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# Global coupled equations for dynamic analysis of planishing mill①

CAI Gan-wei(蔡敢为)[1], ZHONG Jue(钟 掘)[2]
(1. College of Mechanical Science & Engineering,
Huazhong University of Science and Technology, Wuhan 430074, China;
2. College of Mechanical & Electrical Engineering,
Central South University, Changsha 410083, China)

**Abstract:** The dynamic properties of rolling mill are significantly influenced by many coupling factors. According to the coupled mechanical and electric dynamics theory, the global coupled equations for the dynamic analysis of planishing mill CM04 of Shanghai Baosteel Group Corporation were derived, by using finite element methods. These elasto-dynamic equations establish the coupling relations among the stand vibration system, torsional vibration system, driving motors, etc. It provides theoretical basis to a certain extent for globally dynamic simulation, analysis of stability of motion, prediction of abnormal operating mode, globally optimum design and control, etc.

**Key words:** rolling mill; dynamic analysis; electromechanical coupling
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## 1 INTRODUCTION

Modern high-speed rolling mill is a complex electromechanical system in which many coupling factors exist. These factors have significant influence on the dynamic properties of the rolling mill, which can't be explained only with separate coupling or their superposition. For example, the influence on dynamic properties brought about by electromechanical coupling and interface coupling can hardly be explained by the superposition of electromechanical coupled forced vibration and self-excitation vibration of rolling interface. During the last few years, a lot of works on the worldwide difficult problem of the vibration of rolling mill have been done by many specialists from different countries, but few of them had attached enough importance to the many coupling factors which do exist in the rolling mill, and the dynamic model of rolling mill established by them can be classified into two kinds: stand vibration system and torsional vibration system of main drive shafting[1-8]. Although great success have been made on the study of such stand and torsional vibration subsystem, there are still many vibration behaviors can't be explained reasonably. Therefore, it is quite necessary to take all kinds of coupling factors into account and research the vibration principle of rolling mill from the viewpoint of the whole system more deeply[9].

The main constituents of planishing mill CM04 of Shanghai Baosteel Group Corporation (CBGS) are shown in Fig. 1. Its mechanical main block is composed of stand vibration system and torsional vibration system, and both of them are coupled with rolling interface. The torsional vibration system is a multi-shaft system with each shafting separately driven by direct-current motors, and they are connected by the strip steel. The electromagnetic fields among the air gap of electromotors couple shaftings with electrical system.

There has been a "ghostly vibration" without any obvious law in the working process of the planishing mill. This often results in the light and shade alternating stripes on the plate being flattened, which significantly influences the quality of products. In this paper, the global coupled equations, which indicate the relations among the stand vibration system, torsional vibration system, driving motors of the planishing mill are established, so as to study more deeply the multi-unit and multi-dimensional relations among all kinds of processes and parameters, the law of feedback between the operation mode and the function of machine set.

## 2 VIBRATION EQUATIONS OF STAND VIBRATION SYSTEM

The stand vibration system of the planishing mill is

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**Correspondence:** CAI Gan-wei, Professor, + 86-27-87543872, caiganwei@163.com

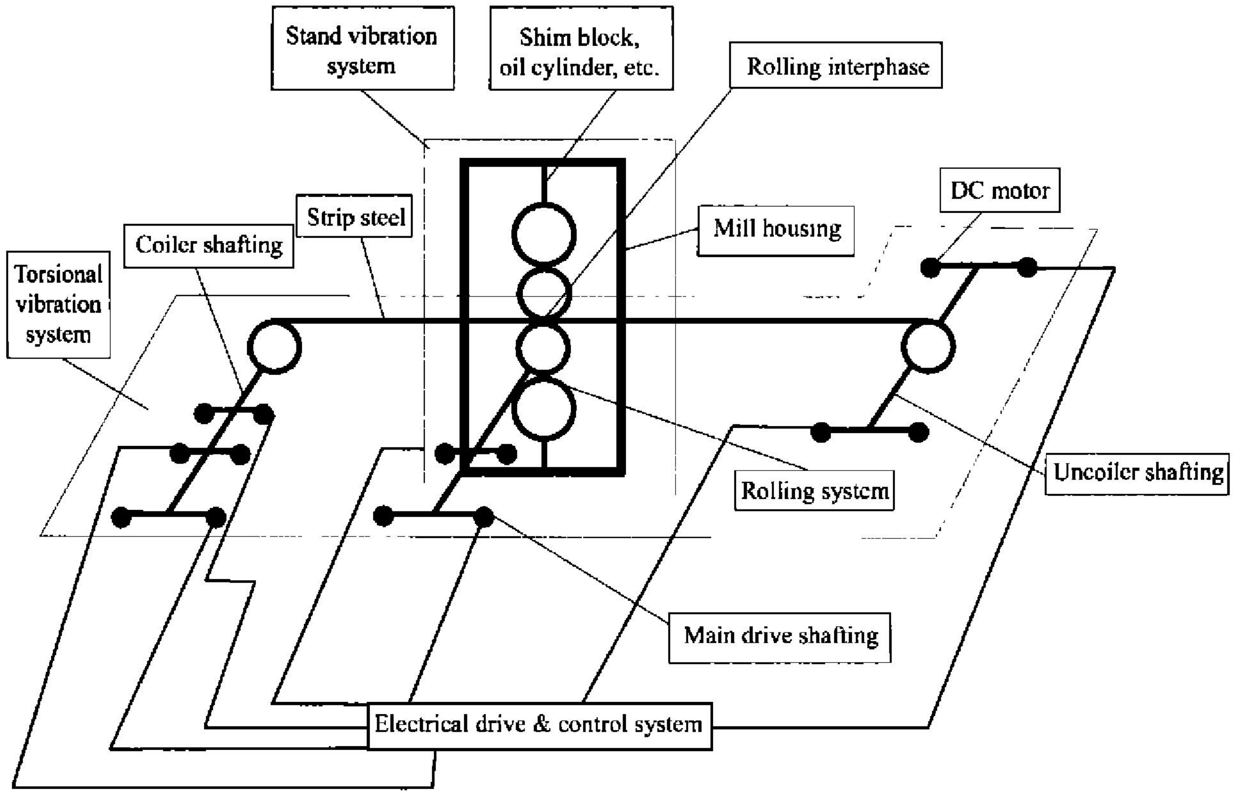


**Fig. 1** Sketch map of planishing mill CM04 system

composed of mill housing, backing roll, working roll, oil cylinder, wedging block, bearings and bearing brackets, etc. Previous studies demonstrate that the finite element method with appropriate discrete scale is suitable for the dynamic analysis of rolling mill[10]. Therefore, the vibration equations for the stand vibration system of four-high mill are established with the finite element model.

The model of the stand vibration system is shown in Fig. 2. The mill housing is divided into 18 beam elements. The oil cylinder and wedging block are regarded as one pole element. The rigidity of the oil film in the oil cylinder, the rigidity of bearings and bearing brackets of upper backing roll and the flexural rigidity of upper backing roll are taken into account in the computation of element stiffness matrix. Each roll set is regarded as a lumped mass element, and these equivalent masses are calculated according to the vibration mode curves of the rollers with conservation of energy principle[10]. The interface between the backing roll and the working roll is respectively simulated as elastic elements whose stiffness is the elastic contact stiffness between the rollers. The constituent between the lower backing roll and the bottom beam is represented by a pole element. According to the real condition of the planishing mill, two constraints are placed on node 1 and node 15.

Each node and generalized coordinate is shown in Fig. 2. Therefore, we can obtain the dynamic equations of the stand vibration system from the finite element model:

$$\boldsymbol{M}_1 \ddot{\boldsymbol{U}}_1 + \boldsymbol{C}_1 \dot{\boldsymbol{U}}_1 + \boldsymbol{K}_1 \boldsymbol{U}_1 = \boldsymbol{Q}_1 \tag{1}$$

where $\boldsymbol{M}_1$, $\boldsymbol{C}_1$ and $\boldsymbol{K}_1$ respectively represent the mass matrix, damping matrix, and stiffness matrix of the stand vibration system; $\boldsymbol{U}_1$, $\dot{\boldsymbol{U}}_1$ and $\ddot{\boldsymbol{U}}_1$ are the column matrix of the generalized coordinates of the system and its first and second derivatives; $\boldsymbol{Q}_1$ is the column matrix of generalized force whose elements are composed of the draught pressure $p$.

## 3 DYNAMIC EQUATIONS OF TORSIONAL VIBRATION SYSTEM

The torsional vibration system of the planishing mill is composed of uncoiling shafting, main drive shafting and coiling shafting. Each shafting is independently driven by DC motor, and then is jointed together through the strip. The dynamic analysis of torsional vibration system is as follows.

### 3.1 Dynamic equations of strip

The thickness and elongation of the strip being planished are both very small (0.3 − 3.5mm, 0.5% − 2.1%), so the variation of the thickness of the strip is neglected in the global model of planishing mill. On both ends of the strip there are the tension forces $\boldsymbol{T}_1$, $\boldsymbol{T}_2$ offered by the uncoiling and coiling shaftings, and on the neutral point there is the frictional force $\boldsymbol{F}$ provided by the working rolls. The strip is simulated with finite pole ele ments, as shown in Fig. 3. The dynamic equation

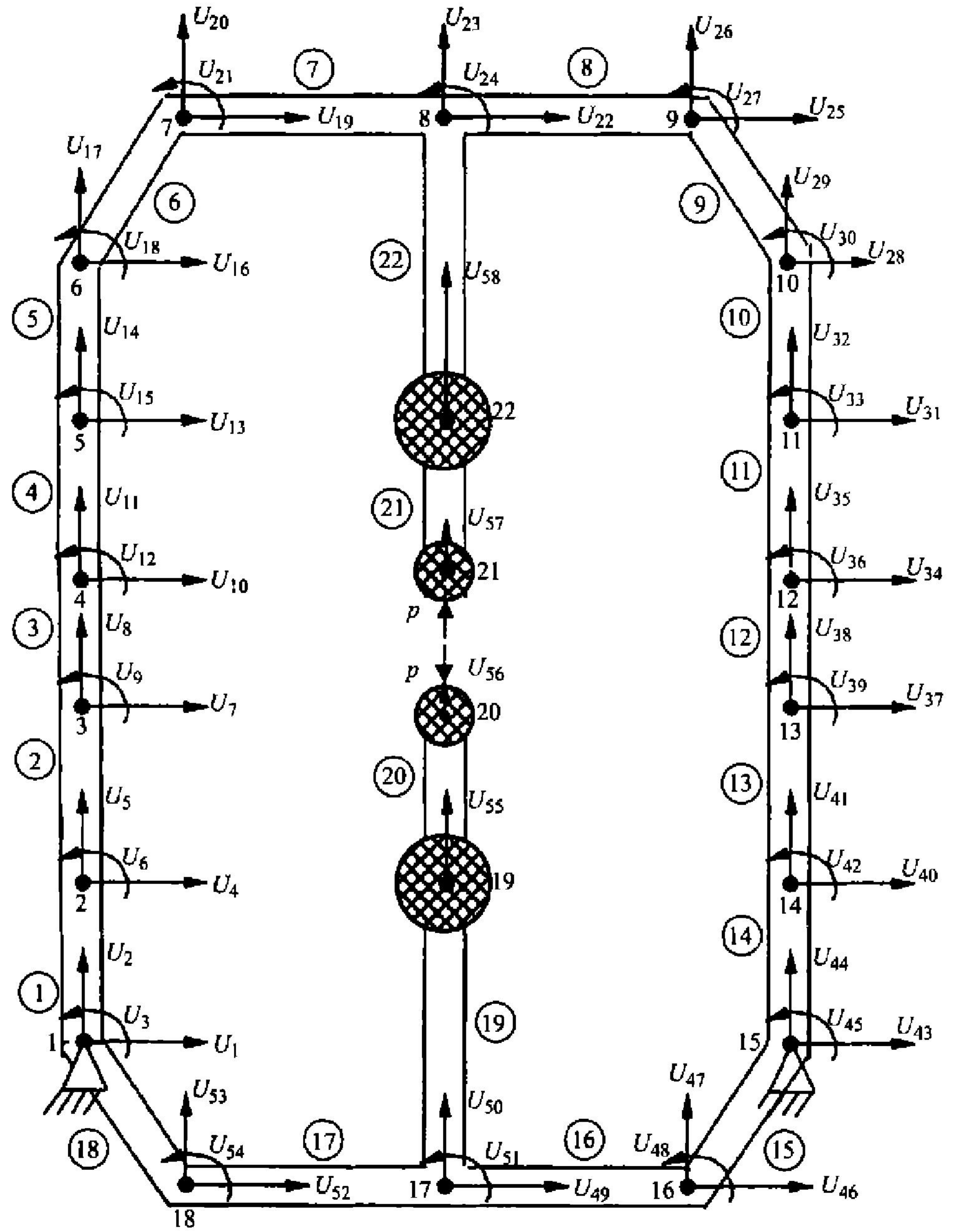


**Fig. 2** Calculation model of stand vibration system of planishing mill

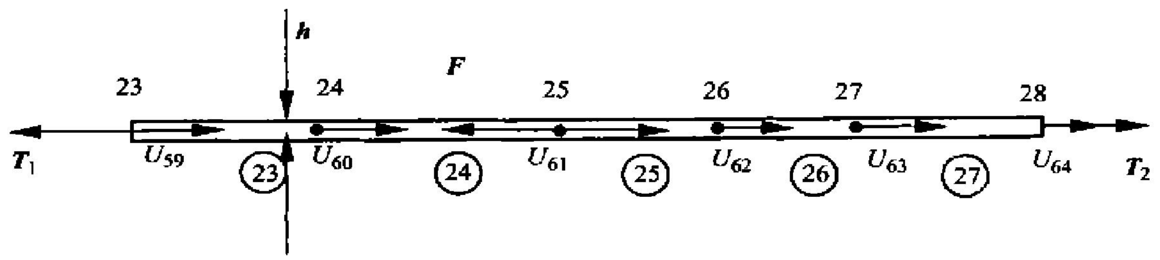


**Fig. 3** Finite element model of strip

of the strip being composed of certain number of pole elements is:

$$M_2\ddot{U}_2 + C_2\dot{U}_2 + K_2U_2 = Q_2 - M_2\ddot{U}_{r2} \qquad (2)$$

where $M_2$, $C_2$ and $K_2$ are mass matrix, damping matrix and stiffness matrix of the strip system; $U_2$, $\dot{U}_2$ and $\ddot{U}_2$ are column matrix of elastic coordinate displacements of the system and its first and second derivatives; $\ddot{U}_{r2}$ is the column matrix of rigid-body translational acceleration of every element of the strip, which can be regarded as zero in stable rolling; $Q_2$ is the generalized force column matrix being composed of tension force $T_1$, $T_2$ and frictional force $F$.

### 3.2 Dynamic equations of main drive shafting

The main drive shafting is composed of several inertia components (including electromotor, clutch, working roll, etc) and elastic components (including coupling spindle, etc), which can be simplified as shaft and board torsional vibration system, as shown in Fig. 4. When establishing the dynamic equation, the thick and short components are simplified as inertia, and their torsional rigidity is neglected. The slender components are simplified as a torsional spring, and their rotary inertia is neglected. $M_{dc1}$, $M_{dc2}$ are electromagnetic torque of electromotor, $N_3$ is working torque:

$$N_3 = fRp \tag{3}$$

where $R$ is the radius of working roll, $f$ is the friction coefficient, $p$ is the draught pressure.

According to Ref. [11], electromagnetic torque $M_{dc,i}$ is made up of the torque resulted from DC component $I_d$ and the torque resulted from harmonic component of commutating current[11]:

$$M_{dc,i} = C_{m,i}\Phi_i I_{d,i} + C_{m,i}\Phi_i \sum_{j=6}^{\infty} I_{m,j} \cos(j\omega t - \theta_j) \tag{4}$$

where $C_{m,i}$ is the constant of electromagnetic torque of electromotor $i$, $\Phi_i$ is the magnetic flux of electromotor $i$; $I_{m,j}$ is the amplitude of harmonic component $j$; $\omega$ is the angular frequency of fundamental component; $\theta_j$ is the phase angle of harmonic component $j$.

From this "mass-spring system", dynamic equations of main drive shafting is established:

$$\boldsymbol{M}_3\ddot{\boldsymbol{U}}_3 + \boldsymbol{C}_3\dot{\boldsymbol{U}}_3 + \boldsymbol{K}_3\boldsymbol{U}_3 = \boldsymbol{Q}_3 - \boldsymbol{M}_3\ddot{\boldsymbol{U}}_{r3} \tag{5}$$

where $\boldsymbol{M}_3$, $\boldsymbol{C}_3$, $\boldsymbol{K}_3$ are respectively mass matrix, damping matrix and stiffness matrix of the main drive shafting; $\ddot{\boldsymbol{U}}_{r3}$ is the column matrix of rigid-body rotational acceleration of the main drive shafting; $\boldsymbol{U}_3$ is generalized coordinate, which represents the elastic torsion motion that overlaps on the rigid-body rotation of the main drive shafting; $\boldsymbol{Q}_3$ is the generalized force column matrix being composed of the torque of working roll, electrical torque of the electromotor, etc.

### 3.3 Dynamic equations of uncoiler and coiler shafting

Uncoiler and coiler shafting is also a torsional vibration system being composed of inertia elements and elastic elements, as shown in Fig. 5 and Fig. 6. $M_{dc,i}$ is the electromagnetic torque that the electromotor acts on the

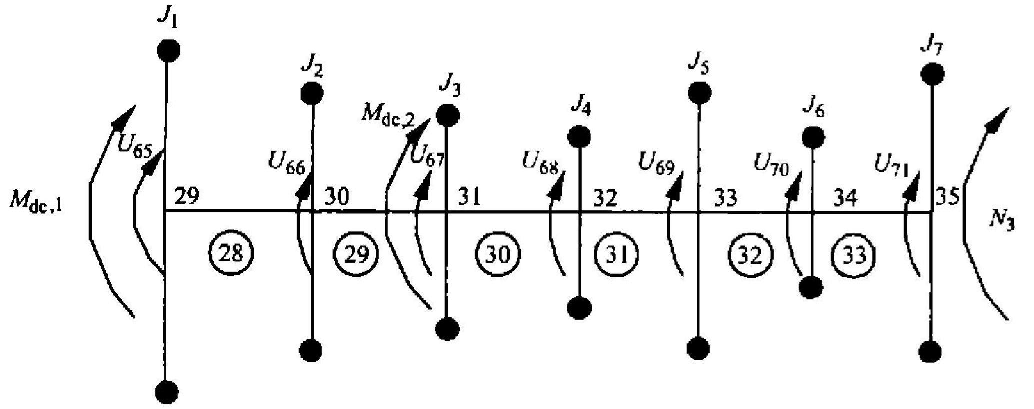


**Fig. 4** Calculation model of main drive shafting

( $J_i$ —Rotational inertia of the first electromotor, electromotor shaft coupling, the second electromotor, shaft coupling of output shaft, intermediate shaft, spindle, working roll; O—Corresponding torsional elastic element; $M_{dc1}$, $M_{dc2}$ —Electromagnetic torque of electromotor; $N_2$ —Torque of working roll)

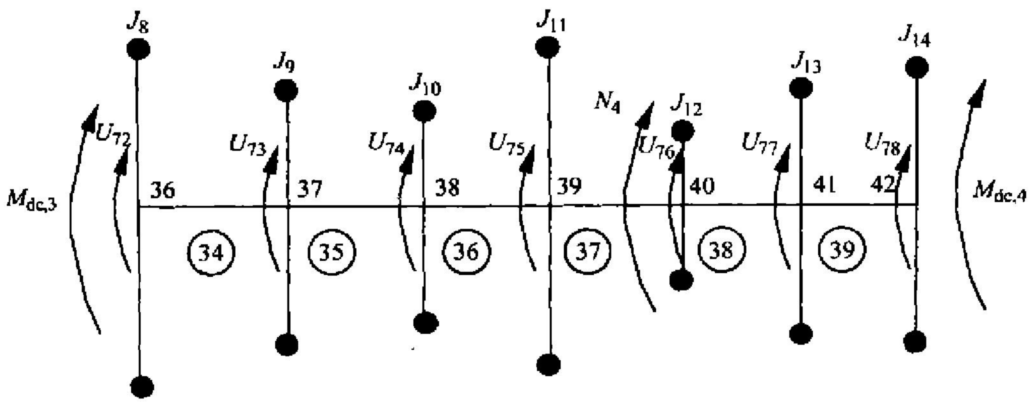


**Fig. 5** Calculation model of uncoiler shafting

( $J_i$ —Rotational inertia of electromotor, shaft coupling of output shaft of electromotor, intermediate shaft, uncoliling roll; O —Corresponding torsional elastic element; $M_{dc,i}$ —Electromagnetic torque of electromotor $i$; $N_4$ —Uncoliing torque)

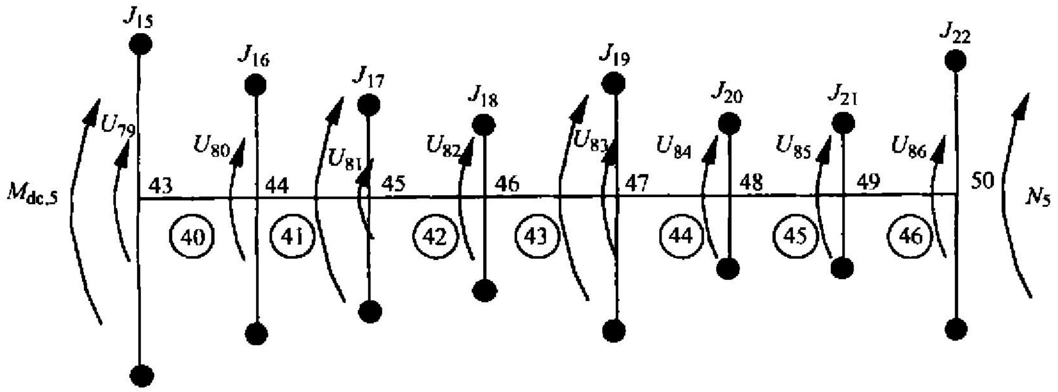


**Fig. 6** Calculation model of coiler shafting

($J_i$ —Rotational inertia of the first electromotor, electromotor shaft coupling, the second electromotor, electromotor coupler, the third electromotor, shaft coupling of output shaft of electromotor, intermediate shaft, coiling roll; ○ —Corresponding torsional elastic element; $M_{dc,\ i}$ —Electromagnetic torque of electromotor $i$; $N_5$ —Coiling torque)

shafting, which is decided by Eqn. (4). $N_4$ is the working torque of uncoiler shafting and $N_5$ is that of coiler shafting.

$$N_i = R_i T_i \qquad (i = 4,\ 5) \qquad (6)$$

where $R_4$ and $R_5$ are respectively the uncoiling radius and coiling radius, which are variable in rolling process. They are considered as constants when doing elasto-kinetic analysis on this time dependent system with the assumption of temporal structure. From each "mass-spring system", the correspondent dynamic equation is derived:

$$\boldsymbol{M}_i \ddot{\boldsymbol{U}}_i + \boldsymbol{C}_i \dot{\boldsymbol{U}}_i + \boldsymbol{K}_i \boldsymbol{U}_i = \boldsymbol{Q}_i - \boldsymbol{M}_i \ddot{\boldsymbol{U}}_{r,\ i} \qquad (i = 4,\ 5) \qquad (7)$$

where $\boldsymbol{M}_i$, $\boldsymbol{C}_i$, $\boldsymbol{K}_i$ ($i = 4,\ 5$) are respectively mass matrix, damping matrix and stiffness matrix of uncoiler and coiler shafting; $\ddot{\boldsymbol{U}}_{r,\ i}$ ($i = 4,\ 5$) is the column matrix of rigid-body rotational acceleration of uncoiler and coiler shafting; $\boldsymbol{U}_i$ ($i = 4,\ 5$) is the elastic rotating motion being overlapped on the rigid-body rotation of uncoiler and coiler shafting; $\boldsymbol{Q}_i$ is generalized force column matrix being composed of $M_{dc,\ i}$ and $N_i$.

## 4 COUPLING RELATION BETWEEN STAND VIBRATION SYSTEM AND TORSIONAL VIBRATION SYSTEM

The stand vibration system and torsional vibration system of the planishing mill are connected through the rolling interface, in which many factors interact complexly. $\Delta p$, variable of draught pressure, is a dynamic load of stand vibration system. $\Delta F$, variable of frictional force of rolling, is a dynamic load of torsional vibration system. Besides, $\Delta p$ and $\Delta F$ interact with each other, therefore, from influence of the stand vibration and torsional vibration on the draught pressure $p$ and frictional force $F$, the coupling relation between the stand vibration system and torsional vibration system is analyzed.

### 4.1 Factors led to variation of draught pressure

There are many factors that lead to the vibration of draught pressure. With regard to planishing mill, down pressure rate, length of contact arc and tensile force are mainly taken into consideration, namely

$$\Delta p \approx \Delta p_r + \Delta p_l + \Delta p_q \qquad (8)$$

Variable of draught pressure brought about by down pressure rate is[12]

$$\Delta p_r = -\Delta K_r (\dot{U}_U - \dot{U}_L) \qquad (9)$$

where $\Delta K_r$ is increment of system stiffness resulted from the variation of down pressure rate[12].

Variable of draught pressure resulted from the length of contact arc is[12]

$$\Delta p_l = -B(k_m - q_m) Q_p \sqrt{\frac{R}{\Delta h_m}} (U_U - U_L) \qquad (10)$$

where $U_U$ and $U_L$ in Eqns. (9), (10) are respectively displacements of the center of upper roller and lower roller. The other parameters can be referred to Ref. [14].

The variation of tensile force leads to the variation of draught pressure, which is brought about by the elastic vibration of strip.

The back tensile force is

$$\Delta q_b = \frac{E}{l} | U_C - U_{C+1} | \qquad (11)$$

The front tensile force is

$$\Delta q_f = \frac{E}{l} | U_C - U_{C-1} | \qquad (12)$$

where $E$ is the modulus of elasticity of strip, $l$ is the length of pole element of strip, $U_C$ is the nodal displacement at rolling neutral point of strip, $U_{C-1}$ and $U_{C+1}$ are respectively the nodal displacements at previous point and next point.

According to rolling theory, the variation of draught pressure resulted from $\Delta q_b$, $\Delta q_f$ is

$$\Delta p_b = -0.7 \Delta q_b Q_p B \sqrt{R \Delta h_m} \quad (13)$$

$$\Delta p_f = 0.3 \Delta q_f Q_p B \sqrt{R \Delta h_m} \quad (14)$$

The parameters in above equations have been expressed in Ref. [12].

### 4.2 Factors led to variation of rolling frictional force

Rolling frictional force comes into being under the influence of the friction coefficient $f$ and draught pressure $p$.

$$F = fp \quad (15)$$

Therefore

$$\Delta F \approx dF = p \Delta f + f \Delta p \quad (16)$$

Variation of draught pressure $\Delta p$ has been discussed above, and friction coefficient has relations with the thickness of oil film on the deformation zone. According to Roberts friction coefficient formula[12]:

$$f = \sqrt{\frac{(U_U - U_L)}{D}} [0.5 + (G_1 - 0.5) \exp(-G_2 v)] \quad (17)$$

where $G_1$, $G_2$ are constant, $D$ is the diameter of roll, $v$ is rolling speed. The variation of draught pressure is taken from Ref. [12] as:

$$\Delta p_f = -Q_f a_7 (U_U - U_L) \quad (18)$$

where $Q_f$, $a_7$ have been expressed in Ref. [12].

## 5 SUMMARY

In this paper, the global coupled equations for dynamic analysis of planishing mill CM04 are established. With these equations, we can study not only the interrelation between the stand vibration system and torsional vibration system, but also the interrelationship between electromagnetic parameters of driving motors and dynamic parameters of stand or torsional vibration system, and the mutual effects among electromagnetic parameters of driving motors or dynamic parameters of shaftings. Besides, we can do synchronously comprehensive research on the coupling relations and dynamic properties of each subsystem of the planishing mill; study the multi-unit and multi-dimensional relation and constraint principle among all kinds of parameters. Moreover, on the basis of this paper, we can study the uncoupling and solutions of the global coupled model, do dynamic simulation, analyze global motion stability, investigate relevant law of coupling parameters and main body motion and functions of the system, predict abnormal operating mode. Then examine relevant theory through physical model and industrial practice. Furthermore, the model given in this paper will also be perfected and corrected with the evolution of the research of rolling interface and experimental result. The work done in this paper can be extended to the research on cold mills and hot mills.

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