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Forecasting method of fatigue life test data for metal materials①

ZHANG Huai-liang(张怀亮), QIU Xian-yan(邱显焱), TAN Guan-jun(谭冠军)
(College of Mechanical and Electrical Engineering, Central South University,
Changsha 410083, P. R. China)

[Abstract] GM(1, 1) model of grey system theory is used to forecast fatigue life test data for metal materials. The method can reduce test time and save test cost, and reliability indexes of metal materials can be obtained quickly. The results of an example show that grey system theory has a high precision for forecasting fatigue life test data for metal materials. A valuable method is put forward, which can effectively reduce the fatigue life test time for metal materials.

[Key words] fatigue life of materials; test data; forecasting method
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1 INTRODUCTION

The fatigue life of the metal materials is the basic data for calculating the intensity of the parts of the mechanical products. In general, the fatigue life test data are obtained through the ruinous test on the material specimen. But the time of the fatigue life test is long and the cost is also high. If an appropriate method of data fitting and forecasting is found, i. e., the model of the forecasting life test data is established with a few of test data obtained, and it is used to forecast the subsequent test data, then more data for statistical analyzing are obtained and the reliability of the analyzing results is improved except that the test time is saved and the cost is reduced.

If the grey forecasting model of the grey theory is introduced to the fields of the reliability projects, the system of the reliability life test is regarded as a grey system including some unknown information and the test data is regarded as the grey value. The life test data are queued from small to big and added up once, then the data sequence varies in the form of exponential functions. So the life test data can be fitted and forecasted by using the grey forecasting model of GM(1, 1) (it is a exponential model). The test method is called the accelerated forecasting method with consideration of the test system disturbed little from environment, the system being steady, the test data being more than those in energy system in board sense and the data extending well.

2 MODEL OF GREY GM(1, 1)

Assuming that there is a original sequence $x^{(0)}$, and $x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(N)\}$. The sequence $x^{(0)}$ is added up, then $x^{(1)}$ is obtained, that is, $x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(N)\}$, where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$.

The albino form of differential equation of GM(1, 1) model is

$$dx^{(1)}/dt + a \cdot x^{(1)} = b \qquad (1)$$

The difference form of GM(1, 1) model is also called the grey differential equation, which is as follows:

$$x^{(0)}(k) + a \cdot z_n^{(1)}(k) = b$$
$$k = 2, 3, \cdots, N \qquad (2)$$

where $z_n^{(1)}(k)$ is the background value, and it is calculated by Eq. (3)[1~3]:

$$z^{(1)}(k+1) = \frac{1}{2n}[(n+1) \cdot x^{(1)}(k) + (n-1) \cdot x^{(1)}(k+1)] \qquad (3)$$

where $k = 1, 2, \cdots, (N-1)$ and n is an undetermined constant.

Eq. (2) is also expressed by:

$$\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(N) \end{bmatrix} = \begin{bmatrix} -z_n^{(1)}(2), & 1 \\ -z_n^{(1)}(3), & 1 \\ \vdots & \vdots \\ -z_n^{(1)}(N), & 1 \end{bmatrix} \times [a, b]^T \qquad (4)$$

If $\boldsymbol{y}_N = [x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(N)]^T$, $\zeta = [a, b]^T$, and

$$\boldsymbol{B} = \begin{bmatrix} -z_n^{(1)}(2), & 1 \\ -z_n^{(1)}(3), & 1 \\ \vdots & \vdots \\ -z_n^{(1)}(N), & 1 \end{bmatrix} \qquad (5)$$

where ζ is a pending identified parameter vector, a and b are pending identified constants, then

$$\boldsymbol{y}_N = \boldsymbol{B} \times \zeta \qquad (6)$$

The pending identified parameter vector ζ is obtained by the use of the least square method:

$$\zeta = (\boldsymbol{B}^T \cdot \boldsymbol{B})^{-1} \cdot \boldsymbol{B}^T \cdot \boldsymbol{y}_N \qquad (7)$$

The discrete response of Eq. (1) is obtained as

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follows:

$$\hat{x}^{(1)}(k+1) = [x^{(0)}(1) - b/a] \cdot \exp(-a \cdot k) + b/a \qquad (8)$$

The result of fitting $\hat{x}^{(0)}(k+1)$ of the origin data $x^{(0)}(k+1)$ is as follows:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$$

In course of calculating, firstly the undetermined constant n in Eq. (3) is determined, then the matrix $\boldsymbol{B}$ is calculated according to Eq. (5). Secondly the pending identified parameter vector ζ is calculated by Eq. (7), that is, the pending identified parameters a and b are obtained. At last the results of fitting and forecasting are calculated by Eq. (8) and Eq. (9).

Now to a certain sequence, an appropriate value of n is obtained. Then the background value is calculated according to Eq. (9) and the matrix $\boldsymbol{B}$ is obtained. In this way the most appropriate model of GM(1, 1) and the best forecasting precision are obtained.

For the original sequence $x^{(0)}$ composed of the fatigue life test data, the type of life distributing function is assumed and the value of n is initialized according to experience. The calculating formula of background value corresponding to the value of n is made certain according to Eq. (3), then GM(1, 1) model is obtained according to Eq. (5), Eq. (7), Eq. (8) and Eq. (9), and the data fitting and forecasting are carried out. The original sequence $N^{(0)}$ and the forecasting data are integrated to a new sequence, and the correlative coefficient ρ_n of the new sequence is calculated by adopting median order and the assumed life distributing function. Whether or not the sequence submits to the assumed distributing function is estimated. If the sequence is in accordance with it, it is reserved. The value of n is added by a step Δn, i. e., $n' = n + \Delta n$, then n' is substituted into Eq. (9), and repeat the above calculating process. The corresponding new sequence and its correlative coefficient ρ_n and $\rho_{n'}$ are obtained. If $|\rho_n - \rho_{n'}| < \sigma$ (σ is a small plus presupposed to calculating precision.), the calculating stops, and the result is the new sequence corresponding the value of n'; if $|\rho_n - \rho_{n'}| > \sigma$, the value of n is added again, and the iterative calculating is repeated until the difference of the correlative coefficient of the corresponding value of n neighboring meets the calculating precision presupposed.

3 EXAMPLE

The fatigue life of a certain steel products submit to Weibull distribution. The stress circulation times of the fatigue life N of 8 specimens are 0.39M, 0.52M, 0.60M, 0.73M, 0.80M, 0.90M, 106M, 130M, respectively. Now what are the distributing parameters estimated?

3.1 Establishment of model of GM(1, 1) for forecasting fatigue life test data

The sufficient and necessary condition for the established GM(1, 1) model to be extended (i. e. the forecasting character) is the sequence dimension $N \geqslant 4$. 5~6 data are usually used to establish the model.

In this example, assuming that the test stops when the fifth test datum is obtained, $x^{(0)} = \{0.39M, 0.52M, 0.60M, 0.73M, 0.80M\}$ is regarded as the original established model sequence, then the sequence is used to establish the GM(1, 1) model to forecast the three subsequent test data. That can save the test time. According to the above method to establish the model, the values are initialized at first, i. e., $n = 2$, $\sigma = 0.0010$, $\Delta n = 0.5$. When the marked level $\alpha = 0.05$, $\rho_\alpha = 0.7067$, the correlative coefficient of the sequence $\rho \geqslant \rho_\alpha$, and the sequence submits to Weibull distribution. By calculating, if $n = 8$, $\rho_8 = 0.96761$; if $n = 9$, $\rho_9 = 0.96751$. Because $|\rho_8 - \rho_9| \leqslant \sigma$, it meets the calculating precision and the calculating stops.

When $n = 9$, $z_9^{(1)}(k+1) = (10x^{(1)}(k) + 8x^{(1)}(k+1))/18$, $k = 1, 2, 3, 4$.

The corresponding GM(1, 1) model is

$$\hat{x}(1)(k+1) = (8.6019 \times \exp 0.14654k - 7.6019) \times 0.39M$$

$k = 1, 2, \cdots, 7$

The fitting and forecasting data are {0.39M, 0.53M, 0.61M, 0.71M, 0.82M, 0.95M, 1.10M, 1.28M} by using the above model, and the front five data are the fitting value of the original sequence while the three subsequent data are the forecasting value. The correlative error percent between the fitting value and the original sequence $x^{(0)}$ is e= {0, −1.827%, −2.167%, 2.767%, −2.725%}. Assuming that the primary test data sequence is expressed by A = {0.39M, 0.52M, 0.60M, 0.73M, 0.80M, 0.90M, 106M, 130M}, $x^{(0)}$ and the three forecasting data form the sequence B = {0.39M, 0.52M, 0.60M, 0.73M, 0.80M, 0.95M, 1.10M, 1.28M}.

3.2 Checking of calculating results

The forecasting sequence B is obtained by the above calculating. In course of calculating, it is proved that the sequence submits to the Weibull distribution, but do the sequence B and the sequence A whether belong to the same distribution as a whole? If not, the distributing parameters estimated by statistical method in the sequence of B don't represent the whole character of the sequence A of the original test data. Then from this point of view, the forecasting method is disabled. So it is necessary whether the sequence B and A submit to the same collectivity is checked in order to testify the applicability of GM(1, 1) in the forecasting fatigue life test data for metal materials. We testify the equality of both the collectivities by the order method and the test method as follows, respectively.

The distribution functions of the sequences A and B are expressed by $F_A(N)$ and $F_B(N)$ respectively.

Assuming that H_0: $F_A(N) = F_B(N)$. The sum of the order T of the small sample is calculated, then the critical values of the sum of the orders T_1 and T_2 are obtained under the marked level α by looking up the table of the sum of order. If $T_1 < T < T_2$, the assumption H_0 is accepted, both collectivities are not the prominent, and difference between the sequence A and B is considered. By calculating, $T_A = 67.5$, $T_B = 68.5$. If the marked level $\alpha = 0.05$, the critical values of the sum of the orders T_1 and T_2 are: $T_1 = 52$, $T_2 = 84$, respectively. Because $T_1 < T_A < T_B < T_2$, the assumption H_0 is accepted and the forecasting method is practicable.

3.3 Estimating of distribution parameters

The parameters a, b and c of Weibull distribution are shown in Table 1, which are estimated by matrix method. When reliability value $R = 0.9$ and $R = 0.95$, the circulation times are calculated according to the distributing functions of the sequences of A and B, which are shown in Table 2. The reliability values are calculated according to the distribution function of the sequences of A and B when the circulation times are 0.40 M, 0.50 M, 0.60 M, respectively, which are shown in Table 3.

Table 1 Estimated value of Weibull parameters

Sequence	a	b	k
A	0.39 M	0.33 M	1.0827
B	0.39 M	0.35 M	1.1272

Table 2 Circulation times when reliability value $R = 0.9$ and $R = 0.95$

Sequence	$R = 0.9$	$R = 0.95$
A	0.43 M	0.41 M
B	0.44 M	0.42 M

Table 3 Reliability value R

Circulation times	R	
	A	B
0.40 M	0.9776	0.9821
0.50 M	0.7379	0.7637
0.60 M	0.5422	0.5720

From the above data in three tables, the various indexes of the sequence of A and B are very approximative. In view of the discrete character of the fatigue life test data for material, such forecasting results are acceptable. The above calculating and analyzing results indicate that the following method is practicable: firstly GM(1, 1) model is established by the front 5 original test data to forecast the subsequent three test data, then the forecasting sequence B is obtained; secondly the distribution function is made certain through the forecasting sequence; at last the intensity index is analyzed by the distribution function, and the analyzing results are credible. In this example, the method is adopted. Then the circulation times are reduced 0.50 M, and the test time is saved 38.46%. That demonstrates the effectiveness of the method.

[REFERENCES]

[1] TAN Guan-jun. The establishing method of the background and the application of the GM(1, 1) model(Ⅰ) [J]. The Theory and the Practice of the System Engineering, 2000, 20(4): 98- 103.

[2] TAN Guan-jun. The establishing method of the background and the application of the GM(1, 1) model (Ⅱ) [J]. The Theory and the Practice of the System Engineering, 2000, 20(5): 125- 127.

[3] TAN Guan-jun. The establishing method of the background and the application of the GM(1, 1) model(Ⅲ) [J]. The Theory and the Practice of the System Engineering, 2000, 20(6): 70- 74.

[4] DENG Ju-long. The Grey Controlling System (revised edition) [M]. Wuhan: Central China Science and Engineering University Press, 1985. 329- 380.

[5] ZHUANG Chun-qiang, WE Ya-sheng. The Basis of the Applicable Symbolic Stat [M]. Guangzhou: South China Science and Engineering University Press, 1992.

[6] LIU Xi-qiang. The grey GM(1, 1) model and its application [J]. The Theory and the Practice of the System Engineering, 1995, 15(1): 59- 62.

[7] GU Jing. The new format of calculating and its application of the grey establishing model [J]. The Controlling Theory and its Application, 1993, 10(2): 224- 229.

[8] ZHANG Tong. The improved calculating method of the parameters of the GM(1, 1) model [J]. The System Engineering and the Electron Technology, 1998, 20(8): 58- 60.

[9] TAN Guan-jun. The study of the grey GM(1, 1) model applied to the reliability engineering [J]. J of Central South University of Technology, 1998, 4(3): 270- 272.

[10] TAN Guan-jun. The grey transacting of the reliability test data on mechanical products [J]. The Mining Mechanics, 1998, 8(7): 65- 66.

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