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Bundle adjustment for data processing of theodolite industrial surveying system①

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[Abstract] The photogrammetric bundle adjustment was used in data processing of electronic theodolite industrial surveying system by converting angular observations into virtual photo coordinates. The developed algorithm has ability of precision estimation and data-snooping, do not need initial values of exterior orientation elements and object point coordinates. The form of control condition for the system is quite flexible. Neither centering nor leveling is the theodolite needed and the lay-out of theodolite position is flexible when the system is used for precise survey. Experiments carried out in test field verify the validity of the data processing method.

[Key words] electronic theodolite; industrial surveying system; bundle adjustment
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1 INTRODUCTION

The industrial surveying system has gained more and more applications in the positioning, installing and calibrating of industrial equipment as well as in the quality check of industrial products[1]. Generally speaking, the system may assume one of two forms: the first one consists of the electronic theodolite or the total station; the second one is the photogrammetric industrial surveying system. The former is more popular in practice, it can be a real time system using at least of two sets of the electronic theodolites and a computer to conduct space intersection. Before the system is operated, the horizontal distance and the height difference between two theodolites' optical centrals of collimation, the zero direction of the line passing through the centrals of the horizontal circles of the theodolites must be determined. This kind of pre-surveying work should affect the accuracy of the system to a great extent, at the same time, the distance between two stations may be quite short in most cases, some special procedures must be adopted in the work. Moreover, if the industrial object is a bulky one, it is necessary to establish several stations around it and to conduct coordinates transformation between different coordinate systems in data processing.

The bundle adjustment is a rigorous and effective method used in analytical photogrammetry[2]. The method of using bundle adjustment to process theodolite observations was presented by Obidowski and Chapman[3], in this approach, the initial values of both the exterior orientation elements of photos and the coordinates of object points determined by other methods must be input, and the form of control condition is also rigorous. A software of ZEBEND, which has been developed as a part of the Digital Close-Range Photogrammetric System[4], is one featuring being able to join a set of photos taken around the object with arbitrary direction of the camera optical axe and with adequate overlap, to form an integral model of the object, to compute automatically the initial values for both exterior orientation elements of photos and the coordinates of object points which in turn be used in the bundle adjustment to get the final results. The software has been adopted in this paper for data processing of a theodolite industrial surveying system. The value of the angle observations have to be converted to the virtual photo coordinates in advance due to the inherent requirement of the bundle adjustment.

2 GENERATION OF VIRTUAL PHOTOS

To let the theodolite's optical center of collimation be projection center of a virtual photo, a direction of an average of the maximum and the minimum of horizontal observations of the station, which is perpendicular to the standing axis, be its virtual optical axis, any positive value, for the sake of easy computation, say $f = 206.265$ mm, be the principal distance of the virtual photo.

Suppose an object point j is observed from a theodolite station i with its horizontal direction being d_{ij} and its zenithal angle β_{ij}, then its photo coordinates on a virtual photo i would be calculated as[5]

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$$x_{ij} = f \cdot \tan \alpha_{ij} \quad (1)$$

$$z_{ij} = \sqrt{f^2 + x_{ij}^2} \cdot \tan(90° - \beta_{ij}) \quad (2)$$

with

$$\alpha_{ij} = d_{ij} - r_i \quad (3)$$

where r_i represents the horizontal observation of the optical axis of the virtual photo i.

With the known standard deviation of the angle observation, the standard deviation for the virtual photo coordinates can be calculated by applying the law of error propagation[6,7]. Attention must be paid to a fact that, different from the case in photogrammetry, each virtual image point has a different standard deviation, as a result of the fact, the weights for the virtual image points used in bundle adjustment are different from point to point.

It is clear from the discussion above that for the purpose of generation of a virtual photo the standing axis of the theodolite need not be coincided with the vertical direction, so it is not necessary to level the theodolite when observing.

3 DATA PROCESSING OF VIRTUAL PHOTOS

3.1 Automatic computation of initial values required for bundle adjustment

Let the virtual photo generated by the left-most theodolite station be the first photo, independent stereomodels can be created by relative orientation using every two adjacent virtual photos from left to right. After joining model one by one to form an integral model and performed absolute orientation of the model using field controls, the exterior orientation elements for each virtual photo as well as the three-dimensional coordinates of the object points can be determined, and these intermediate results could be used as the initial values in the subsequent bundle adjustment.

A key technique adopted in the relative orientation is to use the algebraic parameters instead of trigonometric functions to form a rotation matrix[8], so that, without any initial values and regardless of lay-out of the theodolite position and the leveling of the instrument (equivalent to arbitrary convergent photography and arbitrary configuration of the stations), the exterior orientation elements of the virtual photos can be reliably derived for the late use.

3.2 Bundle adjustment

Due to the features of the virtual photos, in the bundle adjustment the principal distance and the coordinates of principal points are regarded as constants, the weight for each virtual image point is determined by its estimated standard deviation, the software used should be able to do data-snooping so that appreciable virtual photo coordinate error resulted from the theodolite observation with gross error, if any, can be sift out for assuring the reliability of the results. The standard deviation of the coordinates for each object point can also be estimated easily from the adjustment.

3.3 Control requirement

In the terrestrial photogrammetry, at least two points with known horizontal as well as vertical coordinates and one elevation control point are needed in order to carry out the rotating, leveling and scaling of the stereomodel. In the industrial surveying, if the positions of the industrial objects with respect to an object-space reference coordinate system are desired, the minimum control requirement is the same as that for terrestrial photogrammetry. When some kinds of frequent industrial surveying should be made in the same place, it is highly recommended that the coordinates of the needed ground control points with a satisfactory configuration be determined with high precision. If the relative positions of the characteristic points of the industrial objects are the only concerns, then no complicated control condition is required. A known horizontal length of a line is enough for scaling the model. The length may be provided by an invar wire, an invar subtense bar or an invar leveling rod, etc.

In the bundle adjustment, the relative controls such as that provided by a standard length rule, can be easily introduced. This kind of relative control is helpful to secure the better performance of a high precision industrial surveying system.

4 RESULTS OF TEST

In order to meet the requirements for the control condition in the adjustment and to compare the results of tests, the three-dimensional coordinates of 31 points located in an in-door test field were measured in advance. The instrument used was an electronic theodolite of Wild T2002 (its nominal standard deviation for the angle measurement is $\pm 0.5''$). An invar wire was put horizontally through a bubble and 2.5 m away from the theodolite. From two stations in turn, the electronic theodolite was used to point at four points on the invar wire (with one point on each end of the scale and the other two points evenly spaced on the scale), then the horizontal positions of the standing axis of theodolite could be gotten from resection, and the height difference between the theodolites' optical center of collimation in two station could be determined by horizontally pointing theodolite at a vernier rod put vertically in a suitable place. Finally, the coordinates of all 31 points were obtained by space intersection from the two theodolite stations[9,10]. The estimated precision of X and Y coordinates for the stations were ± 0.066 mm and ± 0.046 mm respectively, while the estimated mean square error of point for the 31 intersection points were all less than ±0.20 mm.

Three cases were tested in the experiment. The instrument for the observation is Wild T2002 electronic theodolite. The observation were carried out from three theodolite stations, which covered the whole area of the test field, with the average distance between the object points and the stations being about 5 m. In order to meet the requirement for generating the virtual photos, the automatic vertical circle compensator or automatic vertical circle indexing of the electronic theodolite was switched off when observations were being made. In the first case, the theodolite was leveled in all 3 stations, and 4 of 31 intersection points were used as control points. The difference between the second case and the first one is that it uses only a standard length provide by an invar wire as a mean of control while the theodolite is also being leveled. In the third case the control conditions are the same as that in the first case but deliberately leave the theodolite being unleveled. For the purpose of comparing the test results, the coordinates of the object points calculated in the second case were transformed to the same coordinates system as that used in the other two cases. Due to the limit of space, only parts of the results are given in the following tables where X, Y, Z are coordinates of object points, m_X, m_Y, m_Z denote the mean square error of coordinates, m_p the mean square error of points.

From the above mentioned tables it can be known

Table 1 Adjustment results for case Ⅰ

Point	X/mm	Y/mm	Z/mm	m_X/mm	m_Y/mm	m_Z/mm	m_p/mm
1	-1728.634	581.695	1223.630	±0.059	±0.114	±0.066	±0.144
4	-1251.328	1113.921	-929.816	±0.049	±0.125	±0.048	±0.142
7	-739.713	1699.620	-526.725	±0.045	±0.133	±0.046	±0.148
11	-234.372	2275.733	725.685	±0.050	±0.171	±0.061	±0.188
14	449.383	2741.470	1212.195	±0.059	±0.201	±0.080	±0.224
17	956.431	2408.549	-1051.792	±0.059	±0.174	±0.068	±0.196
21	1500.240	1928.527	-504.660	±0.058	±0.142	±0.051	±0.162
24	2043.141	1443.232	704.661	±0.060	±0.120	±0.048	±0.142
27	2544.064	1000.818	1202.116	±0.065	±0.112	±0.057	±0.141

Table 2 Adjustment results for case Ⅱ

Point	X/mm	Y/mm	Z/mm	m_X/mm	m_Y/mm	m_Z/mm	m_p/mm
1	-1728.442	581.647	1223.507	±0.135	±0.179	±0.113	±0.251
4	-1251.212	1113.850	-929.701	±0.130	±0.174	±0.092	±0.236
7	-739.639	1699.464	-526.657	±0.141	±0.192	±0.076	±0.250
11	-234.342	2275.506	725.609	±0.164	±0.234	±0.084	±0.298
14	449.347	2741.196	1212.061	±0.188	±0.282	±0.107	±0.355
17	956.326	2408.286	-1051.680	±0.166	±0.244	±0.091	±0.309
21	1500.069	1928.309	-504.610	±0.140	±0.206	±0.064	±0.257
24	2042.912	1443.082	704.577	±0.129	±0.187	±0.072	±0.238
27	2543.789	1000.729	1201.984	±0.140	±0.188	±0.102	±0.256

Table 3 Adjustment results for case Ⅲ

Point	X/mm	Y/mm	Z/mm	m_X/mm	m_Y/mm	m_Z/mm	m_p/mm
1	-1728.752	581.199	1223.765	±0.102	±0.118	±0.083	±0.177
4	-1251.298	1113.850	-929.909	±0.099	±0.133	±0.065	±0.178
7	-739.657	1699.621	-526.582	±0.088	±0.136	±0.064	±0.174
11	-234.303	2275.650	725.992	±0.097	±0.216	±0.087	±0.252
14	449.718	2741.610	1212.459	±0.092	±0.267	±0.115	±0.305
17	956.065	2408.895	-1052.184	±0.073	±0.235	±0.091	±0.262
21	1500.043	1928.696	-504.606	±0.058	±0.197	±0.065	±0.215
24	2043.041	1443.197	704.813	±0.051	±0.170	±0.066	±0.189
27	2544.007	1000.763	1202.378	±0.056	±0.158	±0.085	±0.188

that the system obtains rather high surveying precision in all 3 cases and the best results would be get in the first case where the theodolite is leveled and coordinates of control points were used as control conditions.

It should be noted that the standard deviation of the image points on the virtual photos generated from the test field is within $\pm 4\ \mu$m, therefore the coordinate's accuracy of the virtual image is quite high, and when being compared with the actual photos, the virtual photos have the advantage of being free from errors caused by camera lens distortions and film flattening.

5 CONCLUSIONS

1) The hardware of the system is only one electronic theodolite, so setting up such a system is easy and not expensive.

2) The field work is reduced considerably as neither centering and leveling the theodolite nor intervisibility, and orientation between stations is needed.

3) The form of control condition is quite flexible, and easy introduction of relative controls would increase the surveying accuracy of the system.

4) Data processing is straight forward due to the software used needs no initial values of the exterior orientation elements of the virtual photos and of coordinates of the object points.

5) High reliability for the industrial surveying system is secured by ability of data-snooping of the software, and a sound precision estimation is also available.

6) The drawback of the system is that data processing can only be made after all of observations are finished, so it is unable to provide real time results.

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