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Effect of thermal residual stresses on yielding behavior under tensile or compressive loading of short fiber reinforced metal matrix composite①

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[Abstract] Using large strain two-dimension axisymmetric elasto-plastic finite element method and the modified law of mixture, the effects of thermal residual stresses on the yielding behavior of short fiber reinforced metal matrix composite and their dependencies on the material structure parameters (fiber volume fraction, fiber aspect ratio and fiber end distance) were studied. It is demonstrated that the stress-strain partition parameter can be used to describe the stress transfer from the matrix to the fiber. The variation of the second derivation of the stress-strain partition parameter can be used to determine the elastic modulus, the proportion limit, the initial and final yield strengths. In the presence of thermal residual stress, these yielding properties are asymmetric and are influenced differently by the material structure parameters under tensile and compressive loadings.

[Key words] metal matrix composite; thermal residual stress; FEM; yielding behavior
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1 INTRODUCTION

Due to special material structure of short fiber reinforced metal matrix composite (SFRMMC), the matrix and the fibre deform evidently non-uniformly. This non-uniform deformation can be further enhanced due to the presence of thermal residual stresses (TRS) developed during cooling of the composite from processing temperature. It is shown that TRS can result in asymmetric elastic modulus and yield strength under subsequent tensile and compressive loadings. Due to the non-uniformity of the matrix plastic deformation, using the traditional 0.2% offset yield strength ($\sigma_{0.2}$) to define the yielding strength of the composite would cause considerable confusion[1~3]. An accurate and reliable method is therefore highly desired to determine the overall composite properties pertaining to the yielding process, such as elastic modulus, proportion limit and yield strengths. It can be noted that in spite of considerable attempts to investigate the deformation and properties of SFRMMC, the investigations of initial yielding behavior are quite limited[1,2]. By examining the variation of stress—strain curves and observing the microstructure of the composite, the yielding behavior of SFRMMC was studied. The yielding points were defined according to the positions of the extreme points in the third derivative of the experimental stress—strain curves[1]. Another method was proposed recently to study the yielding behavior of SFRMMC by using the modified law of mixture and the finite element method[4]. The yielding behavior can be well determined by calculating the variation of the second derivative of the stress-strain partition parameter. However, the effects of TRS were not taken into account.

In the present study, large strain axisymmetric elasto-plastic finite element method and the modified law of mixture were used to study the yielding behavior of SFRMMC in the presence of TRS. The yielding process was analyzed and the method determining the yielding properties, i. e. proportion limit and yield strengths was proposed according to the variation of the stress-strain partition parameter. The dependencies of the yielding properties on the material structure parameters (fiber aspect ratio, fiber volume fraction and fiber end distance) were also analyzed.

2 ANALYSIS

2.1 Modified law of mixture

The modified law of mixture is applied to de-

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scribe the tensile stress-strain response in dual-phase materials[5,6]. For SFRMMC, in the presence of TRS, the modified law of mixture can be written as

$$\sigma_C = (1-\varphi_F)\sigma_M + \varphi_F\sigma_F = (1-\varphi_F)(\sigma_M^* + \sigma_M^R) + \varphi_F(\sigma_F^* + \sigma_F^R) \quad (1)$$

$$\varepsilon_C = (1-\varphi_F)\varepsilon_M + \varphi_F\varepsilon_F = (1-\varphi_F)(\varepsilon_M^* + \varepsilon_M^R) + \varphi_F(\varepsilon_F^* + \varepsilon_F^R) \quad (2)$$

where φ_F is the fiber volume fraction, σ_C and ε_C, σ_M and ε_M, σ_F and ε_F are the axial stresses and strains of the composite, the matrix and the fiber respectively; σ_M^* and σ_F^*, ε_M^* and ε_F^* are the axial stresses and strains of the matrix and the fiber due to the applied tensile or compressive loading, respectively; σ_M^R and σ_F^R, ε_M^R and ε_F^R are the axial residual stresses and strains of the matrix and the fiber, respectively.

The self-equilibrium of TRS in the matrix and the fiber requires

$$\sigma_C^R = (1-\varphi_F)\sigma_M^R + \varphi_F\sigma_F^R = 0 \quad (3)$$

For the convenience of investigating the stress-strain response of composite under tension or compression, the strain at room temperature is used (at which the applied strain equal to zero). So Eqns. (1) and (2) can be expressed as

$$\sigma_C = (1-\varphi_F)\sigma_M^* + \varphi_F\sigma_F^* \quad (4)$$

$$\varepsilon_C^* = (1-\varphi_F)\varepsilon_M^* + \varphi_F\varepsilon_F^* \quad (5)$$

where ε_C^* is the composite strain due to tension or compression in the absence of TRS. The relationship between the stress and strain can be described by the stress-strain partition parameter[4~6]:

$$q = \frac{\sigma_M - \sigma_F}{\varepsilon_M^* - \varepsilon_F^*} \quad (6)$$

The physical meaning of the modified law of mixture and the stress-strain partition parameter is shown in Fig. 1. The load transfer stress can be defined as $\sigma_{TR} = \sigma_M^* - \sigma_F^* = -q(\varepsilon_M^* - \varepsilon_F^*)$ from Eqn. (6), which is a measurement of the stress transfer from the matrix to the fiber. The meaning of σ_{TR} is also shown in Fig. 1. From Eqns. (4) ~ (6), the composite stress can be written as

$$\sigma_C = \sigma_M^* + (1-\varphi_F)\sigma_{TR} = \sigma_M^* - q(1-\varphi_F)(\varepsilon_M^* - \varepsilon_F^*) \quad (7)$$

The above equations indicate that the stress-strain partition parameter q is an important parameter, which gives the relationship of the stress-strain behaviors among the matrix, the fiber and the composite. Therefore, the overall deformation behavior of the composite can be well described if one knows the variation of the parameter q during the deformation.

2.2 Finite element analysis

In the present finite element analysis, the stress—strain relationships of the matrix and the fiber can be calculated using a volume average approach as the following.

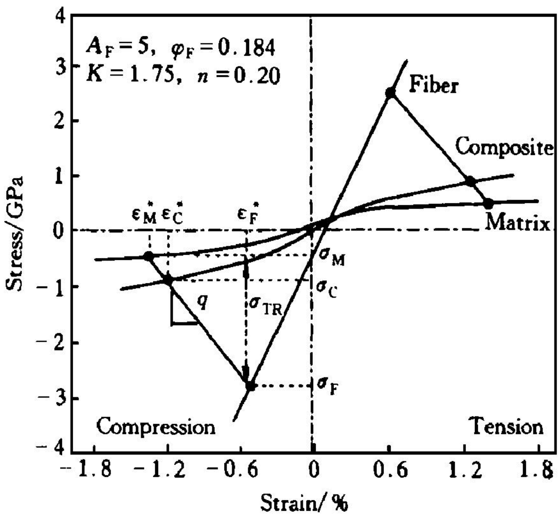

Fig. 1 Schematic of modified law of mixture

$$\sigma_M = \frac{1}{V_{MT}}\sum_{J=1}^{N_M}\bar{\sigma}_J^M \Delta V_{MJ} \quad (8)$$

$$\sigma_F = \frac{1}{V_{FT}}\sum_{J=1}^{N_F}\bar{\sigma}_J^F \Delta V_{FJ} \quad (9)$$

where $\bar{\sigma}_J^M$ and $\bar{\sigma}_J^F$ are the average axial stresses of the matrix and the fiber due to the applied tensile or compressive loadings which are obtained from the Gaussian integration points within each finite element; ΔV_{MJ}, ΔV_{FJ}, N_M and N_F are the volume and number of the finite element of the matrix and the fiber, respectively; V_{MT} and V_{FT} are the volume of the matrix and the fiber in the unit cell. The expressions for ε_M^* and ε_F^* have similar forms to those of σ_M and σ_F. The finite element model and method were specified in Ref. [7].

3 RESULTS AND DISCUSSION

3.1 Effects of TRS on $|q|-\varepsilon_C$ curves and stress—strain curves

The calculated $|q|-\varepsilon_C$ curves in tension and compression for different material parameters (fiber volume fraction φ_F, fiber aspect ratio A_F and fiber end distance K) are shown in Fig. 2. The corresponding stress—strain curves of the composite are shown in Fig. 3. The material data used for the calculations are the same as those in Ref. [7]. For comparison, the corresponding $|q|-\varepsilon_C$ curve and the stress—strain curve in the absence of TRS are also plotted in Fig. 2 and Fig. 3, respectively. As shown in Fig. 2, the shapes of the $|q|-\varepsilon_C$ curves in tension and compression are different and also are different from that in the absence of TRS[4,8]; while in the absence of TRS, the $|q|-\varepsilon_C$ curves are identical in tension and compression. Unlike that in the absence of TRS, the $|q|-\varepsilon_C$ curves in tension and compression do not exhibit a plateau region in lower strain range. The $|q|$ value in compression is larger than that in tension. In compression, the $|q|$ value decreases continuously, while in tension the $|q|$ value first decreases, then increases with increasing ε_C. The variation of $|q|$ in tension is obvious because there is a transition of the fiber axial stress from compression to tension (from negative to positive). Since such a transition does not occur in compression (the fiber axial stress and strain are always negative value), the $|q|$ value in compression decreases continuously with increasing the strain.

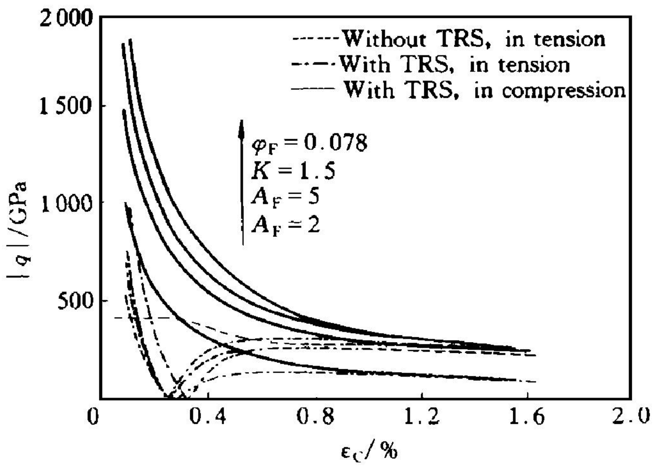

Fig. 2 $|q|$ —ε_C curves in tension and compression

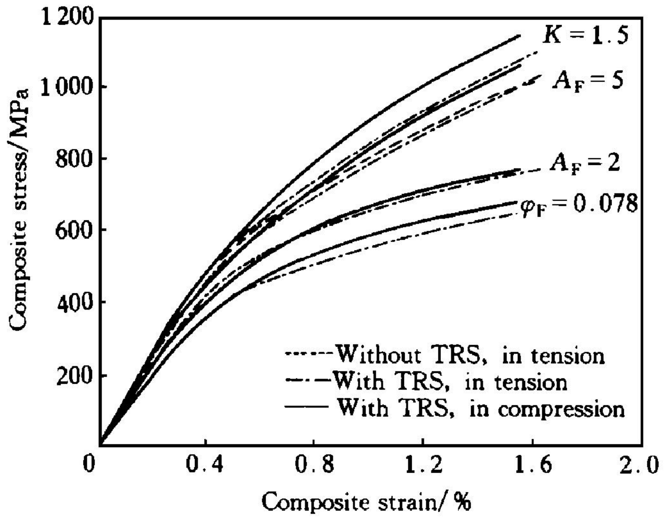

Fig. 3 Stress —strain curves of composite in tension and compression

Fig. 2 also shows that, the $|q|$ value in compression is larger than that in tension, and the effects of the three material parameters on $|q|$ value is similar to their effects on fiber and matrix axis stress distributions[7], which indicates that $|q|$ can be used to describe the stress transfer from the matrix to the fiber in the procedure of composite deformation.

It can be observed from Fig. 3 that the stress — strain curves of the composite in tension and compression are asymmetric, the stress value in compression is higher than that in tension for the same material structural parameter. Furthermore, the stress — strain curves in tension and compression exhibit the deviation from the linearly elastic behavior at lower strains, but the exact deviation points are difficult to be detected. Obviously, the above asymmetry and early deviation from the linear elastic behavior in the stress —strain curves are due to asymmetric stress distributions, matrix plastic behavior and the early plastic deformation in the matrix caused by TRS.

In addition, it can be seen from Fig. 3 that the material structure parameters, φ_F, A_F and K have important effects on the stress —strain curves of the composite. Compared with the results in the absence of TRS[4], the presence of TRS does not change the tendency of stress —strain curve on the material structure parameters.

3.2 Determination of proportional limit and yielding strengths

Since the plastic deformation is a gradual transition process from local to whole matrix region, the deviation of the stress —strain curve of the composite from the linearly elastic behavior is related to not only the amount of the plastic deformation, but also the distribution of the plastic deformation in the matrix. Therefore, to explore the essence of the yielding process and to determine the composite properties in the yielding stage, such as the elastic modulus, the proportional limit and the yield strengths, it is essential to know the connection of the overall yielding behavior of the composite with the matrix plasticity.

It can be clearly seen from Eqn. (6) that the parameter q is very sensitive to the change of the stress-strain behavior of the matrix and the fiber. It is possible to use q to describe the yielding behavior of the composite by examining the variation of the parameter q with the strain. The detail calculations show that the second derivative of q has a sharper variation with the applied strain than q and its first derivative. Two examples of the calculated q'' —ε_C curves for tension and compression and the corresponding stress—strain curves are given in Fig. 4. It can be seen from the q'' —ε_C curves that there are two distinct extreme points at A and B. Between points A and B, the q'' —ε_C curve has sharper variation. After point C the q'' value keeps nearly constant with increasing strain. To establish the connection of the variation in the q'' —ε_C curves with the development of the matrix plasticity, the contours of the von Mises effective stress in the matrix and the fiber are calculated and shown in Fig. 5 for the corresponding A, B and C points. As shown in Fig. 4 and Fig. 5, at point A, the plastic region forms in smaller region near the fiber end face both in tension and compression. In tension the plastic deformation occurs around the fiber, while in compression the matrix around the fiber is still elastic. At point B, most regions in the fiber end enter the plastic state both in tension and

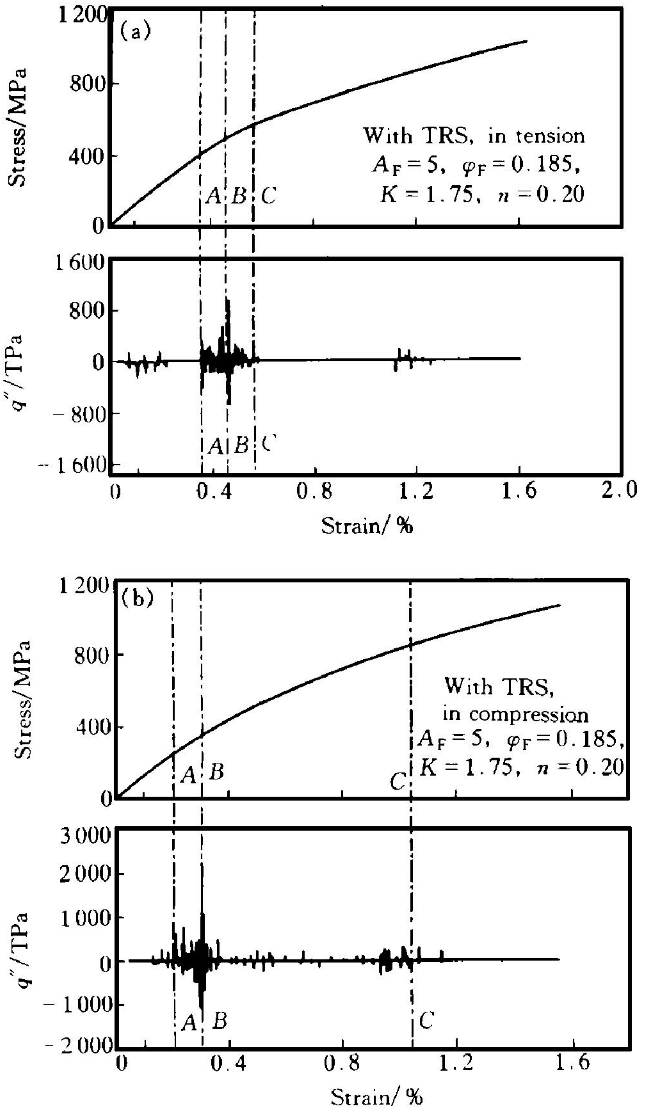

Fig. 4 q'' —ε_C curves and corresponding stress—strain curves in tension and compression

(a) —In tension; (b) —In compression

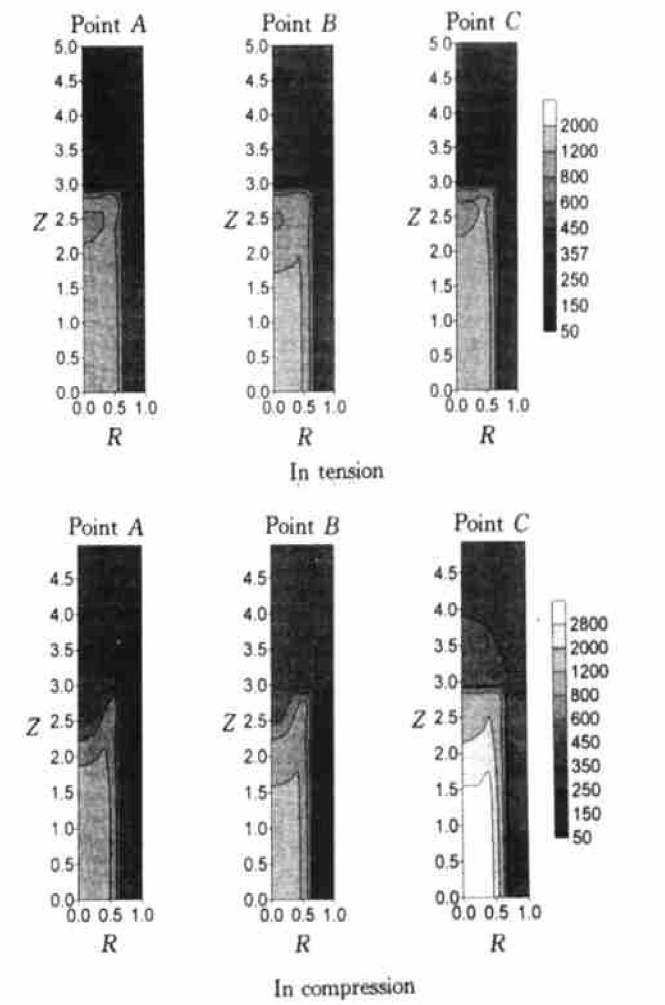

Fig. 5 Contours of von Mises effective stress in matrix and fiber in tension and compression

compression. Between points A and B, the strain-hardening rate of the composite (the slope of the stress—strain curve) becomes small. At point C, the plastic deformation occur in the entire matrix including the fiber region and the fiber end region, the strain-hardening rate of the composite becomes markedly small.

According to the results in Fig. 4 and Fig. 5, the properties of the composite in the yielding stage can be well determined. It can be thought that point A is a point before which the overall deformation of the composite is basically elastic although there exist local plastic regions in the matrix. This argument is supported by the results in Fig. 3 and Fig. 4 that the slope of the stress—strain curve keeps basically constant before point A. So phenomenally, the strain corresponding to point A can be defined as the maximum elastic strain of the composite, the stress at this point can be defined as the proportional limit of the composite. The elastic modulus of the composite can be thus determined as the average value of the slope of the stress—strain curve before point A. Since significant plastic deformation occurs in the fiber end region, the stress at point B can be considered the initial yield strength; while the stress at point C can be defined as the final yield strength because the deformation of the matrix is fully plastic after this point. The further calculations show that the material structure parameters change the positions of three points, but do not change basic development process of the plastic deformation.

It should be pointed out that the proportional limit and the yield strengths determined by the present method are different from the traditional 0.2% offset yield strength ($\sigma_{0.2}$)[1, 2, 9] which is obviously vague in the physical meaning since there is no direct connection of $\sigma_{0.2}$ with the development of the matrix plasticity.

3.3 Effects of material structural parameters on elastic modulus and yield strengths

Since the fiber end distance K is of statistical feature, only the effects of the fiber volume fraction φ_F and the fiber aspect ratio A_F on the elastic modulus and the yield strengths of the composite are analyzed here. In the calculation, the material parameters are taken from a typical whisk reinforced aluminum matrix composite (SiC_w/6061Al, T6 treatment)[7]. K is taken as 1.75 from the relation $K = L_C/L_F = R_C/R_F$.

Fig. 6 shows the variation of the normalized elastic modulus in tension and compression E_C^T/E_M and E_C^C/E_M with A_F for three given φ_F, where E_M is the elastic modulus of the matrix. For comparison, the variation of the normalized elastic modulus E_C/E_M in the absence of TRS is also shown in Fig. 6. As revealed, the variations of E_C^T and E_C^C with φ_F and A_F are similar to that of E_C. E_C^C is smaller than E_C^T and both E_C^C and E_C^T are smaller than E_C for given φ_F and A_F. These results are in agreement with the previous results[9].

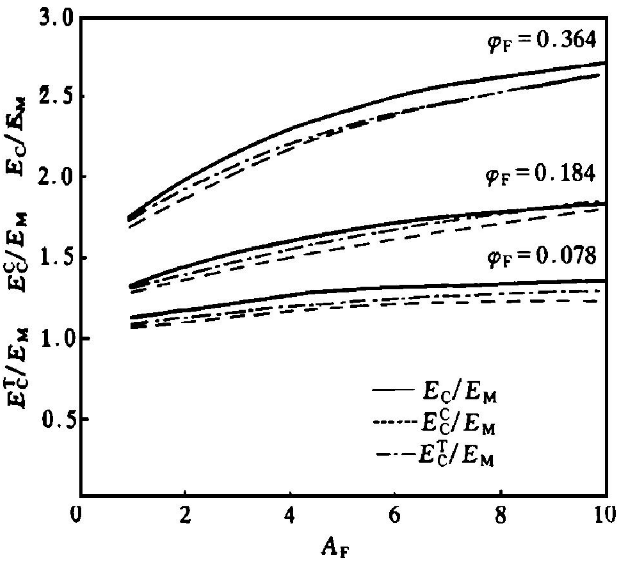

Fig. 6 Variations of E_C^T/E_M, E_C^C/E_M and E_C/E_M with A_F and φ_F

Fig. 7 shows the variations of the calculated proportion limit σ_P, the initial yielding stress σ_{IY} and the final yielding stress σ_{FY} with A_F and φ_F. In tension, σ_P, σ_{IY} and σ_{FY} increase with increasing φ_F and A_F. In compression, σ_P and σ_{IY} decrease slightly and σ_{FY} increases with increasing φ_F and σ_P, σ_{IY} and σ_{FY} increase with increasing A_F. Furthermore, it can be seen that in the presence of TRS, σ_P and σ_{IY} in tension are higher than those in compression, σ_{FY} in tension is lower than that in compression.

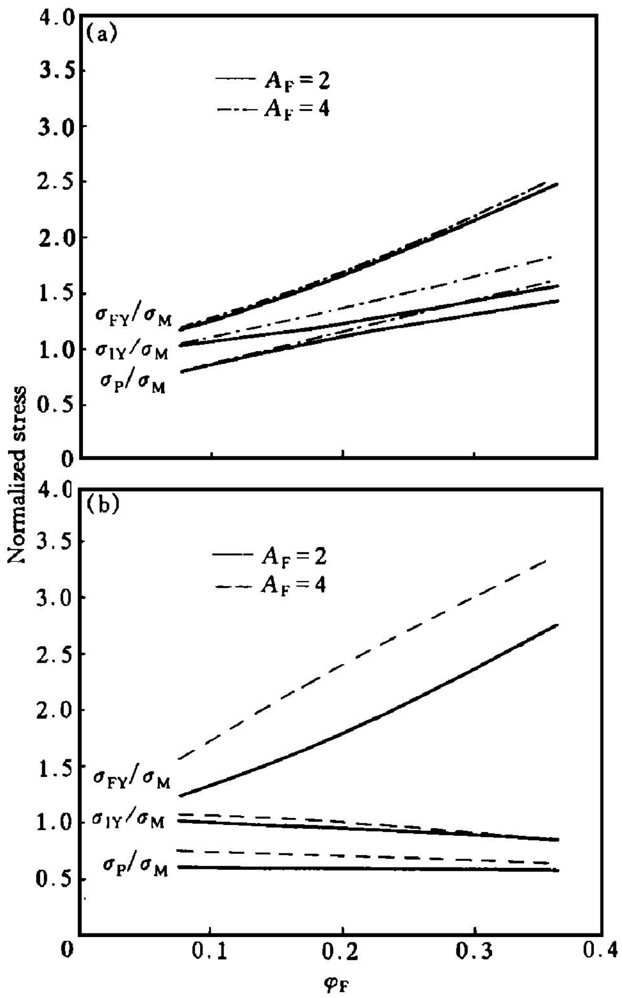

Fig. 7 Variations of σ_P, σ_{IY} and σ_{FY} with A_F and φ_F

($A_F/A_C = L_F/L_C$, $n = 0.20$)

(a) —In tension; (b) —In compression

In addition, the experimental tensile yield strengths ($\sigma_{0.2}$) of several SiC/Al composites are compared with the calculated σ_P and σ_{IY} in Table 1. It can be seen that the experimental $\sigma_{0.2}$ is larger than the corresponding σ_p and is smaller than the corresponding σ_{IY} and has same varying tendency with φ_F as σ_P and σ_{IY}.

Table 1 Comparisons of experimental yield strength with proportional limit and initial yield strength

Material	φ_F	A_F	$\sigma_{0.2}$/MPa	σ_P/MPa	σ_{IY}/MPa	E_M/GPa	E_F/GPa	σ_M/MPa
SiC_d/Al 2124, T6[10]	0.20	4	497	367	506			
SiC_w/Al 6061, T6[11]	0.10	2	312	238	322			
SiC_w/Al 6061, T6	0.20	2	328	281	356			
SiC_w/Al 6061, T6	0.30	2	356	304	379			
Al2124, T6						69	483	400
Al6061, T6						68.8	450	280

T6 treatment temperature of all composites is 170 ℃

4 CONCLUSIONS

1) The stress-strain partition parameter can describe the stress transfer from the matrix to the fiber in tension and compression in the presence of TRS qualitatively.

2) The variation of the second derivative of stress-strain partition parameter is sensitive to the initial yielding behavior of SFRMMC in tension and compression in the presence of TRS. The composite properties in the yielding range, i. e. the elastic modulus, the proportion limit, the initial and final yield strengths determined by the method have more exact physical meanings.

3) Due to the presence of thermal residual stresses, the elastic modulus, the proportional limit and the initial and final yield strengths are asymmetric in tension and compression. The material structure parameters have important effects on these properties and the effects are quite different in tension and compression.

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