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Effect of loading point position on fracture mode of rock①

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[Abstract] Anti-symmetric four-point bending specimen with different loading point positions was used to study effect of loading point position on fracture mode of rock in order to explore a feasible method for achieving Mode Ⅱ fracture and determining Mode Ⅱ fracture toughness of rock, K_{IIC}. Numerical and experimental results show that the distance between the inner and outer loading points, L_1+L_2, has a great influence on stresses at notch tip and fracture mode. When $L_1+L_2>0.5L$ or $0.1L<L_1+L_2<0.5L$, maximum principal stress σ_1 exceeds the tensile strength σ_t. The ratio of τ_{max}/σ_1 is relatively low or high and thus Mode Ⅰ or mixed mode fracture occurs. When $L_1+L_2<0.1L$, σ_1 is smaller than σ_t and the ratio of τ_{max}/σ_1 is much higher, which facilitates the occurrence of Mode Ⅱ fracture.

[Key words] fracture mode; loading point position; stress analysis; rock
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1 INTRODUCTION

Mode Ⅱ (shear) fracture is often detected in rock engineering, e.g. in earthquake zones or in rock bridges between two adjacent discontinuities in rock masses, especially in underground excavations and in the toe of rock slopes. An increasing interest in Mode Ⅱ fracture of rock in recent years has resulted in a great demand to determine Mode Ⅱ fracture toughness, K_{IIC}, since K_{IIC} is an important and useful parameter which represents the resistance of rock to the initiation of shearing fracture. Although a number of testing methods providing pure shear loading on pre-existing crack plane have been proposed to measure K_{IIC}[1~3], there is no standard method available for K_{IIC} to date.

Anti-symmetric four-point bending test is a common method for studying fracture under pure shear loading. Unlike the fracture under pure tensile loading which has a fixed trajectory vertical to the loading direction, the fracture under pure shear loading is usually in an indefinite direction, depending on the specimen dimension, the notch length, in particular, the loading point position[4~6]. The effect of loading point position on fracture mode has not been quite clear. The aim of this paper is to study this problem experimentally and numerically in order to find out a favorable loading point position for achieving Mode Ⅱ fracture and explore a feasible method to determine K_{IIC} of rock.

2 TEST ARRANGEMENT AND RESULT

2.1 Materials

The rock type used in this study is marble A and B. The mechanical properties of these rocks, i.e. the tensile strength σ_t, uniaxial compressive strength σ_c, internal friction angle ϕ, cohesion C, elastic modulus E and Mode Ⅰ fracture toughness K_{IC}, were obtained by ISRM (the International Society for Rock Mechanics) standard testing methods. These values are listed in Table 1.

Table 1 Mechanical properties of rocks

Rock type	σ_t/MPa	σ_c/MPa	ϕ/(°)
Marble A	3.1	79	41
Marble B	4.7	61	47
Rock type	**C/MPa**	**E/GPa**	**K_{IC}/(MPa·m$^{1/2}$)**
Marble A	25	25	1.26
Marble B	30	21	0.94

2.2 Specimens and methods

Fig. 1 illustrates the configuration of anti-symmetric four-point bending specimen and the loading form. The load is applied to the beam by two upper and bottom steel rollers. The notch is located in the symmetric plane of the specimen, where there is only shear force without any bending moment. Therefore the notch is subjected to pure shear loading. Table 2 gives the dimensions of these specimens and the load-

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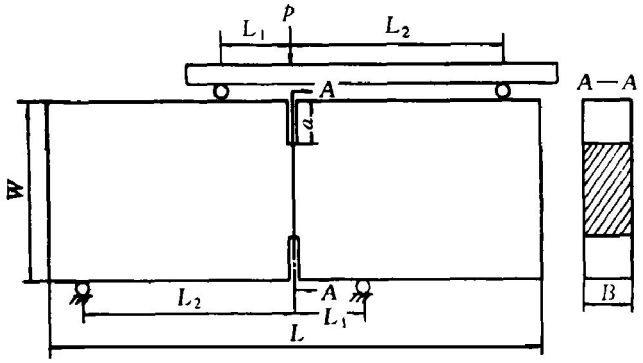

Fig. 1 Anti-symmetric four-point bending test

Table 2 Dimensions of specimens and loading point positions

Specimen No.	Dimension $L \times W \times B$ /mm	Notch length a /mm	a/W	Loading point position	
				L_1/mm	L_2/mm
A1(5)*	220×60×15	30	0.5	30	95
A2(4)*	220×60×15	2×20	0.3	5.5	95
A3(5)*	220×60×15	30	0.5	5.5	75
B1(3)*	220×60×15	2×24	0.4	5	10

* The number in the parentheses denotes how many specimens were tested for each sample

ing point positions. There are four kinds of loading point positions used in the tests. L_1 and L_2 are 30 mm and 95 mm, 5.5 mm and 95 mm, 5.5 mm and 75 mm, 5 mm and 10 mm, respectively. All of the tests were carried out on a servo-controlled Instron 1342 press with a capacity of 100 kN under position control. The peak load was measured and the fractured trajectory was carefully analyzed after tests.

2.3 Test results

For the single-notched specimen, if $L_1/W \geqslant 0.33$, the stress intensity factor K_m (m = Ⅰ, Ⅱ or mixed mode Ⅰ-Ⅱ) can be calculated by[7]

$$K_m = \frac{Q}{BW^{1/2}}\left[1.47 - 5.1(a/W - 0.5)^2\right] \cdot \sec\frac{\pi a}{2W}\sqrt{\sin\frac{\pi a}{2W}} \quad (1)$$

$$Q = \frac{L_2 - L_1}{L_1 + L_2}p$$

where p is thrust load.

If $L_1/W < 0.33$, K_m can be deduced by the boundary disposition method as follows[8]:

$$K_m = \frac{Q}{BW^{1/2}}\left|-3.40\left(\frac{a}{W}\right)^4 + 15.78\left(\frac{a}{W}\right)^3 - 16.04\left(\frac{a}{W}\right)^2 + 9.70\left(\frac{a}{W}\right) - 0.85\right| \quad (2)$$

For the double-notched specimen, when $0.167 \leqslant 2a/W \leqslant 0.883$, K_m can be determined by[9]

$$K_m = \frac{2Q}{BW}\sqrt{\pi a}F\left(\frac{2a}{W}\right) \quad (3)$$

where F is a shape factor and listed in Table 3.

Table 3 Shape factor F

$2a/W$	F	$2a/W$	F
0.2	0.33	0.6	0.93
0.3	0.52	0.7	0.17
0.4	0.69	0.8	0.156
0.5	0.82		

Table 4 shows the test results of the crack initiation angle θ_{mC} and the fracture toughness K_{mC}, where the subscript m denotes the fracture mode, i.e. m = Ⅰ (Mode Ⅰ fracture) or m = Ⅰ-Ⅱ (Mixed Mode Ⅰ-Ⅱ) when $\theta_{mC} \neq 0$ and m = Ⅱ (Mode Ⅱ fracture) when $\theta_{mC} = 0$. The typical fracture trajectories of these specimens are displayed in Fig. 2.

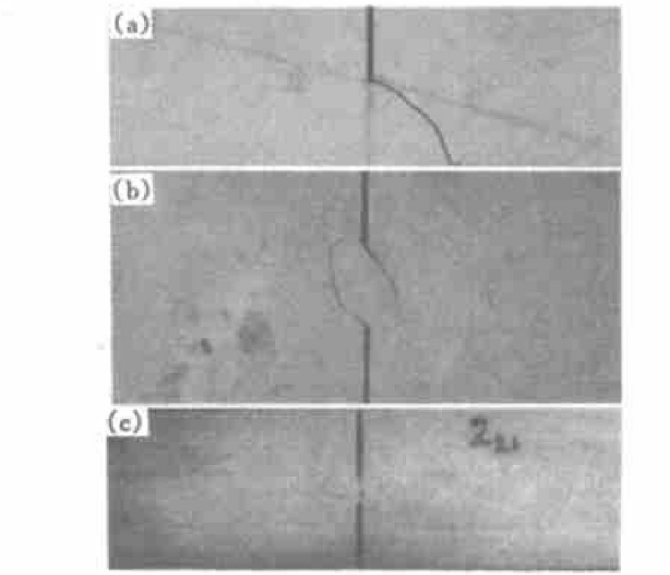

Fig. 2 Fracture trajectories of specimens with different loading point positions
(a) —Specimen A1 (L_1 = 30 mm, L_2 = 95 mm);
(b) —Specimen A2 (L_1 = 5.5 mm, L_2 = 95 mm);
(c) —Specimen B1 (L_1 = 5 mm, L_2 = 10 mm)

3 NUMERICAL CALCUALTION

A series of finite element calculations were conducted to study the stress distribution in the specimens with different loading point positions. Six-noded triangular plane elements with isotropic properties were used. Mesh refinement was performed in the vicinity of the notch tips in order to improve the accuracy of calculation. The boundary conditions were defined by applying node forces at the upper support points and zero vertical displacements at the bottom support points.

Figs. 3 and 4 illustrate two typical contours of maximum principal stress σ_1 and maximum shear stress τ_{max} for the specimens A1 and B1, where the tensile stress is defined as positive. Table 5 lists the maximum values of σ_1 and τ_{max} at the notch tips, as well as the ratios of σ_1/σ_t and τ_{max}/σ_1.

Table 4 Test results of specimens (mean value)

Specimen No.	Peak shear force Q_m/kN	Crack initiation angle $\theta_{mC}(1)$/(°)	Crack initiation angle $\theta_{mC}(2)$/(°)	Fracture toughness K_{mC}/(MPa·m$^{1/2}$)	$\frac{K_{mC}}{K_{IC}}$	Fracture mode
A1	1.65	68.8	–	0.74	0.59	Mode I
A2	3.25	51.3	54.3	1.58	1.26	Mixed mode I - II
A3	3.23	57.9	–	1.45	1.15	Mixed mode I - II
B1	4.20	0	0	3.28	3.49	Mode II

Table 5 Numerical results of tested specimens (mean value)

Specimen No.	L_1+L_2/mm	$(L_1+L_2)/L$	σ_1/MPa	σ_1/σ_t	τ_{max}/MPa	τ_{max}/σ_1
A1 (Single notch)	125	0.57	3.5	1.13	3	0.86
A2 (Double notches)	100.5	0.46	4.0	1.29	7	1.75
A3 (Single notch)	80.5	0.37	4.5	1.45	7	1.56
B1 (Double notches)	15	0.07	3.0	0.64	7	2.33

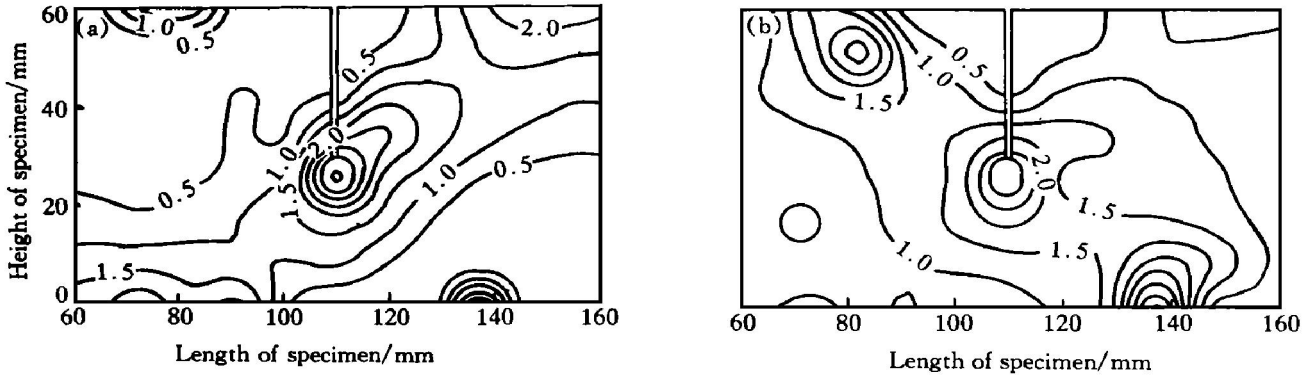

Fig. 3 Stress contours for specimen A1
(a) —Maximum principal stress σ_1(MPa); (b) —Maximum shear stress τ_{max}(MPa)

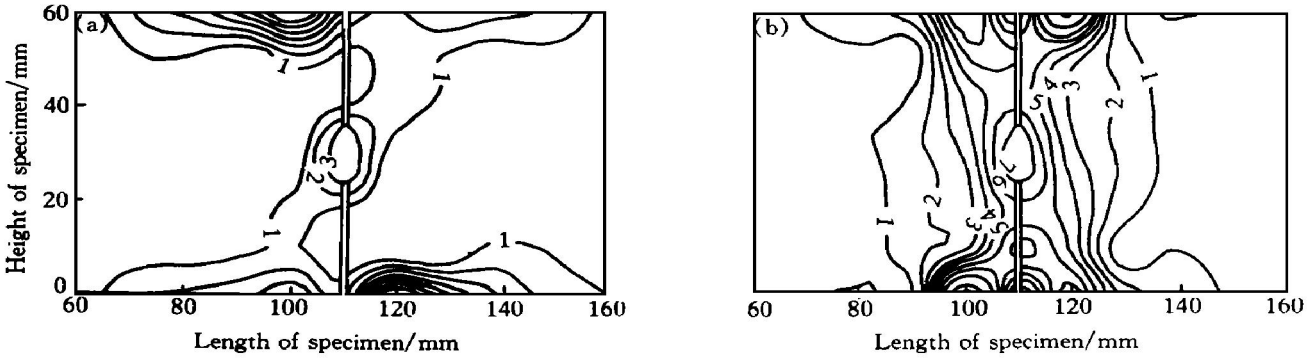

Fig. 4 Stress contours for specimen B1
(a) —Maximum principal stress σ_1(MPa); (b) —Maximum shear stress τ_{max}(MPa)

4 DISCUSSION

4.1 Effect of loading point position on fracture trajectory

The fracture trajectories of the specimens with different loading point positions are quite different (see Fig. 2 and Table 4). For the single-notched specimen, when the distance between the inner and outer loading points is relatively long, L_1+L_2 is equal to 80.5 mm ($0.1L < L_1+L_2 < 0.5L$) and 125 mm ($L_1+L_2 > 0.5L$), the crack is usually initiated at an angle, 57.9° (specimen A3) or 68.8° (specimen A1), to the notch plane and then rapidly reaches the edge of the specimen in a curve line.

For the double-notched specimen, when L_1+L_2 is 100.5 mm ($0.1L < L_1+L_2 < 0.5L$), two cracks initiate at 51.3° and 54.3° simultaneously at the two

notch tips, then stretch to a certain length and finally abruptly connect with the opposite notch tip (specimen A2). Noticeably, when L_1+L_2 is very small, 15 mm ($L_1+L_2<0.1L$), the crack is initiated and propagated in the original notch plane (specimen B1).

It is concluded that for both the single- and double-notched specimens, the crack initiation angle θ_{mC} decreases as L_1+L_2 decreases. This is attributed to the change of stresses at the notch tip when the distance between the two loading points becomes shorter, which will be discussed as follows.

4.2 Effect of loading point position on stresses at notch tip and on fracture mode

Numerical results indicate that under the pure shear loading, both tensile and shear stresses exist at the notch tips and they have the same order of magnitude, see Table 5. When the distance between the inner and outer loading points decreases, the maximum principal stress σ_1 may increase (for the single-notched specimen) or decreases (for the double-notched specimen), but the ratio of τ_{max}/σ_1 increases for both the single- and double-notched specimens.

In specimen A1, the magnitude of σ_1 exceeds its tensile strength σ_t and thus causes the fracture initiation. Since the ratio of τ_{max}/σ_1 is lower, the tensile stress is predominant during the fracture process and the fracture is of Mode Ⅰ. The crack initiation angle, 68.8°, is in good agreement with the theoretical crack initiation angle of Mode Ⅰ fracture under the pure shear loading, 70.5°, based on the maximum circumferential stress criterion (σ_θ-criterion)[10].

For specimens A2 and A3, since σ_1 is larger than σ_t and the ratio of τ_{max}/σ_1 is relatively higher than that in specimen A1, the fracture initiation is caused by the tensile stress, while the fracture propagation results from the co-action of both the tensile and shear stresses. It may be thought that the fracture is of mixed mode Ⅰ-Ⅱ. The crack initiation angle, 51.3° ~ 57.9°, is smaller than the theoretical crack initiation angle 70.5°, which corresponds to that of Mode Ⅰ fracture under the pure shear loading. The measured fracture toughness K_{mC} is slightly larger than K_{IC} and about 1~ 2 times K_{IC}.

Specimen B1 is an exception. When the inner and outer loading points are very close to the notch plane, the notch tip is subjected to an extremely high shear stress, 7 MPa, and relatively low tensile stress, 3 MPa (smaller than σ_t). This leads to a high ratio of τ_{max}/σ_1, 2.33 in the specimen. The lower tensile stress and higher shear stress facilitate the occurrence of Mode Ⅱ fracture. The obtained Mode Ⅱ fracture toughness K_{IIC} is much larger than K_{IC} and about 3.5 times K_{IC}.

5 CONCLUSIONS

1) Loading point position has a great influence on fracture mode of rock. When the distance between the inner and outer loading points is very long, e.g. $L_1+L_2>0.5L$, Mode Ⅰ fracture occurs.

2) When the distance between the inner and outer loading points becomes shorter, e.g. $0.1L<L_1+L_2<0.5L$, mixed mode Ⅰ-Ⅱ fracture occurs.

3) Mode Ⅱ fracture occurs when the distance between the inner and outer loading points is very short, e.g. $L_1+L_2<0.1L$.

4) The ratios of Mode Ⅱ to Mode Ⅰ fracture toughness, K_{IIC}/K_{IC}, and mixed mode Ⅰ-Ⅱ to Mode Ⅰ fracture toughness, K_{mC}/K_{IC}, are different. The former is about 2~ 4 and the latter is 1~ 2.

[REFERENCES]

[1] Awaji H, Sato S. Combined mode fracture toughness measurement by the disk test [J]. J of Eng Mater Tech, 1978, 100: 175- 182.

[2] Richard H A. A new compact shear specimen [J]. Int J Fract, 1981: R105- 107.

[3] Banks-Sills L, Arcan M. An edge-cracked mode Ⅱ fracture specimen [J]. Exp Mech, 1983, 23(3): 257- 261.

[4] Carpinteri A, Ferrara G, Melchiorri G. Single edge notched specimen subjected to four point shear: an experimental investigation [A]. Shah S P, Swartz S E, Barr B. Fracture of Concrete and Rock: Recent Developments [C]. 1989. 605- 614.

[5] Jung S J, Enbaya M, Whyatt J K. The study of fracture of brittle rock under pure shear loading [A]. Mybe L R, Tsang C F, Cook N G W, et al. Fractured and Jointed Rock Masses: Proceedings of the Conference on Fractured and Jointed Rock Masses [C]. Lake Tahoe, California, USA, 1992. 457- 463.

[6] RAO Qiu-hua. Pure Shear Fracture of Brittle Rock, a Theoretical and Laboratory Study [D]. Sweden: Lulea University of Technology, 1999. 54- 77.

[7] Li L Y. A simple method for calculation of K_I and K_{II} for four-point bending specimen [J]. J Water Conservancy, (in Chinese), 1989, 9: 54- 59.

[8] WANG Gui-yao. Study of Mode Ⅱ Fracture of Rock and its Engineering Application [D]. Changsha: Central South University of Technology, 1996.

[9] Otsuka A. Investigation of Mode Ⅱ fatigue characteristics of aluminum alloy weldments using four-point shear loading test technique [A]. Proceedings of the 16th fatigue symposium [C]. Soc Mater Sci, Japan, 1982.

[10] Erdogan F, Shi G C. On the crack extension in plate under in plane loading and transverse shear [J]. J Basic Eng, 1963, 85(4): 519- 527.

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