (\rho v_j v_i + p \delta_{ij} - S_{ij})}{\partial x_j} = 0 \quad (6)$$

3) Energy conservation

$$\frac{\partial (\rho E)}{\partial t} + \frac{\partial (\rho v_i E)}{\partial x_i} = \frac{\partial [v_i (S_{ij} - p \delta_{ij})]}{\partial x_j} \quad (7)$$

where  $\rho$ ,  $v_i$ ,  $S_{ij}$ ,  $p$  and  $E$  are respectively the density, a component of the velocity vector, a component of the Cauchy stress deviators, the pressure and internal energy.

4) Constitutive relation

$$\begin{cases} \frac{\partial \sigma_{ij}}{\partial t} = G \left[ \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right] + \Lambda \frac{\partial v_k}{\partial x_k} \delta_{ij} \\ \dot{\epsilon}_{ij}^p = \Lambda \frac{\partial f}{\partial \sigma_{ij}} \end{cases} \quad (8)$$

where  $\dot{\epsilon}_{ij}^p$  is a component of the plastic strain rate tensor, and parameter  $\Lambda$  is a function depending on stress, strain rate and strain. The Von Mises isotropic hardening rule is used here as the yield function  $f$ . Here the stress tensor is defined as

$$\sigma_{ij} = -p \delta_{ij} + S_{ij}$$

Under the assumption that the mass density is constant during one acoustic step, the volume integral

from Eqns. (5) and (6) can be represent as

$$\int_V \frac{\partial v_i}{\partial t} dV = \frac{1}{\rho} \int_V \frac{\partial \sigma_{ij}}{\partial x_j} dV \quad (9)$$

Using Gauss' theorem, the volume integral is transformed in the surface integral form in order to allow the application of the finite volume method:

$$\int_V \frac{\partial \sigma_{ij}}{\partial x_j} dV = \int_S \sigma_{ij} n_j dS \quad (10)$$

where  $V$  is the element volume,  $S$  is the area of the element faces and  $n_i$  is a component of the unit vector normal to the surface.

The face values of the finite volume elements can be solved from the Riemann problem, posed at the element faces. The state variables in the governing equations are evaluated in a Runge-Kutta time integration scheme<sup>[9]</sup>. The updated velocity field is then used in the advection step.

#### 3.2 Thermal formulations

The temperature field is affected by the heat generation converted from mechanical work. So the heat conduction equation is derived by considering the total energy balance<sup>[10]</sup>:

$$\rho c_p \dot{T} - \frac{\partial}{\partial x_i} \left( k \frac{\partial T}{\partial x_i} \right) - \sigma_{ij} \dot{\epsilon}_{ij}^p = 0 \quad (11)$$

where  $k$  denotes the coefficient of heat conduction,  $T$  is the temperature and  $c_p$  is the specific heat capacity.

In addition to the heat conduction within the material, heat transfer from the surface of the workpiece and the dies is treated using Newton's law of cooling:

$$k \frac{\partial T}{\partial x_i} n_i = -h(T - T_w) \quad (12)$$

where  $h$  is the heat transfer coefficient, and  $T_w$  is the ambient or die temperature.

### 4 COMPOUND SIMULATION TECHNIQUES

#### 4.1 Application scopes of FEM and FVM in compound simulation system

In aluminum extrusion processes, it is very difficult for the metal to flow directly through the die land for too large extrusion ratio. So preform is always necessary. In the preform process, materials are allocated into the preform die firstly. After the preform die is filled out, materials begin to come through the die land. Thus, the extrusion ratio is divided into two parts and each part decreases correspondingly.

Simulation of metal forming with FEM has certain advantages. In the simulation process with FEM, surface meshes represent the free surface of the workpiece. So the free surface of the deforming mate-

rial can be tracked in real time through the movement of nodes. Meanwhile, surface nodes can not only discriminate the die, which means that they can touch the die or separate from the die, but also discriminate themselves. So materials can be welded when they meet each other after materials enter the welding room. It is very useful to simulate the porthole extrusion process.

On the other hand, rezoning of the FEM mesh occurs frequently when material flows through the die land because the die cavity is very narrow and the billet is too large relatively. Sometimes, the simulation with FEM is no longer carried out for the troubleshooting of rezoning. So FEM is not suitable for the simulation of aluminum extrusion processes in the shape extrusion stage.

In the simulation process of metal forming with FVM, the Euler meshes must cover the whole area that the material flows through. Thus, a large number of meshes are needed in the simulation of aluminum extrusion which leads to a long time computation because the wall of the product is very thin and the dimension of the billet is great contrarily. Meanwhile, the metal welding process in the porthole extrusion cannot be simulated as materials cannot discriminate themselves in the FVM simulation system. So FVM is not suitable for the simulation of aluminum extrusion processes in the preforming stage, especially for porthole extrusions.

The most important advantage of FVM simulation is that rezoning can be avoided validly. That means the simulation of the shape extrusion stage can be finished by FVM. Meanwhile, the calculation time of FVM will be shortened if the simulation result of FEM can be inherited by FVM.

Therefore, FEM is adapted to the simulation of aluminum extrusion processes in the pre-forming stage. When materials flow through the die land, FVM is recommended.

#### 4.2 Data transformation from FEM to FVM

Since die geometries in both FEM and FVM simulation systems are represented by triangle facets, they can be transferred directly through STL files. The geometry of workpiece in the FVM simulation system is also described with triangle facets. The deformed surface meshes in FEM system can be translated into triangle facets firstly and exported to a STL file. Thus the geometry data of work-piece can also be imported into the FVM system.

In order to transfer variables such as stress, strain, temperature, etc, the locations of FVM grid points in the FEM meshes must be searched firstly. After the local coordinate of a FVM grid point in a FEM element is calculated, the variables of this grid point can be interpolated as

$$f_i(\xi, \eta, \zeta) = \sum_{j=1}^m N_j(\xi, \eta, \zeta) f_j \quad (13)$$

where  $f_i(\xi, \eta, \zeta)$  is the variables of the FVM grid,  $f_j$  is the variables of nodes in the correspondent FEM element,  $m$  is node numbers of the element and  $N_j(\xi, \eta, \zeta)$  is the FEM interpolation function.

#### 4.3 Configuration of compound simulation system

According to the above discussions, the compound simulation system of aluminum extrusion processes is consisted of three parts. They are the FEM simulation system, the data transformation system and the FVM simulation system. The flow chart illustration of the compound simulation system is represented as shown in Fig. 1.

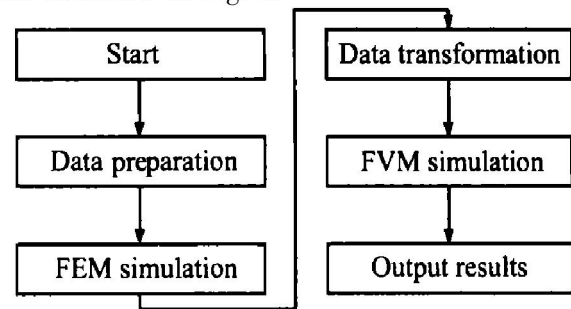


Fig. 1 Flow chart illustration of compound simulation system

### 5 EXAMPLE

#### 5.1 Model

Fig. 2 shows an aluminum extrusion product of tube with the profile shown in Fig. 3. A porthole die for preform is used, which is shown in Fig. 4. The dimensions of the column billet for simulation are  $d77.8 \text{ mm} \times 55.0 \text{ mm}$ . The total extrusion ratio is 45.06. The material is 6061 aluminum alloy. The deformation temperature of billet is  $480^\circ \text{C}$  and the temperature of die is  $460^\circ \text{C}$ . The coefficient of friction is 0.45. The extrusion punch moving speed is  $10 \text{ mm/s}$  along  $Z$  direction.

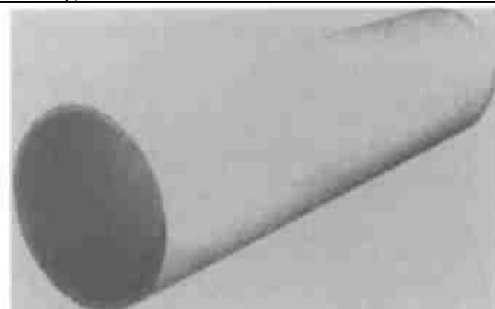
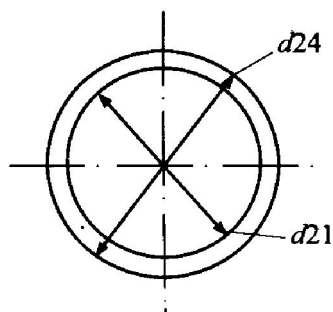


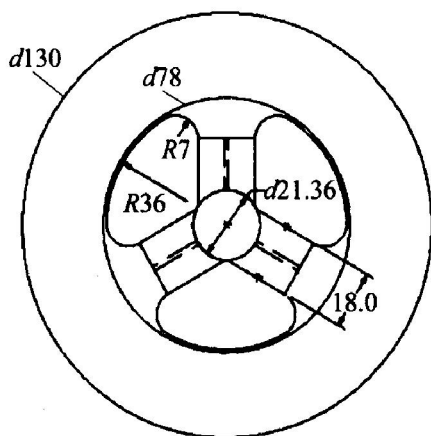
Fig. 2 Aluminum extrusion tube

#### 5.2 Simulation parameters

The initial billet is discretized with 23 406 elements and 5 297 nodes for FEM simulation. The stroke is 37.86 mm in the preform stage and the time increment is 0.04 s for each FEM iteration step. The



**Fig. 3** Profile of tube



**Fig. 4** Porthole die of tube

deforming material is discretized with 490 000 Euler elements during FVM simulation process. The stroke is 6.67 mm in the shape extrusion stage.

### 5.3 Simulation results

#### 5.3.1 FEM simulation

The flow and welding processes of materials are shown in Fig. 5. It can be seen from Fig. 5(b) that the metal separates and flows into the holes of the preform die under the push of the punch firstly. After the materials arrive the bottom of the preform die, they begin to gather each other (Fig. 5(c)~(e)) and weld at last (Fig. 5(f)) under the constraint of the die. This process demonstrates successfully the performance of deformation material tracking in FEM simulation system.

When the metal welding process is over, materials will flow into the shape extrusion die. The FEM simulation process for preforming stage is finished correspondently.

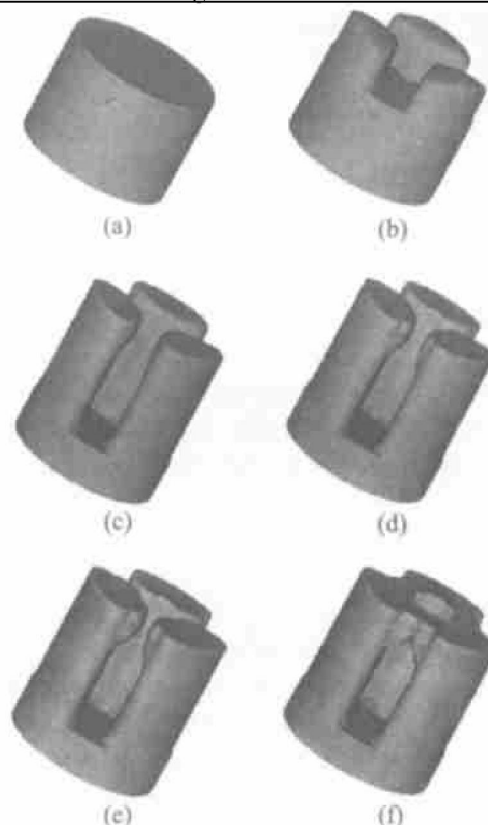
#### 5.3.2 Data transformation

In order to simulate the whole extrusion process, the FEM simulation results must be transferred into the FVM simulation system. The transformation results of geometry and effective stress data are shown in Fig. 6 and Fig. 7 respectively. It can be seen from these figures that the data are inherited very well and the transformation is successful.

#### 5.3.3 FVM simulation

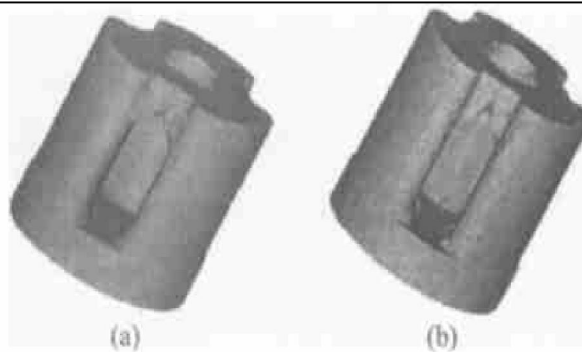
The metal flow process in the shape extrusion

stage simulated with FVM is shown in Fig. 8. It can be seen from these figures that materials with



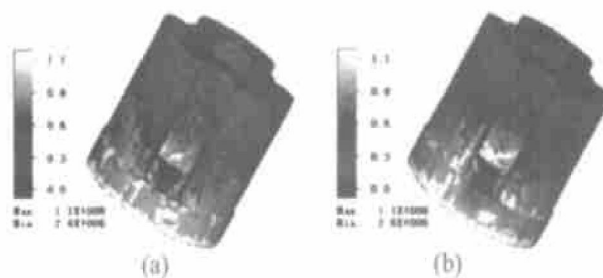
**Fig. 5** Metal flow and welding process in FEM simulation

(a) —Stroke 0 mm; (b) —Stroke 11.53 mm;  
(c) —Stroke 34.53 mm; (d) —Stroke 35.33 mm;  
(e) —Stroke 36.13 mm; (f) —Stroke 37.53 mm



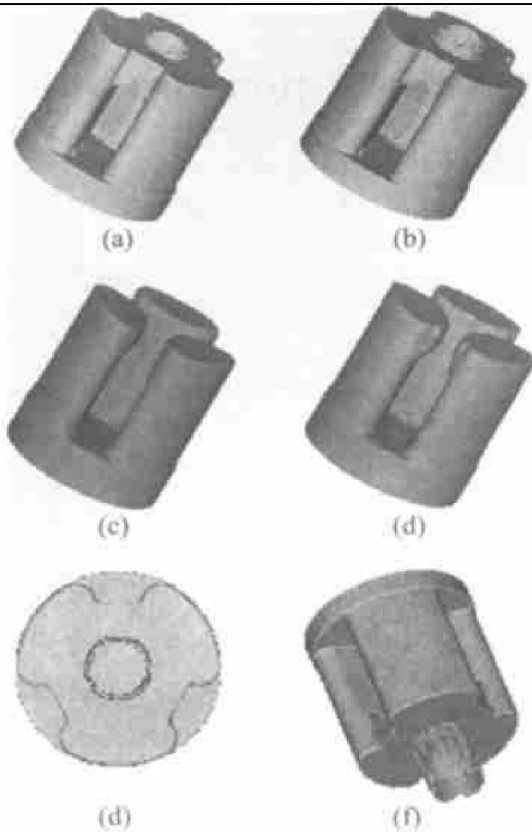
**Fig. 6** Geometry data transformation from FEM to FVM (stroke 37.86 mm)

(a) —FEM data; (b) —FVM data



**Fig. 7** Effective stress field transformation from FEM to FVM (stroke 37.86 mm) (Unit: MPa)

(a) —FEM data; (b) —FVM data



**Fig. 8** Metal flow process in FVM simulation

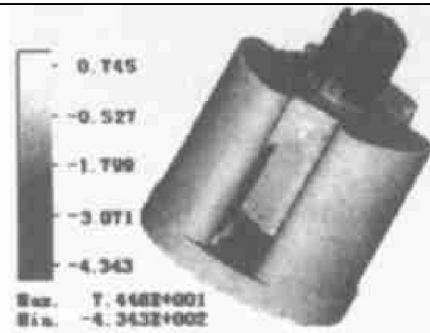
- (a) —Stroke 40.08 mm; (b) —Stroke 41.20 mm;  
 (c) —Stroke 42.31 mm; (d) —Stroke 44.53 mm;  
 (e) —Stroke 44.53 mm; (f) —Stroke 44.53 mm

welding behaviors flow slower than others in the welding room because there is an additional friction between two welded parts. Fig. 8(e) and Fig. 8(f) show that the extruded tube is similar to the practical product.

Fig. 9 shows the distribution of effective stress with stroke of 44.53 mm. The distribution of velocity along the extrusion direction with stroke of 44.53 mm is shown in Fig. 10. It can be seen from Fig. 10 that the velocity at the front end of the tube in the steady extrusion process is 434.3 mm/s. Its theoretic velocity is 450.6 mm/s, which can be calculated according to the extrusion velocity of punch and the extrusion ratio. The error between the simulation result and the theoretic result is 3.6%. Therefore, the simulation results are helpful for the design of products and dies.



**Fig. 9** Distribution of effective stress  
 (Unit: MPa)



**Fig. 10** Distribution of velocity along extrusion direction (Unit: mm/s)

## 6 CONCLUSION

During the numerical simulation of aluminum extrusion processes, FEM is used in the preforming stage and FVM is used in the shape forming stage. Meanwhile, data are effectively transferred from FEM simulation system to FVM simulation system. The simulation results show that the free surface of deforming material is tracked accurately and the simulation is carried out thoroughly. Since both advantages are taken and shortages are avoided, the technology of compound simulation of FEM and FVM is especially valid for aluminum extrusion processes.

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