[Article ID] 1003- 6326(2001)02- 0297- 04

Inverse approach to determine piston profile from impact stress waveform on given non-uniform rod①

ZHU Ping-yu(朱萍玉), LIU De-shun(刘德顺), PENG You-duo(彭佑多), CHEN An-hua(陈安华)
(Institute of Vibration Impact and Diagnosis, Xiangtan Polytechnic University,
Xiangtan 411201, P. R. China)

[Abstract] An essential problem in the design of mechanical impact systems is the impact of a piston on a rod. The impact of a semi-finite cylindrical piston on a non-uniform rod was studied. Based on wave mechanics and characteristic line theory, an inverse numerical approach to determine the piston profile was proposed, by means of which the geometry of an impact piston may be determined from the given stress waveform for a given rod profile. Numerical results show that the given stress waveform may be produced by means of the alternatives of design of piston and rod. There is good agreement between the experimental results and numerical results.

[Key words] impact; piston profile; non-uniform rod; inverse numerical approach; stress waveform

[CLC number] TH113; TD421; TU67 **[Document code]** A

1 INTRODUCTION

In percussive drilling of rock, a piston is accelerated by a low-level force while the transit time is much longer than that of elastic waves through the piston. The piston impacts on a long, slender drill rod. Through the impact process a high-level force wave is generated which has duration of orders of the transit time. This high-level short-duration force wave performs work in rock fragmentation process. It is necessary to produce high-level stress wave if hard rocks are penetrated, but this can be done only at the cost of increased energy losses in the system, resulting in plastic deformation and fatigue.

The geometrical shape of the piston determines the stress waveform produced. Therefore the design of piston geometry is of great significance both from the viewpoints of rock penetration performance and of the system fatigue life, and now becomes a topic of more than academic importance. LIU, et al[1, 2] presented the inverse design of impact pistons based on stress waveforms, by means of which the piston geometry can be determined directly from the stress waveform given. Similarly, Nygren, et al[3] proposed the optimal design of elastic junctions with regard to transmission of wave energy. It must be mentioned that the rod impacted by the piston is uniform in the model presented by LIU. Although the drill rod is uniform along most of its length, for practical reasons its tail portion whose end-face is impacted by the piston is commonly non-uniform.

2 INVERSE NUMERICAL APPROACH

2.1 Mathematical model

The system to be studied is depicted in Fig. 1. A short truncated conic piston, of length L_1, impacts a semi-infinitely cylindrical rod of radius r_0, with a waist-drum like tail of length L_2. And it is assumed that a spring, of stiffness k, exists between the piston and the rod to simulate the impact contact of them.

The motion of the impact system is governed by the one-dimensional wave equation:

$$\left.\begin{aligned}
&\rho A(x)\frac{\partial v}{\partial t}+\frac{\partial F}{\partial x}=0\\
&\rho c^2 A(x)\frac{\partial v}{\partial x}+\frac{\partial F}{\partial t}=0\\
&\text{Whose spring equations are}\\
&\quad F(x=L_1^-,t)=F(x=L_1^+,t)\\
&\quad \frac{\mathrm{d}F(x=L_1,t)}{\mathrm{d}t}=k[v(x=L_1^-,t)-\\
&\qquad\qquad v(x=L_1^+,t)]\\
&\text{Boundary conditions are}\\
&\quad F(x=0,t)=0\\
&\quad F(x=x^*,t)=F^*(t)\\
&\quad v(x=x^*,t)=F^*(t)/(\rho c A_0)\\
&\text{Initial conditions are}\\
&\quad v(0\leqslant x\leqslant L_1,t=0)=v_0\\
&\quad v(x>L_1,t=0)=0\\
&\quad F(x\geqslant 0,t=0)=0\\
&\quad A(x)=\pi r^2(x),\ 0\leqslant x\leqslant L_1+L_2\\
&\quad A(x)=A_0,\qquad x>L_1+L_2
\end{aligned}\right\}\quad(1)$$

where F is the normal force, positive in pressure; v is the particle velocity, positive in the direction of increasing x; ρ, c are density and elastic wave speed of piston and rod; $A(x)$, $r(x)$ are cross-sectional area and radius which are function of the coordinate x;

① **[Foundation item]** Project (59875073) supported by the National Natural Science Foundation of China and project (99JJY20050) supported by the National Science Fund of Hunan Province **[Received date]** 2000- 03- 20; **[Accepted date]** 2000- 10- 08

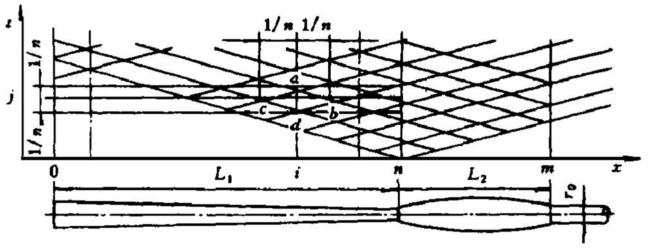

Fig. 1 Impact piston characteristic line grid

A_0, r_0 are cross-sectional area and radius of rod; v_0 is impact velocity of piston; $F^*(t)$ is force wave at the coordinate x^* $(x^* > L_1 + L_2)$ propagating in the uniform portion of the rod; Symbols " − " and " + " represent the impact endfaces of the piston and the rod respectively.

For the simplicity, the above equations can be re-written in non-dimension as follows.

Let

$$x = x/L_1,\ t = ct/L_1,\ v = v/v_0,$$
$$x^* = x/L_1,\ k = kL_1/\rho c A_0,$$
$$F^* = F^*/\rho c A_0 v_0,\ r = r/r_0,$$
$$\lambda = L_2/L_1,\ F = F/\rho c A_0 v_0$$

obtain

$$\left.\begin{aligned}
& r^2\frac{\partial v}{\partial t} + \frac{\partial F}{\partial x} = 0 \\
& r^2\frac{\partial v}{\partial x} + \frac{\partial F}{\partial t} = 0 \\
& F(x = 1^-, t) = F(x = 1^+, t) \\
& \frac{\mathrm{d}F(x = 1, t)}{\mathrm{d}t} = \\
& k[v(x = 1^-, t) - v(x = 1^+, t)] \\
& F(x = 0, t) = 0 \\
& F(x = x^*, t) = F^*(t) \\
& v(0 \leqslant x \leqslant 1, t = 0) = v_0, \\
& v(x > 1, t = 0) = 0 \\
& F(x \geqslant 0, t = 0) = 0 \\
& r = r(x),\ 0 \leqslant x \leqslant 1 \\
& r = 1,\ x > 1 + \lambda
\end{aligned}\right\} \tag{2}$$

For given piston geometry $r(x)$, we shall seek the stress waveform produced by the piston, this problem is called as direct problem. Therefore, we pose the inverse design problem: Given $F^*(t)$, find $r(x)$ such that the piston with geometry $r(x)$ will produces the force wave $F^*(t)$ when it impacts on the given rod.

2.2 Inverse numerical method

As a numerical analysis method, characteristic line method is usually used to solve the wave equation according to characteristic line theory, the following differential equation can be obtain:

$$\left.\begin{aligned}
& \nabla t + \nabla x = 0 \\
& r^2 \nabla v + \nabla F = 0
\end{aligned}\right\} \tag{3}$$

Let a piston occupies $x \in [0, L]$, and consists of N uniform segments with constant cross-sectional area and equal transit time $1/n$ shown in Fig. 1. So a characteristic line grid is established. Introduce three points in the grid, a, b, c. F_{ij}, v_{ij} are normal force and partial velocity at point $a(ij)$, respectively, and so do the other points. For the three points, following equations can be derived from Eqn. (3).

$$\left.\begin{aligned}
& F_{ij} - F_{i-1,j-1} + r_i^2(v_{ij} - v_{i-1,j-1}) = 0 \\
& F_{i-1,j-1} - F_{i,j-2} - r_i^2(v_{i-1,j-1} - v_{i,j-2}) = 0
\end{aligned}\right\} \tag{4}$$

Therefore,

$$\left.\begin{aligned}
F_{i-1,j-1} = {} & [F_{i(j-2)} + F_{ij} + \\
& r_i^2(v_{ij} - v_{i(j-2)})]/2 \\
v_{i-1,j-1} = {} & [v_{i(j-2)} + v_{ij} + \\
& (F_{ij} - F_{i(j-2)})/r_i^2]/2
\end{aligned}\right\} \tag{5}$$

Correspondingly, the spring equation, the boundary conditions and initial conditions in Eqn. (2) are changed.

$$F_{0j} = 0 \tag{6}$$

$$F_{nj} - F_{n(j-1)} = k(v_{nj}^- - v_{nj}^+)/n \tag{7}$$

$$F_{nj}^- = F_{nj}^+ \tag{8}$$

$$v_0 = 1,\ F_{i0} = 0 \tag{9}$$

$$r_i = r(i/n) \tag{10}$$

Based on stress wave theory and initial conditions, we can gain

$$r_i^2 = \frac{F_{i(n+1-i)}}{1 - v_{(n+1-i)}},\quad i = 0, \cdots, n \tag{11}$$

If $F^*(t)$ is known and $r(x)(x > 1)$ is given, the following equations can be obtained

$$F_{mj} = F_j^* = F^*(j/n) \tag{12}$$

$$r_i = r(i/n),\ i = n, \cdots, m \tag{13}$$

Inverse design of the impact piston, in which the piston geometry $r(x)$ can be determined directly from the given stress waveform $F^*(t)$, may be summarized. First, using Eqn. (5) repeatedly, (F_{nj}^+, v_{nj}^+) can be determined from (F_{mj}, v_{mj}). Then, putting (F_{nj}^+, v_{nj}^+) into Eqn. (7) and Eqn. (8), (F_{nj}^-, v_{nj}^-) can be obtained. The radius $\{r_i\}_{i=0}^n$ can be determined as follows: 1) calculating radius of the piston $r_n(x = 1)$ from Eqn. (7); 2) again putting the value of r_n, (F_{nj}, v_{nj}) in Eqn. (5), $(F_{(n-1)j}, v_{(n-1)j})$ can be obtained. By using this procedure repeatedly, the radius of the piston geometry $r_i(i = 0, \cdots, n)$ can be determined successively from $i = n$ to $i = 0$. Finally, we obtain the radius of piston as functions of the coordinate x along the piston.

3 NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

In order to illustrate the use of the method, we designed many kinds of the piston-rod pairs by means of the inverse numerical approach and a computer code called as PID1. The piston-rod pairs shown in

Fig. 2 could produce a same impact stress waveform, i. e. approaching half-sine waveform.

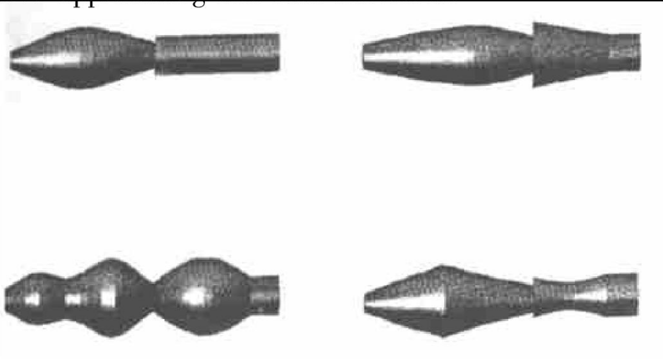

Fig. 2 Designed piston-rod pairs

The above designs were applied to the large Split Hopkinson Pressure Bar (SHPB), illustrated in Fig. 3. As it has been believed that a sine waveform is an ideal loading stress waveform in the measurement of material dynamic properties[1,4]. The impact actions in the SHPB are similar to that in the rock drill, so the SHPB is usually used to perform the impact experiments. These experiments are very necessary in the primary stage to make a comparison between the desired waveform and the practical waveform to improve the design of the piston and the rod.

In this experimental set-up, cylindrical steel piston was accelerated by means of an airgun, then it impacted on a steel rod with length of 540mm and diameter of 75mm. The rods were guided by low-friction slide bearings so that they are free to move axially. A damping device at the end of the rod served to absorb the energy and stop the motion after a test. One pair of diametrically opposite and axially oriented strain gauges were attached to the input rod in order to measure the strains $\varepsilon_1(t)$, which were used to calculate the stress waveforms. Another similar pair of strain gauges were attached to the output rod in order to measure the strain $\varepsilon_2(t)$. The gauges for each pair were connected to a bridge amplifier in opposite branches, so that contributions from small accidental bending strain were suppressed. The strain signals were recorded by means of a two-channel digital oscilloscope with a sampling interval of 1 μs. The recorded signals were transferred to a computer for evaluation. A steel piston "linearized" from designed profile shown in Fig. 4(a) was tested. The SHPB experimental set-up was shown in Fig. 3. The test result shown in Fig. 4(b) is in good agreement with the theoretical curve.

It is clear that we can obtain piston profile according to a given waveform. And there are a number of cases of piston-rod to produce stress waveform of the given character, so that it is possible for designers to obtain satisfactory alternative for a desired stress waveform from the respects of installation, performance and life of the system. This may well be true of most of the drills in practice, but it may not always be desirable. Based on the wave transmission efficiency in percussive drilling of rock, Fairhurst, Simon and Kovalenko expressed the concept of the ideal stress waveform as having an initial exponential rise[5~7]. LIU and LI discussed stress waveforms suitable to impact machines from both energy transmission efficiency of impact loading system and deformation or fracture of working media respectively[8,9]. These studies show that the reasonable stress waveforms are different from the mechanical models of working media and piston geometries allowed in actual impact loading systems. That is to say this inverse numerical approach has more applications.

It has been generally believed that a percussive drill system can produce a unique stress waveform, but determination of some piston-rod would be virtually impossible from the given stress waveforms, which there are not so systems that can produce the given stress waveforms[10]. It must also be mentioned that the stress waveform used to determine the piston geometry is only a portion of the waveform produced. This portion for $t = 0$ to 2 can determine the whole piston geometry, and the other of the stress wave-

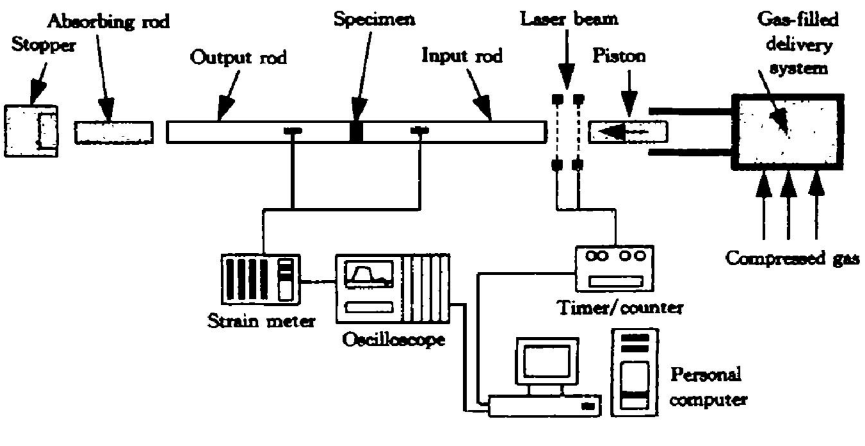

Fig. 3 SHPB experimental set-up

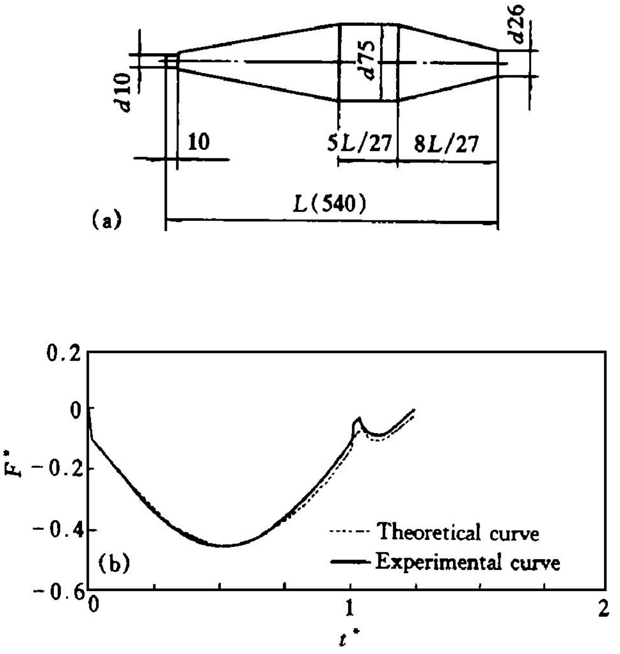

Fig. 4 Impact piston and its stress waveform
(Unit in (a): mm)

form is its consequence. Therefore, it may be said that the portion includes all information about the piston geometry[2].

4 CONCLUSIONS

The inverse numerical approach to determine the piston profile directly from the given stress waveform for the given non-uniform rod are presented. The effects of the restriction to piston-rod with piece-wise constant cross-sectional area can be relieved by choosing the number of segments large. This method make it possible for designers to determine excellent penetration devices operating by mechanical impact, such as rock drill or pile driver. Also, the model used and the results obtained are relevant to other systems such as non-uniform elastic layers and electromagnetic systems.

ACKNOWLEDGEMENTS

The authors thank Dr T S Lok of Nanyang Technological University, School of Civil & Structural Engineering, who afforded the experimental data to the research work.

[REFERENCES]

[1] LIU De-shun, LI Xi-bing and YANG Xiang-bi. An approach for control oscillation in dynamic stress strain measurement [J]. Trans Nonferrous Met Soc China, (in Chinese), 1996, 6(2): 144- 147.

[2] LIU De-shun, PENG You-duo and LI Xue-jun. Inverse design and experimental study of impact piston [J]. Chinese Journal of Mechanical Engineering, 1998, 34(4): 78- 84.

[3] Nygren T, Andersson L E and Lundberg B. Optimization of elastic junctions with regard to transmission of wave energy [J]. Wave Motion, 1999, 29: 223- 244.

[4] Bacan C. Separation of waves propagation in an elastic or viscoelastic Hopkinson Pressure Bar with three-dimensional effects [J]. Int J Impact Engng, 1999, 22: 55- 69.

[5] Fairhurst C. Wave mechanics of percussive drilling [J]. Mine Quarry Eng, 1961, 27: 327- 382.

[6] Simon R. Stress wave energy in the drill stell of a percussive drill [J]. Int J Rock Mech Min Sci, 1964, 1: 187- 411.

[7] Kovalenko O F, Pershin V P and Toporov G V. Optimal shapes of shock impulses [J]. Soviet Mining Science, 1984, 2: 133- 137.

[8] LIU De-shun and YANG Xiang-bi. A mathematical model on the design of impact stress waveforms [J]. Journal of Xiangtan Mining Institute, (in Chinese), 1994, 9(4): 6- 10.

[9] LI Xi-bing, LIU De-shun and LIU Ai-hua. Study on reasonable loading stress waveforms for impact machines [J]. J Cent South Univ Technol, (in Chinese), 1998, 29(2): 119- 121.

[10] LI Xi-bing and GU De-sheng. Rock Impact Dynamics [M]. Changsha: Central South University of Technology Press, 1994.

(Edited by HUANG Jin-song)