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# Theoretical analysis based on a modified mixed-film lubrication model for metal forming processes①

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**[Abstract]** An analytical model for metal rolling in the mixed lubrication regime was developed based on Wilson and Chang's asperity flattening model and Von Mises homogenous deformation model. A more rigorous average Reynolds equation was used to calculate the hydrodynamic pressure. The variations of the yield stress with strain were considered in the model. An efficient iteration procedure was developed to solve the contact area, film thickness and hydrodynamic pressure. The model is more practical with fewer assumption and converges quickly. It is applicable to a wider range of rolling regimes. The calculation results using the model agree well with the literature as well as with measured data from a rolling mill.

**[Key words]** mixed film; rolling; lubrication; cold rolling

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## 1 INTRODUCTION

Rolling oils are widely used in cold rolling to improve the surface quality of the metal products and to reduce the mill power consumption and roll wear. The assumption of hydrodynamic lubrication as adopted by some researchers, in which the contacting surfaces are fully separated by a thin film of lubricant, is not practical for cold rolling. In order to generate adequate friction in the roll bite and improve sheet surface appearance, the film thickness should be around 0.2 μm, which is comparable to the strip and roll surface roughness.

In practice, mixed film regime prevails in the roll bite, where the film is thin so that metal contact occurs at the surface asperities. Significant progress has been made in the analysis of this lubrication regime recently. Sutcliffe and Johnson carried out an inlet analysis, in which the deformation pressure was shared between the contact asperities and fluid film at the surface valleys. And from the asperity crushing rate, the fractional contact area and average film thickness were obtained; the average film pressure was calculated by integrating a simplified first-order Reynolds equation, with two arbitrary constants to be determined. Wilson and Chang[1, 2] developed an analytical model for strip rolling, of which the sheet surface roughness was approximated by a longitudinal sawtooth topography and evolution of surface roughness was estimated, with the relationship between the fractional contact area, average deformation pressure and the film pressure derived by using an upper bound theory (Wilson & Sheu); the film pressure was determined from a simplified first-order Reynolds equation with a flow constant $C$, which was determined by trial and an error method. And since the film pressure is very sensitive to the flow constant $C$, the latter has to be determined with high precision (more than 6 significant digits) which requires many iterations, a relatively time-consuming procedure of integrating four differential equations along the full contact length was used for each iteration. At high rolling speed, convergent solution could not be obtained due to the increasing sensitivity of the pressure to the flow constant.

This paper discusses a similar model as proposed by Wilson and Chang[1, 2], but with a more rigorous second-order average Reynolds equation, hence eliminating the need to introduce an unknown flow constant. The film pressure is solved from average Reynolds equation using an over-relaxation method, with the appropriate boundary conditions applied. The model is then extended to incorporate variable yield stress characteristics of the workpiece in the roll bite to allow for work-hardening effect.

## 2 ROLLING MODEL

### 2.1 Horizontal force equilibrium

The horizontal force equilibrium for an element in the bite is given by

$$\frac{dF}{d\varphi} = -2R(p\sin\varphi + Q\cos\varphi)/y_1 \qquad (1)$$

where $F$, $p$ and $Q$ are, respectively, dimensionless tension force per unit width, rolling pressure, and friction force; and $\varphi$ is angle from exit plane (see the Nomenclature).

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### 2.2 Deformation condition

Under plane strain condition, the von Mises yield criterion:

$$p\cos\varphi + f/y = \sigma_Y$$

or

$$p = (w_r - Fy_1/y)\cos\varphi \quad (2)$$

can be used. Assuming the constrained yield stress satisfies Alexander's empirical relation[3]:

$$\sigma_Y = \sigma_{Y_0} \cdot w_y \quad (3)$$

where $w_y$ is the dimensionless constrained yield strength given by

$$w_y = (1 + c_2\varepsilon)^{c_3}(1 + c_4\dot{\varepsilon})^{c_5} \quad (4)$$

here, $\varepsilon$ and $\dot{\varepsilon}$ are the strain and strain rate:

$$\varepsilon = \frac{2}{\sqrt{3}}\ln\frac{y_0}{y};$$

$$\dot{\varepsilon} = \frac{4}{\sqrt{3}}\frac{u_{w_2}y_2}{y^2}\cdot\tan\varphi$$

and $y_0$ is the workpiece thickness when last in its annealed state.

### 2.3 Friction stress

If the solid contact area ratio is $A$, the frictional stress between two rough surface can be calculated by

$$q = Aq_a + (1 - A)q_f$$

or in dimensionless form:

$$Q = AQ_a + (1 - A)Q_f \quad (5)$$

where the frictional shear stress at the contact area is

$$q_a = c\sigma_Y/2\,\mathrm{sign}(u_w - u_r)$$

or

$$Q_a = cw_r/2\,\mathrm{sign}(u_w - u_r) \quad (6)$$

the shear stress at the fluid valley is

$$q_f = \frac{h_t \cdot dp_f}{2dx} + \eta\cdot\frac{u_w - u_r}{h_t} \quad (7)$$

or in dimensionless form:

$$Q_f = \frac{h_t \cdot dp_f}{2R\,d\varphi} + \frac{\eta_0 e^{\alpha\sigma_{Y_0}p_f}}{\sigma_{Y_0}}\cdot\frac{u_w - u_r}{h_t} \quad (8)$$

The contact pressure $p_a$ and the film pressure $p_f$ at the valley should satisfy

$$p = Ap_a + (1 - A)p_f \quad (9)$$

### 2.4 Contact area and film thickness

For longitudinal roughness on the workpiece surface, Chang[1] proposed a formulation of the contact area from an upper-bound analysis:

$$\frac{dA}{d\varphi} = \frac{-2\varphi R}{\theta_a[2l(1 - A) + yE]} \quad (10)$$

where

$$E = \frac{Aw_y - (p - p_f)f_2}{(p - p_f)f_1} \quad (11)$$

$$f_1 = -0.86A^2 + 0.345A + 0.515 \quad (12)$$

$$f_2 = 1/[2.571 - A - A\ln(1 - A)] \quad (13)$$

For the saw-tooth roughness as in Fig. 1, the RMS roughness before rolling is

$$\delta = \sqrt{\frac{1}{l}\int_0^l |h - r|^2 dz} = \frac{r}{\sqrt{3}} \quad (14)$$

The average film thickness after flattening is

$$H_t = h_t/\delta = \sqrt{3}(1 - A)^2 \quad (15)$$

### 2.5 Lubrication equation

The lubrication pressure for rough surface can be written in a form similar to those proposed by Patir and Cheng[4]:

$$\frac{\partial}{\partial x}\left(\Phi_x\frac{\rho h_t^3}{12\eta}\cdot\frac{\partial p_f}{\partial x}\right) + \frac{\partial}{\partial z}\left(\Phi_z\frac{\rho h_t^3}{12\eta}\cdot\frac{\partial p_f}{\partial z}\right) = \frac{-\partial(\Phi_{ux}\bar{u}\rho\cdot h_t)}{\partial x} - \frac{\partial(\Phi_{uz}\bar{u}\rho\bar{h})}{\partial z} + \frac{\rho d(\Phi_{uz}\cdot h_t)}{dt} + \Phi_{uz}\cdot\frac{h_t\cdot d\rho}{dt} \quad (16)$$

At steady condition, assuming the lubricant is isoviscous and incompressible with constant density, and neglecting the axial flow, the average Reynolds equation can be simplified to

$$\frac{d}{dx}\left(\Phi_x\frac{h_t^3}{12\eta}\cdot\frac{dp_f}{dx}\right) = -\Phi_{ux}\left[\frac{u_r + u_w}{2}\frac{dh_t}{dx} + \frac{h_r}{2}\cdot\frac{du_w}{dx}\right] \quad (17)$$

Considering flow continuity condition,

$$u_w = u_{w2}y_2/y = u_r(1 + s_f)y_2/y \quad (18)$$

gives

$$\frac{du_w}{dx} = -\frac{u_r(1 + s_f)y_2}{y^2}\cdot\frac{dy}{dx} \quad (19)$$

The work piece thickness along the contact arc is

$$y = y_2 + R\varphi^2 \quad (20)$$

thus

$$\frac{dy}{d\phi} = 2R\varphi \quad (21)$$

Under the isothermal condition, lubricant rheological characteristics may be expressed as

$$\eta = \eta_0\exp(\alpha p_f) \quad (22)$$

For the saw-tooth roughness,

$$\Phi_\lambda = \begin{cases} 3\sqrt{3}/H_t & (H_t < \sqrt{3}) \\ 1 + 3H_t^{-2} & (H_t \geqslant \sqrt{3}) \end{cases} \quad (23)$$

Substituting Eqns. (18), (19), and (21) into Eqn. (17) gives

$$\frac{d}{d\varphi}\left(\Phi_x H_t^3\cdot\frac{d\hat{p}_f}{d\phi}\right) = -\frac{6R\eta_0\alpha\cdot u_r}{\delta^2}\cdot\left\{\left[1 + (1 + s_f)\frac{y_2}{y}\right]\frac{dH_t}{d\varphi} - \frac{2(1 + s_f)y_2HR\varphi}{y^2}\right\} \quad (24)$$

From Eqn. (24), the film pressure $\hat{p}_f$ is determined, the shear stress at the fluid is computed from Eqn. (8) and frictional stress $Q$ from Eqn. (5). The rolling pressure $p$ can then be solved from Eqns. (1) and (2).

### 2.6 Boundary condition

Neglecting the elastic deformation regions, the following boundary conditions apply to the plastic region:

$$\begin{cases} 1)\ F(0) = s_2 \\ 2)\ F(\varphi_1) = s_1 \\ 3)\ p(0) = w_y(0) - s_2 y_1/y_2 \\ 4)\ p(\varphi_1) = [w_r(\varphi_1) - s_1]/\cos\varphi_1 \\ 5)\ p_f(0) = 0 \\ 6)\ H_{t1} = (H_t + \sqrt{3})^2/4\sqrt{3} \\ 7)\ A_1 = (\sqrt{3} - H)2\sqrt{3} \\ 8)\ p_f(\varphi_1) = p(\varphi_1) - A_1/f_2 A_1 \end{cases} \quad (25)$$

where

$$\varphi_1 = \cos^{-1}[1 - (y_1 - y_2)/(2R)]$$

$$h \approx \frac{3\eta_0 \alpha u_{w1}}{\tan\varphi_1}$$

$$u_{w1} = [1 + (1 + s_f) y_2/y_1] u_r$$

## 3 NUMERICAL CALCULATION

Eqns. (10) to (24) are solved numerically. To initiate the iteration, the following initial values are used:

1) Rolling pressure set to the constrained yield stress, $p = w_y$;

2) Film pressure $p_f = 0.95p$;

3) Contact area ratio $A = 0.8$.

From these initial values, the rolling pressure $p$, contact area ratio $A$ and film thickness $H_t$ are recalculated. This iterative procedure continues until the convergence criterion is satisfied, which generally requires around 30 iterations. It has found that the solution is insensitive to the initial values, although appropriate values would speed up the convergence of the computation.

A computer program is compiled in Visual Basic to perform all the above calculation as shown by the flow chart in Fig. 1.

## 4 CALCULATION RESULTS AND CONCLUSION

According to the flow chart in Fig. 1, a computer program is compiled in Visual Basic. The calculation results about the mill in Ref. [1] are shown in Figs. 2~ 5. The initial values and parameters of the rolling are the same as the Ref. [1], here they will not be repeated. In Figs. 2~ 5, the Curve 1 is the calculation results by this paper's model, the Curve 2 is the calculation results by the model in Ref. [1], the Curve 3 is the measured data in Ref. [1]. The program in this paper is verified by the example given by Chang[1]. The calculation results are shown in Figs. 2 ~ 5. An analytical model for metal rolling in the mixed lubrication regime was developed. The model is based on Wilson and Chang's asperity flattening model and von Mises homogeneous deformation model. A more rigorous average Reynolds equation was used to calculate the

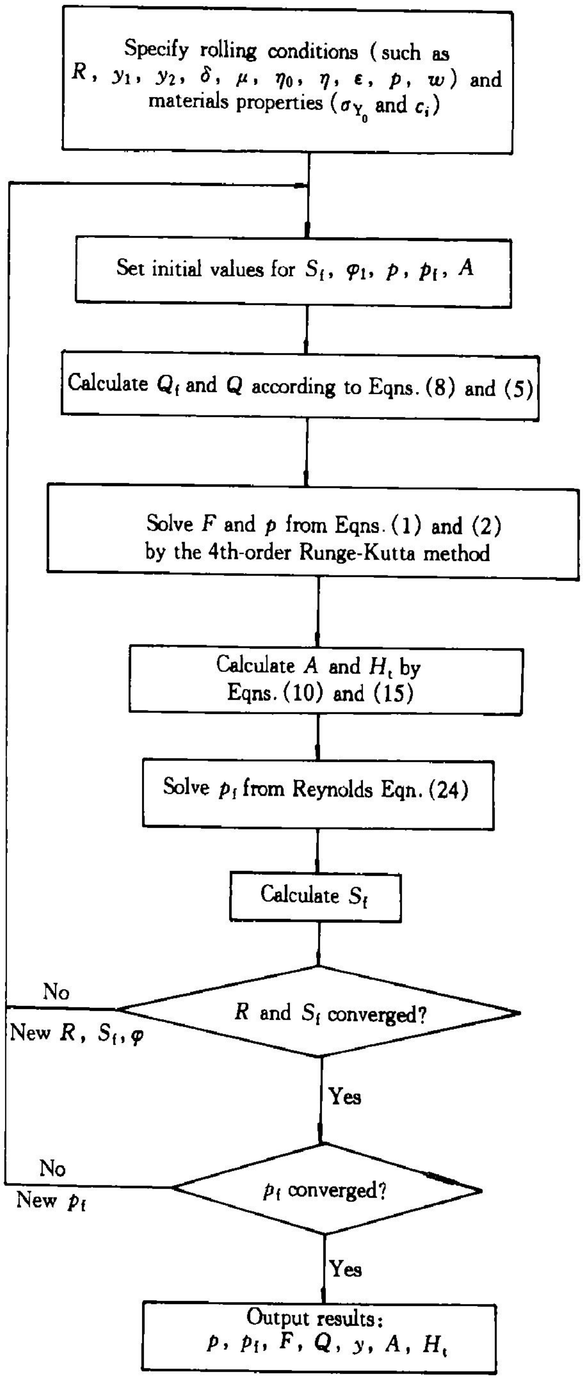


**Fig. 1** Flow chart of computation program

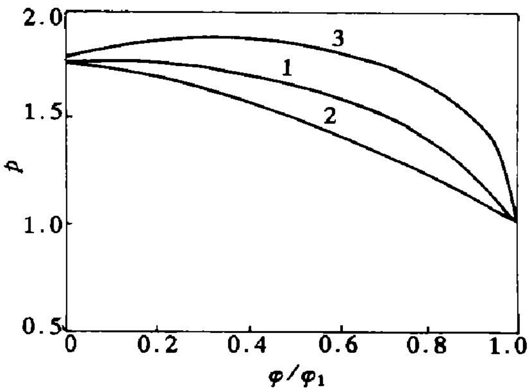


**Fig. 2** $p - \varphi/\varphi_1$ curves

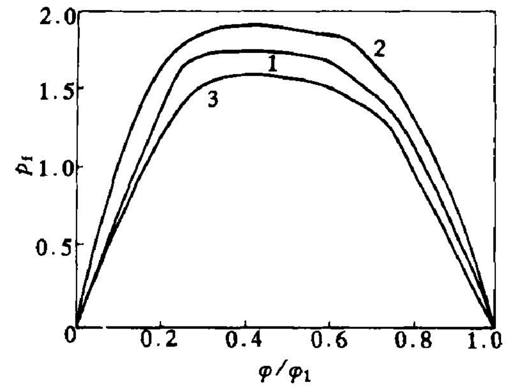


**Fig. 3** $p_f - \varphi/\varphi_1$ curves

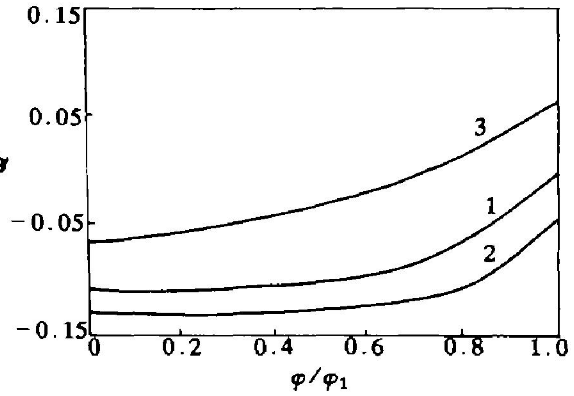


**Fig. 4** $Q - \varphi/\varphi_1$ curves

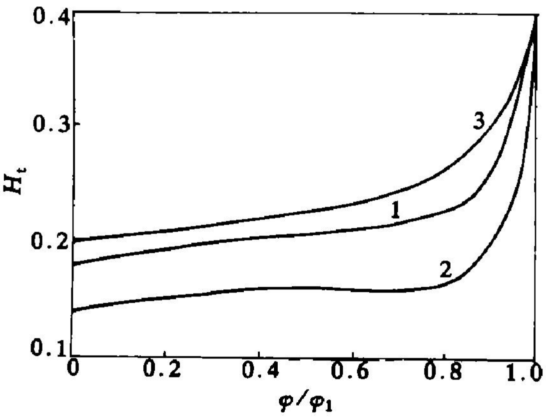


**Fig. 5** $H_t - \varphi/\varphi_1$ curves

hydrodynamic pressure. The variations of the yield stress with strain were considered in the model. An efficient iteration procedure was developed to solve the contact area, film thickness and hydrodynamic pressure. The model is more practical with fewer assumption and converges quickly. It is applicable to a wider range of rolling regimes. The calculation results using the model in this paper agree well with the literature as well as with measured data[1] from a rolling mill. But in the past, the assumption[5~8] of hydrodynamic lubrication as adopted by some researchers, in which the contacting surfaces are fully separated by a thin film of lubricant, is not practical for cold rolling. In practice, mixed film regime[9~12] prevails in the roll bite, where the film is thin so that metal contact occurs at the surface asperities. So through the work of this paper, a general procedure is built to solve the friction, film thickness and hydrodynamic pressure and so on.

## Nomenclature

$A$ —ratio of contact area, dimensionless;
$c$ —adhesive friction coefficient at the asperities;
$c_i$ —yield strength constants;
$E$ —elastic modulus of the roll material;
$f$, $F$ —horizontal tension per unit width;
$F = f / \sigma_{Y_0} y_1$, dimensionless;
$f_y$, $F_y$ —vertical rolling force;
$F_y = f_y / \sigma_{y_0} y_1$, dimensionless;
$h$, $H$ —nominal surface separation;
$H = h/\delta$;
$h_t$, $H_t$ —average film thickness;
$H_t = h_t/\delta$, dimentionless;
$l$ —half pitch of asperity or surface tooth;
$p$ —deformation or rolling pressure;
$p_f$ —average film pressure;
$\hat{p}_f = 1 - e^{-2p_f}$ —transformed film pressure, dimentionless
$p_a$ —average asperity contact pressure;
$q$ —friction force;
$Q = q/\sigma_{Y_0}$, dimentionless;
$q_a$ —friction force at contact area;
$Q_a = q_a/\sigma_{Y_0}$, dimentionless;
$q_f$ —friction force at film valley;
$Q_f = q_f/\sigma_{Y_0}$, dimentionless;
$r$ —half height of the surface tooth;
$R$, $R'$ —roll radius and deformed roll radius respectively;
$s_f = (u_{w2} - w_r)/u_r$ —forward slip at the exit plane;
$s_1$, $s_2$ —backward and forward tension respectively;
$u_r$ —roll velocity;
$u_w$ —workpiece velocity;
$u_{w1} = [1 + (1 + s_f) y_2/y_1)] u_r$ —workpiece entry velocity;
$u_{w2} = (1 + s_f) u_r$ —workpiece exit velocity;
$w_y = (1 + c_2 \varepsilon)^{c_3} \cdot (1 + c_4 \dot{\varepsilon})^{c_5}$ —workpiece constrained yield stress, dimentionless;
$x = R\varphi$ —contact length from the exit plane;
$y$ —workpiece thickness at arbitrary contact angle $\varphi$;
$y_1$, $y_2$ —workpiece thickness at entry and exit plane, respectively;
$\delta = \sqrt{\delta_r^2 + \delta_w^2}$ —RMS surface roughness;
$\sigma_Y$, $\sigma_{Y_0}$ —contrained yield strength of the workpiece;
$\varepsilon = (2/\sqrt{3}) \ln(y_0/y_1)$ —strain, dimentionless;
$\dot{\varepsilon} = (4/\sqrt{3}) u_{w2} y_2 / y^2 \mathrm{tg}\,\varphi$ —strain rate;
$\varphi$ —angle from the exit plane;

$\varphi_1$ —bite angle;

$\varphi_\lambda$ —flow factor in $X$ direction;

$\theta_a$ —asperity slope;

$\alpha$ —viscosity pressure factor;

$w_1$ —relaxation factor;

$\eta$, $\eta_0$ —lubricant viscosity.

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