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Temperature simulation of EMC aluminum ingot with induced heat ①

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[Abstract] The numerical simulation of temperature field of electromagnetic casting (EMC) aluminum ingots is an effective and also necessary approach to study the temperature field and forecast the quality of EMC ingot, or optimize the technological parameters. In EMC, the alternating electromagnetic field can produce induced current and heat within the surface layer. To calculate the temperature field precisely, the induced heat should be taken into account. The induced heat has been coupled into the calculation formula of temperature field of unit volume per unit time, which provides a convenient and also precise method to calculate the temperature field. Besides, the effect of induced heat on the temperature field of ingot has been simulated and discussed. The results show that the induced heat has large influences on the height of liquid column and the surface temperature of ingot.

[Key words] electromagnetic casting; temperature field; induced heat; skin effect
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1 INTRODUCTION

From certain point of view, the process of electromagnetic casting (EMC) is also a heat-extracting process. The heat should be extracted, including the overheat of liquid metal, the latent heat and the heat of high temperature solid metal[1]. The temperature field in the cast ingots not only determines the process parameters such as casting speed, but also directly influences the outer and inner quality of ingot. So the study of temperature field of ingot during EMC process is of great significance. The computer numerical simulation can forecast the temperature field of ingot in EMC process quickly and effectively, and help select the technological parameters, calculate the stress field and even predict the quality of ingots, so it is now widely used and has been given much attention.

Before numerical simulation, a mathematical model should be founded from the corresponding physical model, and then the approximate solution can be got with some discrete method. Generally, there is no question in numerical simulation of temperature field, but for EMC process, there are some particular problems to be solved.

In EMC, the alternating electromagnetic field can produce an induced current within the surface layer, which is well known as the skin effect. This kind of short circuit current will yield considerable heat within the surface layer and affect the temperature field. To calculate the temperature field precisely, the induced heat should be taken into account.

Prasso et al[2] have simulated the temperature field of EMC aluminum ingot using finite element method, and the induced heat is pointed out to be calculated based on the solution of Maxwell function. But how it is done hasn't been given in detail.

In this paper, the induced heat has been coupled into the formula of temperature field of unit volume per unit time. Besides, the effect of induced heat on the temperature field of ingot has been simulated and discussed.

2 HEAT TRANSFER EQUATION IN EMC

The governing heat-transfer equation in EMC is

$$c_p\rho\left(\frac{\partial T}{\partial t} + v \cdot \nabla T\right) = \nabla \cdot (k \nabla T) + \dot{q} \qquad (1)$$

where c_p, k and ρ are the special heat, heat conductivity and mass density of the metal respectively, v is the mass transfer speed, T is the temperature, t refers to time, and $\dot{q}$ is the inner heat source, including the latent heat and induced heat.

Usually the horizontal flow speed of the calculated element is ignored and the liquid metal and solid metal are assumed as one block and move downward with the same speed, that is, the casting speed v_c. The facile heat transport in the liquid pool due to the flow of liquid metal is dealt with a simplified method by multiplying a factor to enhance the thermal conductivity. Thus Eqn. (1) can be written as

$$c_p\rho\frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) - c_p\rho v_c\frac{\partial T}{\partial Z} + \dot{q} \qquad (2)$$

The temperature field of EMC ingot in fact is continuously changing in space. The casting speed item in Eqn. (2) makes the heat transfer equation as a three-dimensional, non-steady and non-homogeneous equation. It's very difficult to solve it directly.

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But now it has been verified that this equation can be replaced by a non-steady, homogeneous Eqn. (3)[3].

$$c_p \rho \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \dot{q} \tag{3}$$

The dynamic temperature field at any next time step can be calculated first by using Eqn. (3). Then, the temperature field move down a distance $v\Delta t$, at the same time, the $v\Delta t$ length at the top should be filled with liquid metal.

The methods usually used in numerical simulation of temperature field include finite difference method (FDM), finite element method (FEM) and boundary element method (BEM), and so on. FDM has the advantage of simple formula, easy netting, small calculating amount, and stable converged result, and is quite suitable in calculating the temperature field that has phase change and large temperature change. According to the different dealing methods of time, it is also divided as explicit FDM, implicit FDM and alternate explicit (or implicit) FDM, and so on. Alternate explicit FDM is a kind of modified FDM, which first calculates the temperature of every element in one time step from quite different directions called positive and negative directions, and then makes the average value of the two calculated results as the current temperature of the element. This method has both the advantages of the explicit and implicit FDM. The requirement of time step isn't strict and will converge stably but has no large calculation amount. So in this paper, the alternate explicit FDM is used.

One of the calculation units is shown in Fig. 1. Suppose the special heat and mass density of the metal are c and ρ respectively, the thermal conductivity of the unit itself and its neighbor units, that is, the left, right, front, behind, upper, and down units are k_e, k_{el}, k_{er}, k_{ef}, k_{eb}, k_{eu} and k_{ed} respectively, Δx, Δy and Δz are the sizes of the units at x, y and z direction. And compound thermal conductivities between the neighbor units and the calculated unit k_l, k_r, k_f,

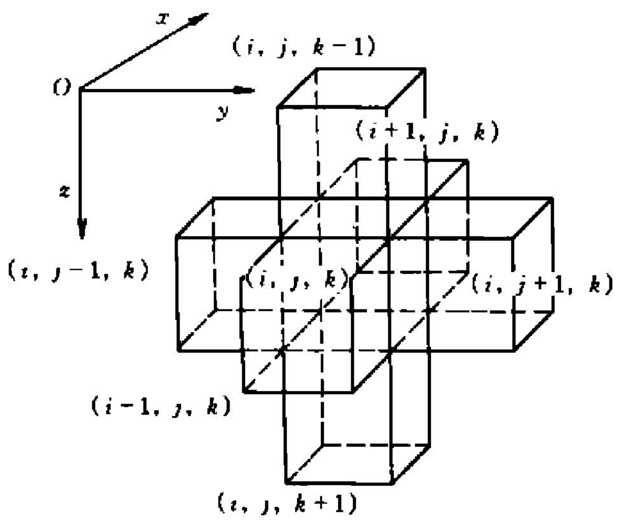

Fig. 1 Schematic drawing of three-dimensional unit

k_b, k_u and k_d are given as follows:

$$k_l = \frac{k_e k_{el}(\Delta y_{j-1} + \Delta y_j)}{k_{el}\Delta y_j + k_e \Delta y_{j-1}},$$

$$k_r = \frac{k_e k_{er}(\Delta y_{j+1} + \Delta y_j)}{k_{er}\Delta y_j + k_e \Delta y_{j+1}},$$

$$k_f = \frac{k_e k_{ef}(\Delta x_{i-1} + \Delta x_i)}{k_{ef}\Delta x_i + k_e \Delta x_{i-1}},$$

$$k_b = \frac{k_e k_{eb}(\Delta x_{i+1} + \Delta x_i)}{k_{eb}\Delta x_i + k_e \Delta x_{i+1}},$$

$$k_u = \frac{k_e k_{eu}(\Delta z_{k-1} + \Delta z_k)}{k_{eu}\Delta z_k + k_e \Delta z_{k-1}},$$

$$k_d = \frac{k_e k_{ed}(\Delta z_{k+1} + \Delta z_k)}{k_{ek}\Delta z_k + k_e \Delta z_{k+1}}$$

According to the theory of energy conversation, the energy change of the unit in a time step of Δt is equal to the energy exchange between itself and its neighbor units. As a result, the following equation is got

$$\begin{aligned}\Delta Q_{i,j,k} &= c\rho(T^{p+1}_{i,j,k} - T^{p}_{i,j,k}) \cdot \Delta x_i \Delta y_j \Delta z_k \\ &= \Delta Q_l + \Delta Q_r + \Delta Q_u + \Delta Q_d + \\ &\quad \Delta Q_f + \Delta Q_b + \Delta Q_m\end{aligned} \tag{4}$$

where $T^{p+1}_{i,j,k}$ and $T^{p}_{i,j,k}$ represent the temperatures of the unit at next time step and the current time step respectively; $\Delta Q_{i,j,k}$ is the energy change of the unit during one time step, while ΔQ_l, ΔQ_r, ΔQ_u, ΔQ_d, ΔQ_f and ΔQ_b are the energies transferred to the unit from its left, right, upper, down, front and behind neighbor units; ΔQ_m is the inner heat source. And there are

$$\begin{aligned}\Delta Q_l &= k_l \mathrm{grad} T\, \Delta x_i \Delta z_k \Delta t \\ &= k_l \frac{T^p_{i,j-1,k} - T^p_{i,j,k}}{1/2(\Delta y_{j-1} + \Delta y_j)} \Delta x_i \Delta z_k \Delta t\end{aligned} \tag{5}$$

$$\begin{aligned}\Delta Q_r &= k_r \mathrm{grad} T\, \Delta x_i \Delta z_k \Delta t \\ &= k_r \frac{T^p_{i,j+1,k} - T^p_{i,j,k}}{1/2(\Delta y_{j+1} + \Delta y_j)} \Delta x_i \Delta z_k \Delta t\end{aligned} \tag{6}$$

$$\begin{aligned}\Delta Q_u &= k_u \mathrm{grad} T\, \Delta x_i \Delta y_j \Delta t \\ &= k_u \frac{T^p_{i,j,k-1} - T^p_{i,j,k}}{1/2(\Delta z_{k-1} + \Delta z_k)} \Delta x_i \Delta y_j \Delta t\end{aligned} \tag{7}$$

$$\begin{aligned}\Delta Q_d &= k_d \mathrm{grad} T\, \Delta x_i \Delta y_j \Delta t \\ &= k_d \frac{T^p_{i,j,k+1} - T^p_{i,j,k}}{1/2(\Delta z_{k+1} + \Delta z_k)} \Delta x_i \Delta y_j \Delta t\end{aligned} \tag{8}$$

$$\begin{aligned}\Delta Q_f &= k_f \mathrm{grad} T\, \Delta y_j \Delta z_k \Delta t \\ &= k_f \frac{T^p_{i-1,j,k} - T^p_{i,j,k}}{1/2(\Delta x_{i-1} + \Delta x_i)} \Delta y_j \Delta z_k \Delta t\end{aligned} \tag{9}$$

$$\begin{aligned}\Delta Q_b &= k_b \mathrm{grad} T\, \Delta y_j \Delta z_k \Delta t \\ &= k_b \frac{T^p_{i+1,j,k} - T^p_{i,j,k}}{1/2(\Delta x_{i+1} + \Delta x_i)} \Delta y_j \Delta z_k \Delta t\end{aligned} \tag{10}$$

$$\Delta Q_m = \dot{q}\, \Delta x_i \Delta y_j \Delta z_k \Delta t \tag{11}$$

$$T^{p+1}_{i,j,k} - T^{p}_{i,j,k} = \frac{2\Delta t}{c\rho \Delta y_j} \cdot \left[k_l \frac{T^p_{i,j-1,k} - T^p_{i,j,k}}{\Delta y_{j-1} + \Delta y_j} + k_r \frac{T^p_{i,j+1,k} - T^p_{i,j,k}}{\Delta y_{j+1} + \Delta y_j} \right] +$$

$$\frac{2\Delta t}{c\rho\Delta x_i}\cdot\left[k_f\frac{T^p_{i-1,j,k}-T^p_{i,j,k}}{\Delta x_{i-1}+\Delta x_i}+k_b\frac{T^p_{i+1,j,k}-T^p_{i,j,k}}{\Delta x_{i+1}+\Delta x_i}\right]+\frac{2\Delta t}{c\rho\Delta z_k}\cdot\left[k_u\frac{T^p_{i,j,k-1}-T^p_{i,j,k}}{\Delta z_{k-1}+\Delta z_k}+k_d\frac{T^p_{i,j,k+1}-T^p_{i,j,k}}{\Delta z_{k+1}+\Delta z_k}\right]+\frac{\dot q\Delta t}{c\rho} \quad (12)$$

Set

$$A_l=\frac{2k_l\Delta t}{c\rho\Delta y_j(\Delta y_{j-1}+\Delta y_j)},\quad A_r=\frac{2k_r\Delta t}{c\rho\Delta y_j(\Delta y_{j+1}+\Delta y_j)},$$
$$A_f=\frac{2k_f\Delta t}{c\rho\Delta x_i(\Delta x_{i-1}+\Delta x_i)},\quad A_b=\frac{2k_b\Delta t}{c\rho\Delta x_i(\Delta x_{i+1}+\Delta x_i)},$$
$$A_u=\frac{2k_u\Delta t}{c\rho\Delta z_k(\Delta z_{k-1}+\Delta z_k)},\quad A_d=\frac{2k_d\Delta t}{c\rho\Delta z_k(\Delta z_{k+1}+\Delta z_k)}$$

Then Eqn. (12) can be written as

$$T^{p+1}_{i,j,k}-T^p_{i,j,k}=A_l(T^p_{i,j-1,k}-T^p_{i,j,k})+A_r(T^p_{i,j+1,k}-T^p_{i,j,k})+A_f(T^p_{i-1,j,k}-T^p_{i,j,k})+A_b(T^p_{i+1,j,k}-T^p_{i,j,k})+A_u(T^p_{i,j,k-1}-T^p_{i,j,k})+A_d(T^p_{i,j,k+1}-T^p_{i,j,k})+\frac{\dot q\Delta t}{c\rho} \quad (13)$$

where $\frac{\dot q\Delta t}{c\rho}=\Delta T_1+\Delta T_2$, ΔT_1 is the temperature increase caused by the thermal release of the latent heat, which can be dealt by the temperature rise method or equivalent special heat method. ΔT_2 is the temperature increase caused by the induced heat and can be dealt according to its distribution rule.

Supposing the functions $U_{i,j,k}$ and $V_{i,j,k}$ satisfy the same initial and boundary conditions as the function $T_{i,j,k}$, Eqn. (13) can be solved using alternate explicit FDM.

When calculated in positive direction, there is

$$U^{p+1}_{i,j,k}-U^p_{i,j,k}=A_l(U^{p+1}_{i,j-1,k}-U^{p+1}_{i,j,k})+A_r(U^p_{i,j+1,k}-U^p_{i,j,k})+A_f(U^{p+1}_{i-1,j,k}-U^{p+1}_{i,j,k})+A_b(U^p_{i+1,j,k}-U^p_{i,j,k})+A_u(U^{p+1}_{i,j,k-1}-U^{p+1}_{i,j,k})+A_d(U^p_{i,j,k+1}-U^p_{i,j,k})+\Delta T_1+\Delta T_2 \quad (14)$$

While calculated in negative direction, there is

$$V^{p+1}_{i,j,k}-V^p_{i,j,k}=A_l(V^p_{i,j-1,k}-V^p_{i,j,k})+A_r(V^{p+1}_{i,j+1,k}-V^{p+1}_{i,j,k})+A_f(V^p_{i-1,j,k}-V^p_{i,j,k})+A_b(V^{p+1}_{i+1,j,k}-V^{p+1}_{i,j,k})+A_u(V^p_{i,j,k-1}-V^p_{i,j,k})+A_d(V^{p+1}_{i,j,k+1}-V^{p+1}_{i,j,k})+\Delta T_1+\Delta T_2 \quad (15)$$

Define $W=1/(1+A_l+A_f+A_u)$, $M=1/(1+A_r+A_b+A_d)$, Eqns. (14) and (15) can then be written as

$$U^{p+1}_{i,j,k}=W[(1-A_r-A_b-A_d)U^p_{i,j,k}+A_rU^p_{i,j+1,k}+A_bU^p_{i+1,j,k}+A_dU^p_{i,j,k+1}+A_lU^{p+1}_{i,j-1,k}+A_fU^{p+1}_{i-1,j,k}+A_uU^{p+1}_{i,j,k-1}+\Delta T_1+\Delta T_2] \quad (16)$$

$$V^{p+1}_{i,j,k}=M[(1-A_l-A_f-A_u)V^p_{i,j,k}+A_lV^p_{i,j-1,k}+A_fV^p_{i-1,j,k}+A_uV^p_{i,j,k-1}+A_rV^{p+1}_{i,j+1,k}+A_bV^{p+1}_{i+1,j,k}+A_dV^{p+1}_{i,j,k+1}+\Delta T_1+\Delta T_2] \quad (17)$$

The average value calculated by these two directions is regarded as the actual temperature of the unit, that is

$$T_{i,j,k}=\frac{1}{2}(U_{i,j,k}+V_{i,j,k}) \quad (18)$$

3 CONSIDERATION ON INDUCED HEAT

The schematic diagram of liquid column H of EMC ingot is shown in Fig. 2. In EMC process, the interface of liquid and solid usually is kept at the mid-height of the inductor, here exists the maximal electromagnetic pressure[4], so it can be considered that the induced heat acts on the zone of $2H$ and its value is symmetrical to the liquid-solid interface.

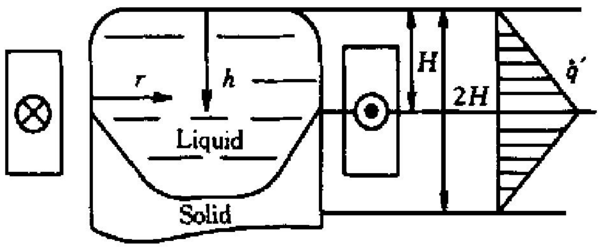

Fig. 2 Schematic diagram of liquid column and distribution of induced heat

The induced heat in the liquid column is

$$\dot q'=\frac{J^2}{\sigma} \quad (19)$$

where J is the effective current density, and σ is the electrical conductivity of liquid aluminum.

Owing to the skin effect, the current density in the liquid column distributes as the following exponential function[5]

$$J=J_0\cdot\exp(-r/\delta) \quad (20)$$

where J_0 is the effective current density at the surface of the liquid column, r is the horizontal distance to the surface of the liquid column, and δ is the skin depth.

During the EMC process, the electromagnetic force acting on the liquid metal should be equal to the metallostatic pressure of the liquid column[6], that is

$$p_E = B_0^2/2\mu = \rho g h \tag{21}$$

where p_E is electromagnetic pressure, B_0 is the effective magnetic flux density, μ is the magnetic permeability with the value about $4\pi \times 10^{-7}$ H/m, ρ is the mass density of liquid aluminum, h is the distance to the top surface of liquid column, and g is the acceleration of gravity.

And because[5, 7]

$$J_0 = \sqrt{2} B_0 / \mu\delta \tag{22}$$

$$J_0 = \frac{2}{\mu\delta} \cdot \sqrt{\mu\rho g h} \tag{23}$$

Combining expressions (19) ~ (23), there is

$$\dot{q}' = \frac{4\rho g h \cdot \exp(-2r/\delta)}{\sigma\mu\delta^2} \tag{24}$$

where the depth of skin effect[8] is

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} \tag{25}$$

Here $\omega = 2\pi f$, f is the frequency of electrical source, usually ranges from 2 000 to 3 000 Hz in EMC. Substituting expression (25) into expression (24), it can be obtained that

$$\dot{q}' = 2\rho g h \omega \exp(-2r/\delta) \tag{26}$$

Hence, ΔT_2 in Eqns. (16) and (17) can be expressed as

$$\Delta T_2 = \frac{\dot{q}' \Delta t}{c\rho} = \frac{2gh\omega \exp(-2r/\delta)\Delta t}{c} \tag{27}$$

Eqn. (27) is the expression of temperature increase due to the induced heat within the liquid column. It is suitable to be used for ingots of any cross section shape.

Obviously, the temperature rise caused by induced heat increases with the height of liquid column, and reaches a maximal value at the liquid-solid interface. This is quite agreeable to the experimental results[9].

According to Eqn. (27), the induced heat mainly depends on the liquid height, and has little relation with the thermal property parameters, except for the skin depth of induced heat, namely $\delta/2$, which is almost the same for the high temperature solid and liquid. Based on this fact, the induced heat of solid ingot below the interface can be treated as being symmetrical to the above liquid column part, although the mass density and the electrical conductivity of solid are different from the liquid.

Practically, considering a horizontal distance of 10 mm away from the surface of the ingot is enough to concern with the induced heat, while the other region can be calculated only through the heat conduction mechanism.

4 RESULTS OF SIMULATION

The effect of induced heat on the temperature field of ingot is shown in Fig. 3. The size of pure aluminum slab is 520 mm × 200 mm. The pouring temperature is 710 ℃ and the casting speed is 1 mm/s. The lines in the figure are isotherms of different temperatures as 660 ℃, 600 ℃, 550 ℃, 500 ℃, 400 ℃ and 300 ℃, and 660 ℃ is known as the solidification point of pure aluminum. The dotted lines represent the calculated isotherms without considering the induced heat in calculation, while the solid lines represent the isotherms for which the induced heat has been considered in calculation. It can be seen that the effect of induced heat on the whole temperature field of ingot is not very obvious. This just reflects the fact that the induced heat is relatively small compared to the overheat of liquid metal and the latent heat. But clearly, the induced heat affects the surface and near surface temperature. It is found that the height of liquid column is 5.5 mm higher than the liquid column calculated without considering the induced heat. In EMC process, the height of liquid column should be controlled very strictly[10]. A fluctuation of 1 mm or more is enough to damage the outer shape of ingot or even break down the casting process. So the influence of induced heat should be paid enough attention. The calculation of induced heat is quite helpful to properly determine the casting speed and the impinging point where the cooling water contacts with the ingot.

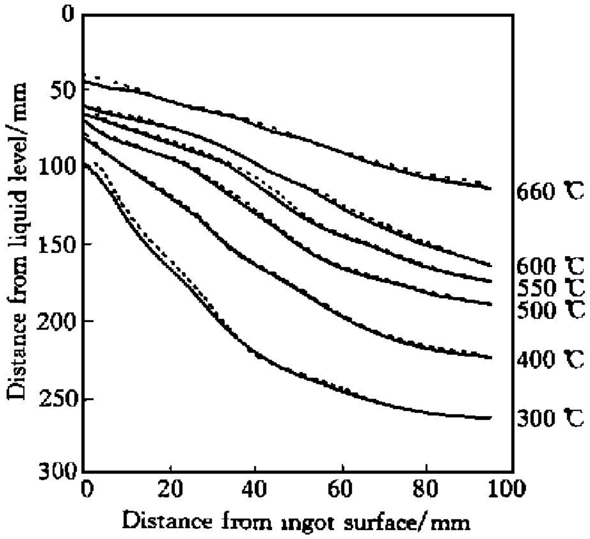

Fig. 3 Comparison of calculated temperature fields near inductor without and with considering induced heat

Solid line—with induced heat;
Dash line—without induced heat

5 CONCLUSIONS

1) According to electromagnetic theory and the

technical parameters, the expression to calculate the temperature field including the induced heat has been deduced.

2) The simulation results of temperature field show that the induced heat has large influences on the height of liquid column and the surface temperature of ingot.

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