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Self-organized criticality of liquefaction in saturated granules ①

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Abstract: Utilizing the dissipative structure theory, the evolutionary process of vibrating liquefaction in saturated granules was analyzed. When the irreversible force increases to some degree, the system will be in a state far from equilibrium, and the new structure probably occurs. According to synergetics, the equation of liquefaction evolution was deduced, and the evolutionary process was analyzed by dynamics. The evolutionary process of vibrating liquefaction is a process in which the period doubling accesses to chaos, and the fluctuation is the original driving force of system evolution. The liquefaction process was also analyzed by fractal geometry. The steady process of vibrating liquefaction obeys the scaling form, and shows self-organized criticality in the course of vibration. With the increment of the recurrence number, the stress of saturated granules will decrease rapidly or lose completely, and the strain will increase rapidly, so that the granules can not sustain load and the "avalanche" phenomenon takes place.

Key words: liquefaction; self-organized criticality; dissipative structure; fractal

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1 INTRODUCTION

Self-organized criticality is a process in which invariable energy is continually given to a system from the outside and is finally dissipated in the form of the "avalanche". Self-organized criticality is a fractal concept first brought forward by Bark et al[1], which has the "avalanche" characteristic. A typical example is the sandy beach model[2-4]. On a sandy beach which has a small angle of slope, and sand grains are laid on it one by one. In the beginning, only a small amount of partial adjustment is caused. If sand grains are continued to lay on it, the slope of sandy beach will reach a critical value. Thereafter, if only one sand grain is laid on it, "avalanche" will be caused. Here, the behavior of this sandy beach is no longer the partial multiple degrees of freedom behavior and can be described as a whole. Because of Bark's pioneering work, the self-organized criticality was applied soon afterwards to many "avalanche" incidents, such as earthquake, forest fire, rainfall, etc. By analyzing this paper it appears that the self-organized criticality can be applied to vibrating liquefaction.

Different kinds of granular media (e. g. broken rocks, sandy soils and tailings) have different formative phenomena, but granules and pores have irregularity, self-similarity, fuzziness, nonlinearity, complexity, etc. Liquefaction of saturated granules is a phenomenon that granular media show similar to liquid characteristic and lose their carrying capacity completely under the action of dynamic force. Seismic wave, vehicle running, machine vibration, piling, explosion, and so on could probably assist in liquefaction of saturated granules[5-7]. This paper utilizes the dissipative structure theory, synergetics and fractal geometry to analyze the process of the saturated granules.

2 ENERGY DISSIPATION OF VIBRATING LIQUEFACTION PROCESS

According to the dissipative structure theory[8-13], the concept of entropy is firstly introduced in order to describe the self-organized criticality of vibrating liquefaction. σ is defined as the entropy at the unit time in the unit area, and is also called the local entropy. The general expression is

$$\sigma = \sum J_k \times X_k \geqslant 0 \qquad (1)$$

where J_k represents the kth irreversible flow, X_k represents the kth irreversible force. Generally speaking, the macroscopic physical quantity can be divided into two kinds. One is the extensive quantity relating to the general mass of the system, such as volume, total energy, force; the other is the intensive quantity, and is also called the "field", such as temperature, displacement, stress. Correspondingly, during the course of the vibrating liquefaction, J_k is an intensive quantity. The displacement, the offset, the instability of the granular frame-

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work, and the deformation are irreversible flows. These also can predict the vibrating liquefaction as for the static parameters of vibrating liquefaction. The irreversible forces X_k in this system mainly are the granular friction, the viscous resistance, the power of pore water, the acting force of vibration to granules, etc.

The total entropy of the system is

$$P = \int_V \sigma \mathrm{d}V \geqslant 0 \tag{2}$$

where V represents the granular volume.

When the granules are in the equilibrium state, there is

$$\sigma = 0;\ J_k = 0;\ X_k = 0 \tag{3}$$

Namely, neither the irreversible flow nor irreversible force exists in the equilibrium state. Once the system is not in an equilibrium state, a macroscopic irreversible process will occur. At this time the irreversible flow and the irreversible force are not zero. The irreversible force is the resource of the irreversible flow, and only where there is the irreversible force, there is the irreversible flow. In other words, when the granules are still in the non-vibrating state, any granule will be in a steady state, and the resultant force is zero. Certainly, there is no any irreversible flow, such as the granular deformation or displacement. Therefore the granules will be in a non-equilibrium state under the action of force during the course of steady vibration, viz, the so-called local non-equilibrium, and the irreversible force will be generated.

From the above analysis, it can be known that the irreversible flow is a certain function of the irreversible force. Suppose that a certain irreversible flow J_k is the function of all irreversible forces($\{X_l\} = X_1, X_2, \cdots, X_n$):

$$J_k = J_k(\{X_l\}) \tag{4}$$

Eqn. (4) is expanded by Taylor's series, and is simplified to obtain the following equation:

$$J_k = \sum_l L_{kl} X_l \tag{5}$$

$$L_{kl} = \left[\frac{\partial J_k}{\partial X_l}\right]_0 \tag{6}$$

During the course of the formation of vibrating liquefaction, because of the continuous vibration, on one hand the granular mechanical energy and the irreversible forces will increase; on the other hand the granular frictional resistance and viscous force will decrease gradually with the change of the granular framework's external form. These two aspects will certainly lead to the increment of the irreversible forces during the course of the vibrating liquefaction. When the irreversible forces are increased to a certain value, there is a nonlinear relation between the irreversible forces and the irreversible flows, and the system will transform into a far-from-equilibrium state. The non-equilibrium zone obeying the nonlinear relation is called the nonequilibrium nonlinear zone, where an ordered structure will occur.

3 EVOLUTIONAL EQUATION OF VIBRATING LIQUEFACTION

The synergetics believe that the nonlinear system with any complex structure is a kind of evolutional self-organized system. Accordingly, the evolutional process of liquefaction can be described by using two variables (y, s), where y represents the slow variable and s represents the quick variable. The system's evolutional equation can be expressed as Langevin equation:

$$y = K(y, s) + F(t) \tag{7}$$

where $K(y, s)$ is the linear function including the slow variable and the quick variable; $F(t)$ is the fluctuation force.

On one hand, the evolution of any nonlinear system is controlled by the inner factors of the system, viz, the nonlinear interaction among all subsystems which can be expressed by the nonlinear function $K(y, s)$. On the other hand, it will be influenced by the external factors, which can be expressed by the fluctuation force $F(t)$. The internal cause is an essential one, but the external actions mainly impel to the change of internal cause and drive the qualitative change to the critical points. Therefore, in a simple situation the external factors, such as $F(t)$, can be ignored.

For the two-dimensional system, Eqn. (7) will become

$$K(y, s) = ay - ys \tag{8}$$

When y becomes small, several integral terms in the above equation can be ignored, then the following equation is obtained:

$$\dot{y} = \frac{\mathrm{d}y}{\mathrm{d}t} = ay - by^3 \tag{9}$$

where a, b are the control parameters.

A difference form of the Eqn. (9) is

$$y_{n+1} = (a+1) y_n \left(1 - \frac{b}{a+1} y_n^2\right) \tag{10}$$

Order $u = a + 1$, $x_n = \sqrt{\frac{b}{a+1}}\, y_n$, so Eqn. (10) will become

$$x_{n+1} = u x_n (1 - x_n^2) \tag{11}$$

Eqn. (11) will generate a doubling period with the increment of u. For $u \to u_\infty$ ($u_\infty \approx 2.3$), the period 2^∞ occurs, namely the evolution of the system will transform from the period area to the chaos area. Therefore, the evolution of the vibrating liquefaction is one process from a doubling period to the chaos. This process is the dynamic evolution process of vibrating liquefaction, and can be simulated by a computer (as shown in Fig. 1).

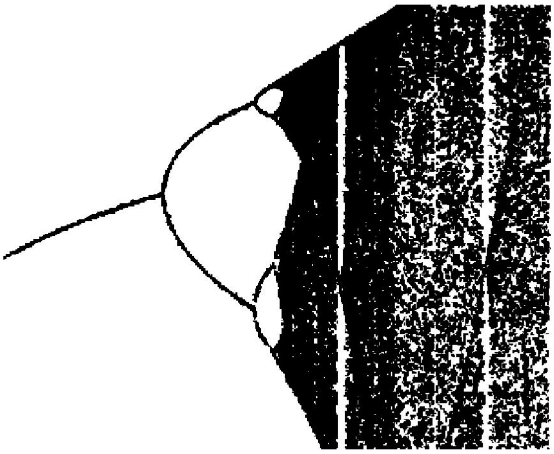

Fig. 1 Dynamic evolution simulation of vibrating liquefaction

4 DYNAMIC ACTION ANALYSIS OF VIBRATION ON LIQUEFACTION

In order to consider the influence of vibration on liquefaction, the control parameter a in Eqn. (9) is regarded as a random variable $\lambda(t)$, so Eqn. (9) is turned into the following random differential equation:

$$\frac{dy}{dt}=\lambda(t)y-by^3 \quad (12)$$

$\lambda(t)$ can be expressed as

$$\lambda(t)=\bar{\lambda}+\xi(t) \quad (13)$$

where $\bar{\lambda}$ is the mean value of $\lambda(t)$, $\xi(t)$ is the fluctuation term.

The general formation of Eqn. (9) is

$$\frac{dy}{dt}=f(y)+\lambda(t)g(y) \quad (14)$$

where $f(y)=-by^3$, $g(y)=y$.

After some mathematical operations, the following equation is obtained:

$$\frac{1}{2}\sigma^2\frac{\partial g^2(y)}{\partial y^2}=f(y)+\bar{\lambda}g(y) \quad (15)$$

where σ^2 is the variance of fluctuation.

By solving Eqn. (15), the following equation is obtained:

$$y_1=0;\ y_{2,3}=\pm\sqrt{\frac{\bar{\lambda}-\sigma^2}{b}} \quad (16)$$

and the solution of Eqn. (9) becomes

$$y'_1=0;\ y'_{2,3}=\pm\sqrt{\frac{\bar{\lambda}}{b}} \quad (\bar{\lambda}=a) \quad (17)$$

Eqn. (17) shows that there is a steady point $y^*=0$ for $\lambda<0$; $y^*=0$ is critical unstable for $\lambda=0$; $y^*=0$ is unstable for $\lambda>0$, and two new stable solutions $y=\pm\sqrt{\lambda/b}$ and a bifurcation are generated. Eqn. (16) has the similar law. The bifurcation occurs in $\lambda=\sigma^2(\sigma\neq0)$ but not in $\lambda=0$, namely, the external fluctuation makes the bifurcation generate a "dislocation". Therefore, the external fluctuation probably changes the evolutional progression of the system.

5 FRACTAL ANALYSIS OF VIBRATING LIQUEFACTION

Fig. 2 shows the vibratory impulse integrated distribution $N(v)$ against v. After the subtraction of k_2, the double logarithmic graph shows an example of the resulting distributions. It is worthwhile to note that there is a slight departure from the scaling behavior at low and high amplitudes[1-3]. Therefore, the relation between $N(v)$ and v obeys the following Equation:

$$N(v)\approx k_1v^{\gamma}+k_2,\quad \gamma=1.7\pm0.2 \quad (18)$$

Both the autocorrelation function and the power spectrum of the vibratory impulse signal time series have been analyzed, and Fig. 3 shows the autocorrelation function for these two typical data sets. The behavior is an algebraic decay over almost 2 orders of magnitude, and the data fit gives the following relation:

$$C_v(t)\sim t^{-\alpha},\quad \alpha=0.4\pm0.1 \quad (19)$$

This corresponds to a spectrum of the type:

$$S_v(f)\sim f^{-\beta},\quad \beta=1-\alpha \quad (20)$$

Fig. 4 shows the experimental results of saturated

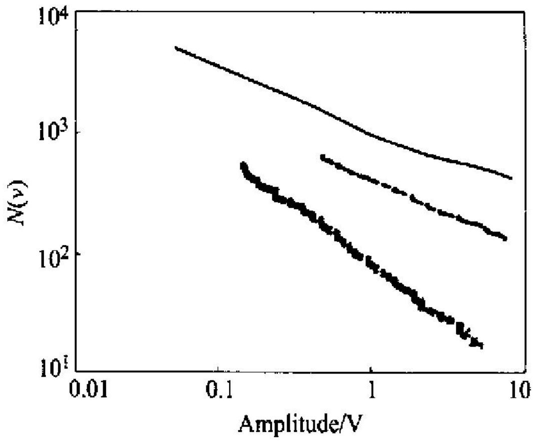

Fig. 2 Relation between $N(v)$ and amplitude(v)

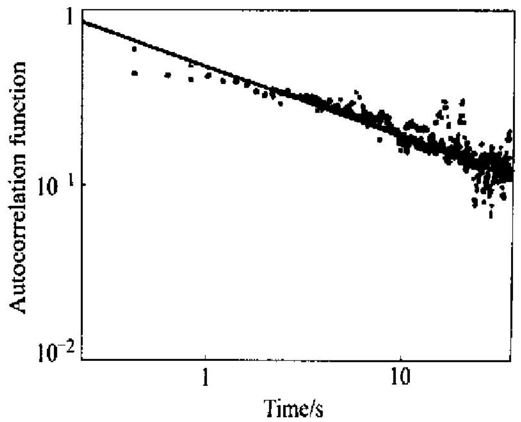

Fig. 3 Autocorrelation functions of vibratory signals time series

granules (silty sand) liquefaction[5]. It can be seen from the relation curve of periodic deviation stress σ_d, dynamic strain ε_d, pore water pressure u_d and cyclic index n that although deviation stress alters within a small range, a certain residual pore water pressure will remain after every stress cycle. With the increment of the stress cycle index, pore water pressure will be continually accumulated and gradually increase, but the effective stress will progressively decrease. Finally when the effective stress is close to zero, granular intensity will suddenly reduce to zero, so the test sample generates liquefaction. Consequently an "avalanche" phenomenon takes place. In the beginning when the strain changes slightly, the dynamic stress maintains an equi-amplitude cycle, and the pore water pressure gradually increases. After a certain number of cycle, the pore water pressure will sharply increase and the strain will also sharply increase. The amplitude value of the dynamic stress will begin to reduce and the granular load capacity will progressively disappear. The above phenomena indicate that liquefaction is in the process. When the pore water pressure is approximately equal to the consolidation pressure, the granules will not be able to bear any load, then the strain will rapidly increase and the dynamic stress will reduce to zero. Subsequently the granules will be in the complete liquefaction and lose the load capacity.

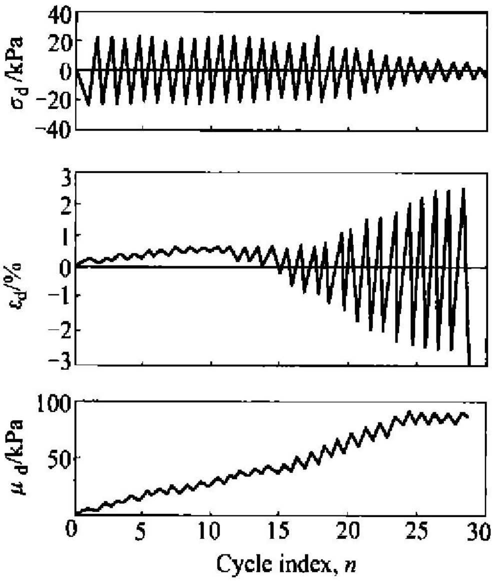

Fig. 4 Liquefaction experimental results of saturated granules

The above analysis shows that the steady state developing process of vibrating liquefaction obeys the power law. Saturated granules have self-organized criticality, which makes the space and time scale invariability of the vibrating liquefaction possible. During the course of vibration with the increment of the cycle index, the granules only have slight adjustment. But after the cyclic index reaches a certain value, at some instant an "avalanche" phenomenon will take place. The granular stress decreases rapidly or loses completely, and the strain increases rapidly. The granules have a typical wholeness characteristic rather than part freedom adjustment, viz, the granules are indicative of obvious self-organized criticality.

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