

Localized shear deformation during shear band propagation in titanium considering interactions among microstructures^①

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Abstract: Closed-form analytical solutions of plastic shear strain and relative plastic shear displacement during shear band propagation are proposed under dynamic loadings based on gradient-dependent plasticity considering the effect of microstructures due to heterogeneous texture of Ti. According to the differences in shear stress levels, Ti specimen is divided into three regions: residual region, strain-softening region and elastic region. Well-developed shear band is formed in the residual region and the relative plastic shear displacement no longer increases. In the normal and tangential directions, the plastic strain and the displacement are nonuniform in the strain-softening region. At the tip of shear band, the shear stress acting on the band is increased to shear strength from the elastic state and the shear localization just occurs. Prior to the tip, Ti remains elastic. At higher strain rates, the extent of plastic strain concentration is greater than that under static loading. Higher strain rate increases the relative plastic shear displacement. The present analytical solution for evolution or propagation of shear localization under nonuniform shear stress can better reproduce the observed localized characteristics for many kinds of ductile metals.

Key words: microstructures of titanium; heterogeneity; gradient-dependent plasticity; shear localization; shear band propagation; nonuniform shear stress; dynamic loading

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1 INTRODUCTION

It is well known that the high specific strength and fracture resistance make titanium and titanium alloys very attractive to aerospace applications. Shear localization is an important and often the dominating deformation and failure mechanism in titanium and titanium alloys subjected to dynamic loadings^[1-11]. Prior to the onset of shear localization, the deformation can be approximately considered to be uniform. Once the shear localization is initiated, the intensely localized shear deformation is accumulated progressively in narrow bands called shear bands or localized bands. The eventual outcome of localized deformation is ductile rupture and material separation. Shear localization occurs and plays an important role in many applications. For example, shear bands can be observed in ballistic impact, explosive fragmentation, high speed machining, metal forming, grinding, interfacial friction, powder compaction, granular flow, and seismic events. It is noted that many experimental results show that after shear localization occurs, shear deformation gradually develops with a certain velocity of propagation^[1-11]. That is to say, in many

tests the plastic shear strain in shear band varies along the shear direction.

To predict the characteristics of shear localization observed in many tests, such as the strain, the displacement, the thickness and the spacing as well as the propagation velocity, and to obtain a full understanding of failure mechanism, numerical simulation and theoretical analysis must be carried out. Unfortunately, the numerical results obtained based on classical elastoplastic theory inevitably suffer spurious mesh sensitivity.

Motivated mainly by difficulties of classical elastoplastic theory describing localized characteristics of heterogeneous materials, some modifications and generalization from the standard continuum description, so-called regularizations, have been proposed during the recent years. One of the most promising approaches is the second order gradient continuum that incorporates the second order spatial gradient of plastic strain in the yield function^[12-20].

In gradient-dependent plasticity, the strain gradient term including the second strain gradient and the characteristic length describes the interactions and interplaying among microstructures. Microstructures are of very importance for Ti and Ti alloys and have

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been studied extensively in experiments^[21-25].

In the paper, firstly, in the context of classical elastoplastic theory a yield function considering strain rate effect is obtained. Secondly, the strain gradient effect is introduced into the classical yield function to consider the heterogeneous texture of Ti. Next, the problem of shear band propagation is regarded as a one-dimensional dynamic shear problem in the shearing direction, and the distributed plastic strain and the thickness of the band are derived. Integrating the plastic shear strain leads to the plastic shear displacement. At last, some new conclusions are drawn.

2 SHEAR BAND PROPAGATION CONSIDERING STRAIN RATE

2.1 Basic assumptions and classical elastoplastic theory

The mechanical model for dynamic propagation of shear band is given in Fig. 1. Ti block with a certain height and length is loaded in horizontal shear stress $\tau(x)$ and vertical compressive stress σ . Fig. 1 shows that the shear stress is different in three regions, i. e. residual region, strain-softening region and elastic region. A horizontal shear band is firstly formed at the tail of shear band and propagates towards right. The total length of the shear band is $L_1 + L_2$ and the thickness of the band is w . L_1 is the length of residual region of shear band, and L_2 is the length of strain-softening region of shear band. At the tip of the shear band, the shear stress equals shear strength τ_c , and prior to the tip, the shear stress is less than τ_c . For simplicity, the lower end of the block is fixed. An important outcome of shear localization is the decrease of the stress-carrying capability of the block, so it can be supposed that localization is initiated at the peak stress, and for simplicity the shear deformation only occurs in the horizontal direction. Some experimental results show that the post-peak behavior of Ti or Ti alloys under dynamic loadings exhibits approximately linear strain-softening^[6], so the constitutive relation for Ti in strain-softening stage can be considered as a descending straight line whose absolute value of the slope is λ called shear softening modulus. In the residual stage the constitutive relation can be considered to be a horizontal line due to the existence of the constant compressive stress. The intercept τ_f of the line depends on the confining pressure σ . The post-peak static constitutive relation for Ti is shown in Fig. 2.

In the context of one-dimensional conventional elastoplastic theory for strain-softening material loaded or unloaded elastically according to shear elastic modulus G , under static loading, the flow shear stress τ

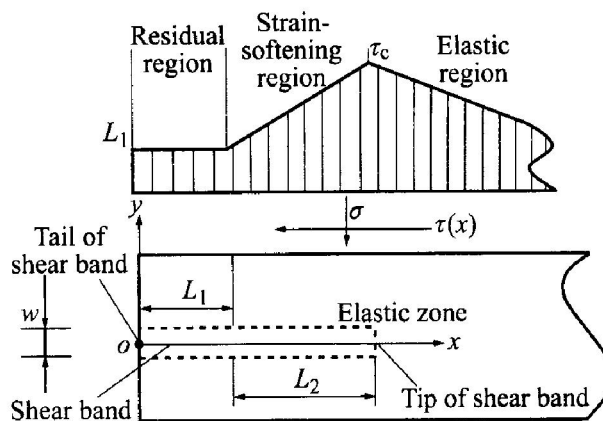


Fig. 1 Stress acting on shear band and mechanical model for propagation of shear band

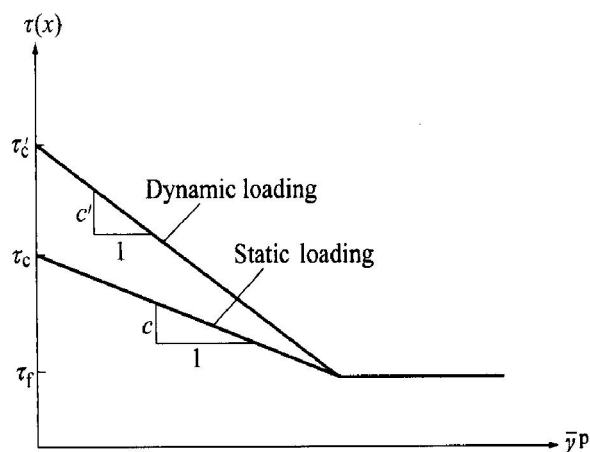


Fig. 2 Comparison of dynamic and static post-peak constitutive relation

(x) depends on shear strength τ_c , λ , G , and the accumulated plastic shear strain $\bar{\gamma}^p$:

$$\tau(x) = \tau_c - \frac{G\lambda}{G + \lambda} \bar{\gamma}^p = \tau_c - c \bar{\gamma}^p \quad (1)$$

2.2 Effect of strain rate

To consider strain rate effect, introducing a function f into the classical elastoplastic theory yields^[26]

$$\tau(x) = (\tau_c - c \bar{\gamma}^p) \cdot f \quad (2)$$

where $f = 1 + C \ln \dot{\gamma} / \dot{\gamma}_0$, $f \geq 1$, C is a material constant, $\dot{\gamma}$ is the average shear strain rate and $\dot{\gamma}_0$ is the average shear strain rate under quasi-static loading conditions. For simplicity, let $\tau_c = f \tau_c$ and $c' = fc$. The dynamic post-peak constitutive relation between shear stress and plastic shear strain is also shown in Fig. 2. It can be seen that the higher strain rate results in steeper post-peak branch and higher shear strength.

2.3 Localized strain and deformation in shear band due to heterogeneity

To investigate the shear strain gradient effect in the context of conventional elastoplastic theory, the

following expression can be obtained according to usual method of introducing the strain gradient into the yield function^[12-20]:

$$\tau(x) = f\tau_c - fc \left| \gamma^p(x, y) + l^2 \frac{d^2 \gamma^p(x, y)}{dy^2} \right| \quad (3)$$

Suppose that the shear band has a width w after localization is initiated. At the boundary of elastic zone and plastic zone, the boundary condition is

$$\gamma^p(x, y) = 0 \text{ at } y = \pm w/2 \quad (4)$$

The plastic shear strain in shear band is an even function of coordinate y . Application of Eqn. (4) results in the following equation:

$$\gamma^p(x, y) = \frac{\tau_c - \tau(x)}{c'} \left[1 - \cos \frac{y}{l} / \cos \frac{w}{2l} \right] \quad (5)$$

The thickness of shear band is determined according to the maximum plastic shear strain:

$$\frac{d\gamma^p(y)}{dy} = 0 \quad (6)$$

So, the shear band thickness can be obtained analytically:

$$w = 2\pi l \quad (7)$$

Substituting Eqn. (7) into Eqn. (5) leads to

$$\gamma^p(x, y) = \frac{\tau_c - \tau(x)}{c'} \left[1 + \cos \frac{y}{l} \right] \quad (8)$$

where $x \in [L_1, L_1 + L_2]$.

In the residual region, $\tau(x)$ equals τ_r . According to Eqn. (8), the plastic shear strain in shear band can be written as

$$\gamma_r^p(x, y) = \frac{\tau_c - \tau_r}{c'} \left[1 + \cos \frac{y}{l} \right] = \gamma_r^p(y) \quad (9)$$

where $x \in [0, L_1]$.

Integrating Eqn. (8) with respect to coordinate y yields

$$s^p(x, y) = \int \gamma^p dy = \frac{\tau_c - \tau(x)}{c'} \left[y + l \sin \frac{y}{l} \right] \quad (10)$$

where $s^p(x, y)$ is the relative plastic shear displacement in strain-softening region.

If $\tau(x)$ equals τ_r , then the maximum relative plastic shear displacement can be expressed as

$$s_r^p(x, y) = \frac{\tau_c - \tau_r}{c'} \left[y + l \sin \frac{y}{l} \right] = s_r^p(y) \quad (11)$$

where $s_r^p(y)$ is the relative plastic shear displacement in residual region.

3 EXAMPLES AND DISCUSSION

3.1 Localized shear strain and deformation in shear band propagation

Shear modulus of Ti and those of most Ti alloys are about 45 GPa, so we take shear modulus $G = 45$ GPa. Shear strength of Ti is about $\tau_c = 280$ MPa ($\tau_c = \sigma_c/2$, where σ_c is the yield stress in uniaxial tensile loading^[1]). The experimentally obtained shear band thickness in Ti is about $10 - 20 \mu\text{m}$ ^[1, 3],

hence we let the thickness of shear band $w = 10 \mu\text{m}$. The characteristic length describing the extent of heterogeneity of Ti is about $l = 1.59 \mu\text{m}$ according to Eqn. (7). It is assumed that the shear softening modulus λ indicating the ductility is 0.45 GPa; the residual shear strength depending on confining pressure is $\tau_r = 140$ MPa; the parameter reflecting the effect of strain rate are $f = 1$; in addition, $L_1 = 0$, and $L_2 = 11 \mu\text{m}$.

A three-dimensional surface plot for distribution of plastic shear strain is shown in Fig. 3. In strain-softening region the extent of concentration of plastic shear strain in shear band is different along the shearing direction. At the tip of shear band where the maximum shear stress is just reached, the plastic shear strain is zero and the shear localization just occurs. Conversely, at the tail of shear band in which the minimum shear stress is attained, the plastic shear strain reaches its maximum, and intense shear localization in the narrow band occurs with a plastic shear strain of 0.6.

For the well-developed shear bands, a wider distribution of strains (0.3 - 0.9) is observed^[1]. If the pre-peak uniform plastic shear strain is 0.3, the post-peak maximum local plastic shear strain due to interactions and interplaying among microstructures is 0.6, which is consistent with the present calculation. It should be noted that for Al-Li alloy, the plastic shear strain of 8 can be attained in the narrow shear bands^[2]. Consequently, compared with shear localization of Al-Li alloy, localized deformation of Ti is less apparent.

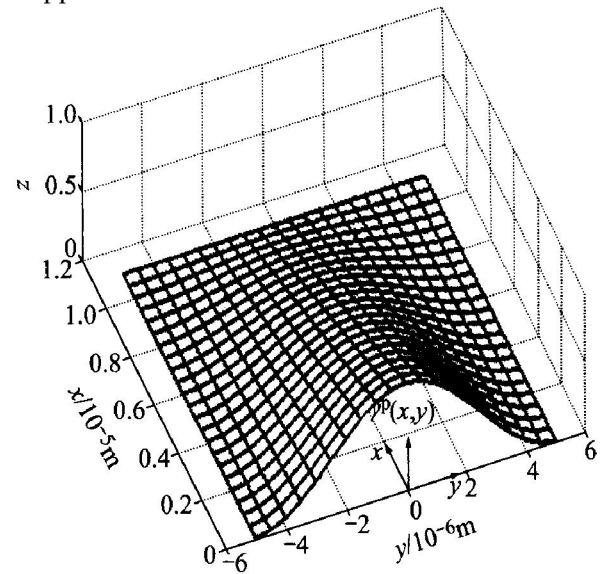


Fig. 3 Distribution of plastic shear strain in strain-softening region

It can be seen from Fig. 4 that the distribution of the relative plastic shear displacement along the shearing direction is also nonuniform for $L_1 = 2 \mu\text{m}$ and $L_2 = 12 \mu\text{m}$. It is noted that the figure depicts the evolution of displacement in all three regions and

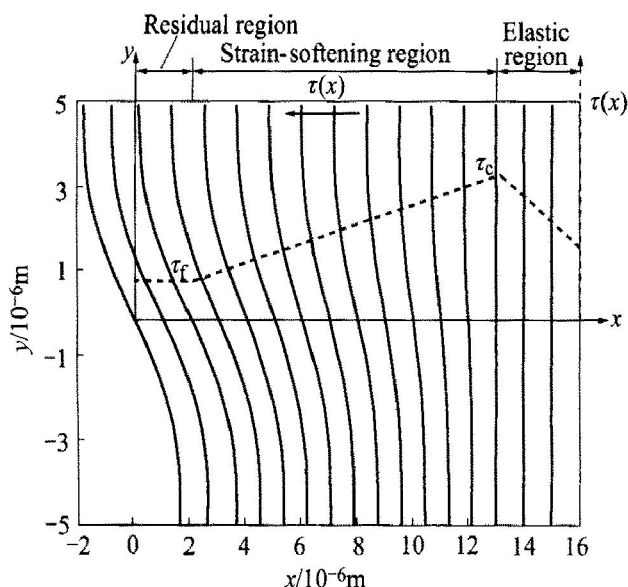


Fig. 4 Distribution of plastic shear displacement and shear stress in three different regions

the levels of shear stress for three different regions (dashed line). In the residual region, well-developed shear band is formed, whereas in the elastic region the plastic shear displacement due to heterogeneity is zero. It can be predicted that in the process of shear band propagation from left to right the elastic region will be covered by the strain-softening region and finally will become the residual region, leading to a great decrease of load-carrying capacity and final macroscopic shear fracture.

Though the present analytical solution for evolution and propagation of shear localization under nonuniform shear stress aims at Ti, it can better reproduce the observed localized characteristics of deformation for many kinds of ductile metal materials under dynamic or static loading.

3.2 Effect of strain rate on plastic shear strain and shear displacement

To investigate the influence of strain rate on the distribution of plastic shear strain and plastic shear displacement at the tail of shear band ($x = L_1$), let $G = 45$ GPa, $\tau_c = 280$ MPa, $w = 10$ μ m, $\lambda = 0.45$ GPa and $\tau_s = \tau_f = 140$ MPa, and the results for different parameter f are shown in Fig. 5. The larger the value of the parameter, the higher the strain rate is. It can be seen from the figure that increasing strain rate leads to steeper profile of plastic shear strain and larger plastic shear displacement.

It should be noted that simplifying the present theoretical analysis by neglecting the strain rate effect and not considering the differences of shear stress acting on the shear band yields the earlier results for static shear localization under uniform shear loading in Ref. [18]. The thermal softening at high strain rates is ignored completely in the present analysis. The factor will be considered in the further investigation.

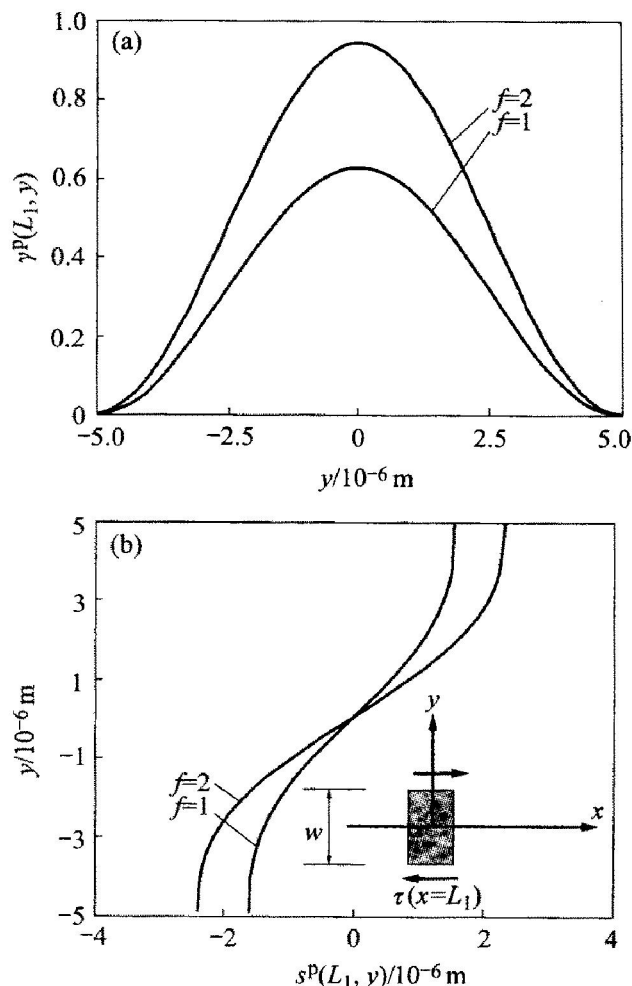


Fig. 5 Effect of strain rate on distribution of plastic shear strain(a) and plastic shear displacement(b) at tail of shear band

4 CONCLUSIONS

Shear deformation of Ti in the direction of shear stress can be divided into three different regions: residual region, strain-softening region and elastic region. In the residual region the well-developed shear band is formed and the relative plastic shear displacement no longer increases. In the process of shear band propagation the length of the region will be increased and can cover the other two regions. In strain-softening region, it is found that the plastic shear strain and the plastic shear displacement are nonuniform both in the normal and tangential directions. At the tip of the shear band, the shear stress acting on the band is increased to shear strength from the elastic stage and shear localization just occurs. Prior to the tip of shear band, Ti remains elastic.

At high strain rates and under the same level of shear stress, the extent of plastic shear strain concentration is higher than that under static loading and the relative plastic shear displacement is increased as

strain rate is increased.

REFERENCES

- [1] Nesterenko V F, Meyers M A, Wright T W. Self-organization in the initiation of adiabatic shear bands[J]. *Acta Mater*, 1998, 46(1): 327 - 340.
- [2] Xu Y B, Zhong W L, Chen Y J. Shear localization and recrystallization in dynamic deformation of 8089 Al-Li alloy[J]. *Mater Sci Eng A*, 2001, A299: 287 - 295.
- [3] Xue Q, Meyers M A, Nesterenko V F. Self-organization of shear bands in titanium and Ti-6Al-4V alloy[J]. *Acta Materialia*, 2002, 50: 575 - 596.
- [4] YU Jir-qiang, ZHOU Hui-hua, SHEN Le-tian. Thermoplastic shear bands induced during dynamic loading in Ti-55 alloys[J]. *Acta Metallurgica Sinica*, 1999, 35(4): 379 - 383. (in Chinese)
- [5] LIU Wen-sheng, HUANG Bai-yun, HE Yue-hui. The room-temperature fracture mechanics of a fully-lamellar TiAl alloy[J]. *J Cent South Univ Technol*, 1997, 28(2): 152 - 155. (in Chinese)
- [6] LI Qiang, XU Yong-bo, SHEN Le-tian. Dynamic mechanical properties and damage characteristics of titanium alloy(Ti-17)[J]. *Acta Metallurgica Sinica*, 1999, 35(5): 491 - 494. (in Chinese)
- [7] YANG Yang, CHENG Xir-lin, LI Zheng-hua. Effects of metallurgical factors on the forming of adiabatic shear band[J]. *Rare Metal Materials and Engineering*, 2003, 32(4): 261 - 263. (in Chinese)
- [8] YANG Yang, WANG Zhao-ming, ZHANG Shao-rui. Some metallurgical behaviours of adiabatic shear band on Ti side in the Ti/mild steel explosive cladding interface [J]. *Rare Metal Materials and Engineering*, 1997, 26(4): 13 - 17. (in Chinese)
- [9] Tamizifar M, Omidvar H, Salehi S M T. Effect of processing parameters on shear bands in non-isothermal hot forging of Ti-6Al-4V[J]. *Materials Science and Technology*, 2002, 18(1): 21 - 29.
- [10] YANG Yang, CHENG Xir-lin. Current status and trends in researches on adiabatic shearing[J]. *The Chinese Journal of Nonferrous Metals*, 2002, 12(3): 401 - 408. (in Chinese)
- [11] Bai Y, Bodd B. *Adiabatic Shear Localization*[M]. Oxford, UK: Pergamon Press, 1992.
- [12] Aifantis E C. On the role of gradients in the localization of deformation and fracture[J]. *Int J Eng Sci*, 1992, 30: 1279 - 1287.
- [13] Zbib H M, Aifantis E C. On the localization and post-localization behavior of plastic deformation(I): on the initiation of shear bands[J]. *Res Mechanica*, 1998, 23: 261 - 280.
- [14] de Borst R, Muhlhaus H B. Gradient-dependent plasticity: formulation and algorithmic aspects[J]. *International Journal for Numerical Methods in Engineering*, 1992, 35: 521 - 539.
- [15] ZHANG H W, Schrefler B A. Gradient-dependent plasticity model and dynamic strain localization analysis of saturated and partially saturated porous media: one dimensional model[J]. *Eur J Mech A/ Solids*, 2000, 19: 503 - 524.
- [16] WANG Xue-bin, YANG Mei, PAN Yi-shan. Analysis of necking under condition of uniaxial tension of low-carbon steel specimen based on gradient-dependent plasticity[J]. *Journal of Plasticity Engineering*, 2002, 9(3): 55 - 57. (in Chinese)
- [17] WANG Xue-bin, PAN Yi-shan, REN Wei-jie. Instability of shear failure and application for rock specimen based on gradient-dependent plasticity [J]. *Chinese Journal of Rock Mechanics and Engineering*, 2003, 22(5): 747 - 750. (in Chinese)
- [18] WANG Xue-bin, DAI Shu-hong, PAN Yi-shan. Analysis of localized shear deformation of ductile metal based on gradient-dependent plasticity[J]. *Trans Nonferrous Met Soc China*, 2003, 13(6): 1348 - 1354.
- [19] WANG Xue-bin, PAN Yi-shan, MA Jin. Analysis of strain (or the ratio of strain) in the shear band and a criterion on instability based on the energy criterion[J]. *Chinese Journal of Engineering Mechanics*, 2003, 20(2): 111 - 115. (in Chinese)
- [20] WANG Xue-bin, PAN Yi-shan, YANG Xiao-bin. Size effect analysis on strain softening of quasi-brittle materials considering strain gradient effects[J]. *Chinese Journal of Rock Mechanics and Engineering*, 2003, 22(2): 188 - 191. (in Chinese)
- [21] DING Hua, BAI Bing-zhe, WANG Ding-liang. Influence of hydrogen on the microstructures and superplasticity of a Ti₃Al-Nb alloy[J]. *Rare Metals*, 2001, 20(2): 87 - 90.
- [22] ZHENG Yong, XIONG Wei-hao. Effect of powder particle size on the properties and microstructure of Ti(C, N)-based cermet[J]. *Rare Metals*, 2001, 20(1): 47 - 51.
- [23] WANG Hao, FU Zheng-yi, GU Ping. Mechanical properties and microstructure of TiB₂ ceramic influenced by ZrB₂ additive[J]. *Trans Nonferrous Met Soc China*, 2002, 12(5): 904 - 908.
- [24] SUN Feng, LIN Dong-liang. Microstructure evolution in a large grained TiAl alloy[J]. *Trans Nonferrous Met Soc China*, 2002, 12(4): 615 - 620.
- [25] WANG Xue-min, SHANG Cheng-jia, YANG Shao-wu. Optimization of RPC technique for refining the intermediate transformation microstructure[J]. *Journal of University of Science and Technology Beijing*, 2002, 9(3): 193 - 196.
- [26] Johnson G R. Fracture characteristics of three metals subjected to various strains, strain rates, temperatures and pressures [J]. *Engineering Fracture Mechanics*, 1985, 21(1): 31 - 48.

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