



Reliability analysis of piles constructed on slopes under laterally loading

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Abstract: Response surface method is used to study the reliability analysis of laterally loaded piles in sloping ground. A development load–displacement (p – y) curve for laterally loaded pile response in sloping ground is used to model the pile–soil system, both the pile head displacement and the maximum bending moment of the piles are used as the performance criteria in this study. The reliability analysis method of the laterally loaded pile in sloping ground under the pile head displacement and the maximum bending moment failure modes is proposed, which is in good agreement with the Monte Carlo method. The influences on the probability index of failure by a number of parameters are discussed. It is shown that the variability of pile head displacement increases with the increase in the coefficients of variation of ultimate bearing capacity factor (N_{pu}), secant elastic modulus at 50% (E_{50}) and level load (H). A negative correlation between N_{pu} and non-dimensional factor (λ) leads to less spread out probability density function (PDF) of the pile head displacement; in contrast, a positive correlation between N_{pu} and λ gives a great variation in the PDF of pile head displacement. As for bearing capacity factor on ground surface (N_{po}) and λ , both negative and positive correlations between them give a great variation in the PDF of pile head displacement, and a negative correlation will obviously increase the variability of the response.

Key words: laterally loaded pile; response surface method; probability; reliability analysis

1 Introduction

The conventional lumped factor of safety approach is widely used in the analysis and design of many geotechnical engineering structures, including laterally loaded piles, high rise buildings, offshore platforms and bridge piers. Although convenient and straight forward, this approach does not take uncertainties of the underlying parameters into consideration. Understanding and applying probability theory and reliability methods help to address such a concern. However, parametric uncertainty is inherent in geotechnical engineering. Traditionally, the uncertainty is dealt by using the global factor of safety approach, which cannot explicitly reflect the uncertainty of the individual underlying parameters and their correlation structure. These shortcomings can

be addressed by adopting probabilistic approaches, such as simulation techniques and reliability methods.

Several methods on the reliability of laterally loaded pile have been proposed [1–3]. The most popular method of these in reliability is the response surface method. The response surface method constructs a polynomial to approximate the performance function by using a number of selected sample points in the neighborhood of the most likely failure point. ZHAO et al [4] proposed an improved response surface method and applied it to the reliability analysis of piles under inclined loads. TANDJIRIA et al [5] compared three different response surface models (linear, quadratic and reciprocal) for the analysis of laterally loaded piles, and showed that the results based on the models were in close agreement with Monte Carlo simulations. Response surface methods were also used in other fields. VAIRAWANI et al [6]

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used the response surface methodology to optimize friction welding parameters to attain a minimum hardness at the interface and a maximum tensile strength of the dissimilar joints of AISI 304 austenitic stainless steel (ASS) and copper (Cu) alloy. BABU and SRIVASTAVA [7] used the response surface method to generate approximate polynomial functions for ultimate bearing capacity and settlement of a shallow foundation resting on a cohesive frictional soil. MOGHADDAM et al [8] used the response surface method to model and optimize the copper extraction in shaking bioreactors. Reliability analysis of laterally loaded piles using the response surface method and considering nonlinearity in both soil and pile behavior was performed [9–11]. Some studies on the reliability analysis of loaded piles in cohesionless soils were conducted [12–17]. SAWANT and SHUKLA [17] illustrated reliability analysis for the lateral capacity of rigid piles based on the response surface method. ZDRAVKOVIĆ et al [18] investigated a reliability-based design approach for estimating pile axial capacity. CHAN and LOW [10] performed probabilistic analysis of laterally loaded piles using response surface and neural network approaches. LOW and TANG [19] investigated the capacity and performance of piles due to major sources of uncertainties. Although several efforts on reliability analysis of laterally loaded piles have been studied [20,21], there is very limited information available for laterally loaded piles in sloping ground.

Hence, the objective of this work is to present an alternative approach of analyzing laterally loaded piles in sloping ground, which couples nonlinear soil behavior (using load–displacement (p – y) curves) with nonlinear flexural rigidity ($E_p I_p$) of the pile. In the present study, response surface methods are proposed to study the reliability analysis of laterally loaded piles in sloping ground.

2 Analysis of laterally loaded pile in sloping ground

Several methods of analyzing a laterally loaded pile are available [15–21]. The pile analysis is carried out by modeling a laterally loaded pile as a vertical beam supported by a series of discrete springs. The discrete springs represent the soil medium surrounding the pile. The discrete springs are assumed to have their load–displacement characteristics or p – y curves. Each p – y curve [16–18] (often nonlinear and depth-dependent) represents the relationship between the lateral deformation of the soil and the mobilized soil resistance. The governing equation of such a model is expressed as

$$E_p I_p \frac{d^4 y}{dz^4} + p(y) = 0 \quad (1)$$

where E_p is the elastic modulus of pile (kPa); I_p is the moment of inertia of pile section (m^4); y is the horizontal displacement (m); z is the depth coordinate (m); $p(y)$ is a function which represents the nonlinear load deflection relationship of the soil surrounding the pile.

2.1 Basic assumptions

As shown in Fig. 1, a pile is completely embedded in clay, H is the pile top level load, L is the pile length and D is the pile diameter. To facilitate the establishment method, the basic assumptions for the proposed method in this paper are as follows.

- 1) The pile is fully rigid so that the pile at some points along the pile shaft rotates under lateral load.
- 2) Slope is cohesive soil, and the drainage of soil does not take into account in the calculation.
- 3) Assume that slope is stable and is not taken into account in the calculation of slope failure and instability.

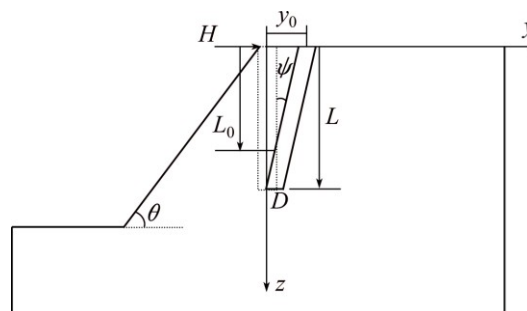


Fig. 1 Laterally loaded pile in sloping ground

2.2 Stress analysis

For the purpose of this work, a laterally loaded rigid pile in sloping ground is shown in Fig. 2. The first kind of circumstance is on both sides of pile foundation under lateral load, no soil reaction reaches the ultimate soil resistance of p_u , as shown in Fig. 2(a), the curve of ABC for pile side soil pressure of p , line AD and line AE are the ultimate resistance of p_u . For the second case shown in Fig. 2(b), a certain depth under the top surface b , pile lateral soil pressure p reaches the limit of soil resistance of p_u , then fails to achieve the ultimate bearing capacity of the soil of p_u along the depth direction. The third condition is possible in the ground of the pile, and the pile lateral soil pressure p reaches the ultimate bearing capacity of soil of p_u . Here, focuses are made on analyzing the first and the second cases.

According to Fig. 2(a), the lateral displacement (y), the soil pressure (p), horizontal force H and moment M can then be expressed by

$$y = \frac{(L_0 - z)y_0}{L_0} \quad (2)$$

$$p = K_h y = \frac{ny_0(L_0 - z)z}{L_0} \quad (0 \leq z \leq L) \quad (3)$$

$$M + \frac{ny_0L^2}{L_0} \left(\frac{L_0}{3} - \frac{L}{4} \right) = 0 \quad (4)$$

$$H - \frac{ny_0L^2}{L_0} \left(\frac{L_0}{2} - \frac{L}{3} \right) = 0 \quad (5)$$

where K_h is the pile lateral soil resistance coefficient, and n is the ratio coefficient.

By solving Eqs. (2)–(5), the pile head displacement y_0 corresponding to force H and moment M can be obtained. After y_0 is obtained, the lateral displacement y , bending moment M and shear force of the pile along the depth can also be obtained.

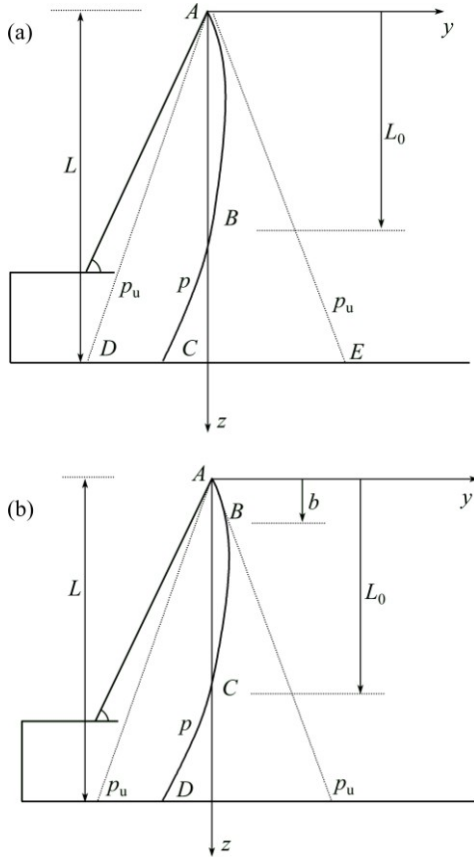


Fig. 2 Pile lateral soil stress model: (a) Lateral soil pressure distribution without yielding; (b) Soil pressure distribution with yielding only in region above rotation point

Figure 2(b) shows the curve of $ABCD$ for pile side soil pressure p . As shown in this figure, at point B , the pile lateral soil pressure reaches the ultimate soil resistance p_u .

$$p = p_u = mz \quad (0 \leq z \leq b) \quad (6)$$

$$p = K_h y = \frac{ny_0(L_0 - z)z}{L_0} \quad (b \leq z \leq L) \quad (7)$$

where m is the depth scaling factor.

At depth $z=b$, we have $p_u = K_h y$. So, b can be expressed as

$$b = L_0 \left(1 - \frac{m}{ny_0} \right) \quad (8)$$

Considering respective bending moment equilibrium and horizontal force equilibrium about point A , we have

$$H - \frac{1}{2}mb^2 + \frac{ny_0}{L_0} \left[\frac{L_0}{2}(b^2 - L^2) - \frac{1}{3}(b^3 - L^3) \right] = 0 \quad (9)$$

$$M + \frac{1}{3}mb^3 - \frac{ny_0}{L_0} \left[\frac{L_0}{3}(b^3 - L^3) - \frac{1}{4}(b^4 - L^4) \right] = 0 \quad (10)$$

In Eqs. (6)–(10), only five parameters of p_u , K_h , y_0 , L_0 and b are unknown, by solving these five equations, y_0 , L_0 , b , bending moment and shear force of the pile along the depth can also be obtained.

3 Parameters determination

3.1 Ultimate pile lateral soil resistance p_u

Several methods are available for determining the ultimate lateral resistance p_u to piles, this paper focuses on discussion of cohesion soils. Therefore, the traditional p – y curve method is introduced, as shown in Fig. 3, where p_u is the pile lateral ultimate soil resistance, p is the pile lateral soil resistance, y is the pile lateral soil horizontal displacement, and K_h is the pile lateral soil resistance coefficient.

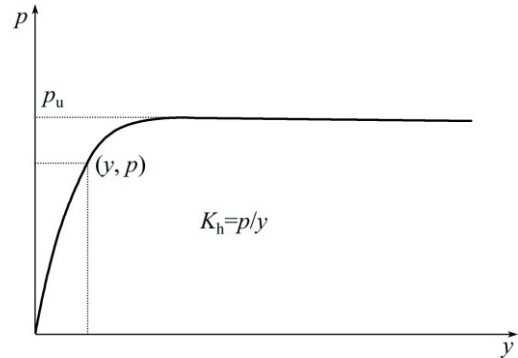


Fig. 3 Load-displacement curve for pile-soil system

For the pile lateral soil resistance of cohesive soil, slope is different from that in the horizontal ground, in order to consider the effect of the slope. For simplicity, the following simple expression proposed by GEORGIADIS et al [11,16] is used to calculate p_u :

$$p_u = N_p c_u D \quad (11)$$

where D is the pile diameter; c_u is the undrained shear strength of the soil; and N_p is the lateral bearing capacity factor.

The lateral bearing capacity factor N_p can be simply derived as

$$N_p = N_{pu} - (N_{pu} - N_{po} \cos \theta) \exp[-\lambda(z/D)/(1 + \tan \theta)] \quad (12)$$

$$N_{po} = 2 + 1.5\alpha \quad (13)$$

$$\lambda = 0.55 - 0.15\alpha \quad (14)$$

$$N_{pu} = \pi + 2\Delta + 2 \cos \Delta + 4 \left(\cos \frac{\Delta}{2} + \sin \frac{\Delta}{2} \right) \quad (15)$$

$$\Delta = \csc \alpha \quad (16)$$

where N_{po} is the bearing capacity factor on ground surface, α is defined as the ratio of the interface shear strength to the undrained shear strength of the soil, $\alpha = \tau_f / S_u$, λ is a non-dimensional factor which varies with the pile-soil adhesion factor from 0.55 for $\alpha=0$ (perfectly smooth pile) to 0.4 for $\alpha=1$ (perfectly rough pile), and N_{pu} is the ultimate bearing capacity factor.

3.2 Foundation resistance coefficient $K_{h\theta}$

For the calculation of ultimate load of pile lateral soil, how to determine the modulus of horizontal subgrade reaction K_h has always been concerned and discussed by researchers, many scholars believe that the modulus of horizontal subgrade reaction K_h , varies linearly with depth, as shown in Fig. 4.

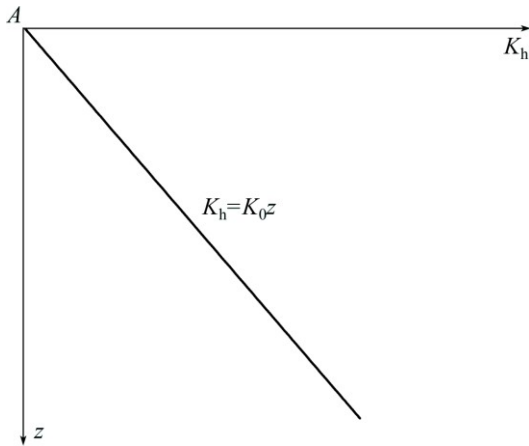


Fig. 4 Variation of K_h with depth z

For considering the effect of slope foundation on resistance coefficient, the variation of $K_{h\theta}$ with depth can be expressed by

$$K_{h\theta} = K_h \left[\cos \theta + \frac{z}{6D} (1 - \cos \theta) \right] \quad (17)$$

The modulus of horizontal subgrade reaction of K_h can be described by the following equation:

$$K_h = 3E_{50} \left(\frac{E_{50} D^4}{E_p I_p} \right)^{1/12} \quad (18)$$

where E_{50} is the secant elastic modulus at 50% of the failure stress, determined from unconsolidated undrained triaxial compression tests, and $E_p I_p$ is the pile flexural stiffness.

4 Probabilistic analysis approaches

4.1 Reliability index

In order to account for the uncertainty occurring in geotechnical structures, the reliability index has been proposed as an alternative. A more flexible and rational alternative is the Hasofer–Lind [15] second-moment reliability index and the first-order reliability method (FORM), which reflects the uncertainty of the parameters as well as their correlation structure. The reliability index for correlated normal random variables, β , is defined in a matrix formulation as

$$\beta = \min_{X \in F} \sqrt{(X - \mathbf{u})^T \mathbf{C}^{-1} (X - \mathbf{u})} \quad (19)$$

where \mathbf{u} is the vector of mean values u_i , \mathbf{X} is the vector representing the set of random variables x_i , F is the limit state boundary, which is the boundary separating the domain of satisfactory performance from the domain of unsatisfactory performance, and \mathbf{C} is the covariance matrix.

The extension of the Hasofer–Lind index to correlated non-normalization is known as the first-order reliability method (FORM).

An expanding ellipsoid leads to a robust and efficient method of computing the reliability index β and design point x^* in the original space of the random variables by using a constrained optimization approach was suggested by LOW and TANG [19]. The reliability index and probability of failure can be expressed as

$$\beta = \min_{X \in F} \sqrt{\mathbf{X}'^T \mathbf{R}^{-1} \mathbf{X}'} \quad (20)$$

$$P_f = 1 - r \quad (21)$$

$$r = \Phi(\beta) \quad (22)$$

where \mathbf{R} is the correlation matrix, \mathbf{X}' is related to x through $\mathbf{X}' = \Phi^{-1}[F(x)]$, $\Phi()$ is the cumulative distribution function (CDF) of the standard normal variance, and $F()$ is the cumulative distribution function (CDF) of the original non-normal variable.

For computing the reliability index by using Eqs. (19) and (20), one needs to distinguish negative indices from positive indices before using Eq. (21) to estimate the probability of failure. The computed β is positive if the performance function is positive at the mean-value point, i.e., the mean-value point is within the safe domain. The computed β is negative if the performance function is negative at the mean-value point, i.e., the mean-value point is within the failure domain.

4.2 Response surface methods

Equations (19) and (20) are used to evaluate β , so that the most critical random variables (\mathbf{X}^*) on the limit state boundary are found. Equations (19) and (20) are

conveniently expressed in an optimization problem as follows:

Minimize β subject to $g(X)=0$

where $g(X)$ represents the performance function of the limit state boundary.

The basic idea of response surface method is to approximate the limit state boundary by an explicit function of the random variables, and to improve the approximation via iterations.

Figure 5 shows the numerical algorithm of the reliability analysis of laterally loaded pile in sloping ground using the response surface method. The response surface method for laterally loaded pile in sloping ground is described as follows:

1) Determine the random variables X and the important pile–soil structural response, and the mean values μ and standard deviations σ of the random variables are also identified.

2) Identify the values of the random variables at the chosen sampling point (s).

3) Using the values of the random variables selected in Step (2) in laterally loaded pile during sloping ground analysis.

4) For the problem, construct the approximate performance function of the laterally loaded pile in sloping ground response.

5) Using the sequential unconstrained minimization technique (SUMT) to minimize β subjected to the performance function=0.

6) If a solution is found which satisfies the specified convergence criterion, the analysis can be stopped. Otherwise, Step (3) is repeated by taking the optimal

values of the random variables obtained from Step (5) as the basis of the next sampling point (s).

5 Example analyses

5.1 Validation

Considering a pile with a diameter of 0.4 m and length of 20 m subjected to a horizontal load, H , at the pile head (Fig. 6), it is supposed that P_{ui} , K_i , H and $E_p I_p$ are random variables. The mean values, coefficients of (COV) variation and standard deviations (SD) of the parameters are listed in Table 1, and the results of the reliability analysis of the laterally loaded pile under the pile head displacement failure mode and the maximum bending moment failure mode are respectively listed in Tables 2 and 3.

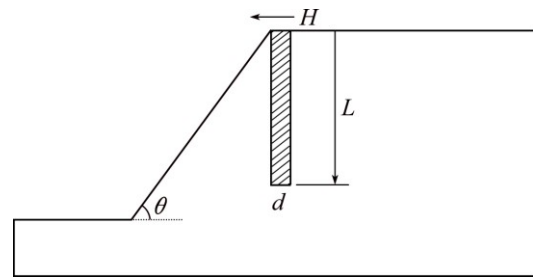


Fig. 6 Laterally loaded pile in sloping ground

Table 1 Random variables and statistical information

Parameter	Mean	COV	SD
$P_{ui}/(\text{kN} \cdot \text{m}^{-1})$	240	0.1	24
K_i/MPa	20	0.2	4
H/kN	120	0.15	18
$E_p I_p/(\text{kN} \cdot \text{m}^2)$	50000	0.15	7500
N_{pu}	10.54	0.2	2.1
N_{po}	2.7	0.1	0.27
E_{50}/kPa	2750	0.2	275
λ	0.5	0.1	0.05

Table 2 Results of reliability analysis of laterally loaded pile under pile head displacement failure mode

Parameter	1st-degree polynomial RS	2st-degree polynomial RS	Monte-Carlo
Reliability index, β	1.923	1.932	1.927
Failure probability, P_F	0.0164	0.0163	0.0175
$P_{ui}/(\text{kN} \cdot \text{m}^{-1})$	203.9	204.1	204.7
K_i/MPa	18.2	18.3	18.6
H/kN	103.4	106.7	107.1
$E_p I_p/(\text{kN} \cdot \text{m}^2)$	47956.63	47968.11	47979.89

Values of P_{ui} , K_i , H and $E_p I_p$ obtained at design point

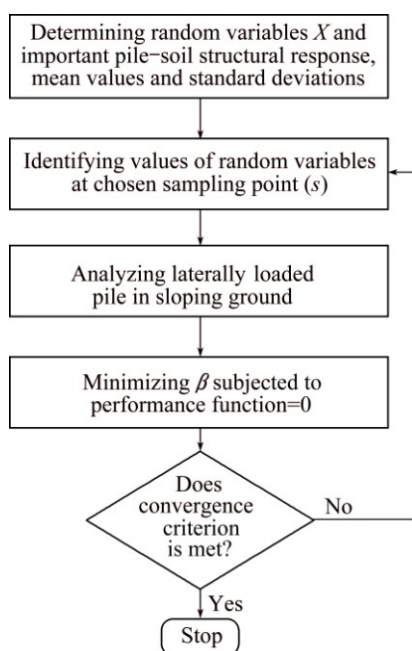


Fig. 5 Numerical algorithm for reliability analysis of laterally loaded pile using response surface methods

Table 3 Results of reliability analysis of laterally loaded pile under maximum bending moment failure mode

Parameter	1st-degree polynomial RS	2nd-degree polynomial RS	Monte-Carlo
Reliability index, β	2.345	2.412	2.433
Failure probability, P_F	0.0061	0.0067	0.0072
$P_{ui}/(\text{kN}\cdot\text{m}^{-1})$	223.8	224.3	224.9
K_i/MPa	17.6	18.0	18.1
H/kN	116.44	117.16	116.98
$E_p I_p/(\text{kN}\cdot\text{m}^2)$	48745.33	48764.24	48788.83

Values of P_{ui} , K_i , H and $E_p I_p$ obtained at design point

The random variables are assumed normally distributed. For simplicity, assume that the undrained elastic modulus $E_u=10$ MPa, undrained shear strength $c_u=50$ kPa, Poisson ratio $\nu_u=0.49$, and bulk unit weight $\gamma=18$ kN/m³. The reliability analysis is conducted for two different failure modes: 1) the maximum bending moment in the pile ≥ 210 kN·m; 2) pile-head deflection ≥ 0.06 m. The performance functions with respect to pile-head deflection and maximum bending moment failure are:

$$g_1(x)=M_p-M_{\max} \quad (23)$$

$$g_2(x)=(y_0)_{\text{limit}}-(y_0)_{\text{computed}} \quad (24)$$

where M_p is the plastic bending moment (300 kN·m); M_{\max} is the maximum computed bending moment; $(y_0)_{\text{limit}}$ is the limiting pile-head deflection (30 mm); and $(y_0)_{\text{computed}}$ is the computed pile-head deflection.

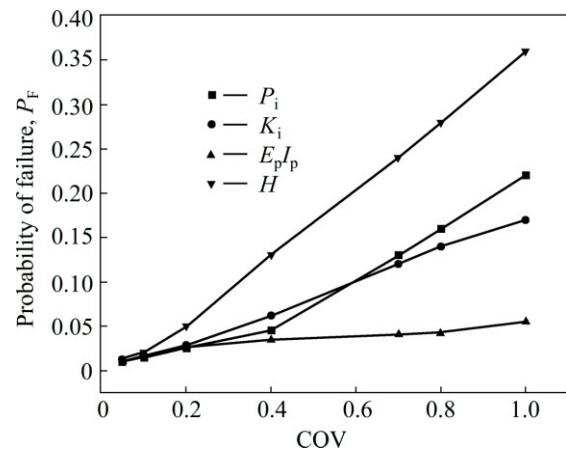
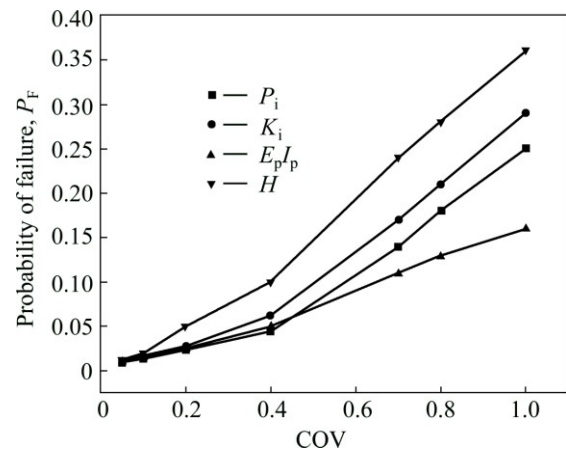
Tables 2 and 3 show that the failure probabilities obtained from the response surface methods are slightly different from those obtained from the Monte Carlo simulation. Under the pile head displacement failure mode, the response surface methods provide a failure probability between 1.63% and 1.64%, while the Monte Carlo simulation gives the failure probability of 1.75%. For the failure mode specified in terms of the maximum bending moment, the failure probability of the response surface methods is between 0.61% and 0.65%, while the failure probability of the Monte Carlo simulation is 0.72%.

5.2 Sensitivity of failure probability to parameters

The failure of laterally loaded piles in sloping ground is influenced by the characteristics of the pile-soil parameters used in the probabilistic analysis. In order to study the effects of the pile-soil parameters on failure probability, a number of parametric studies have been carried out. The mean values and the coefficients of variation (COV) as listed in Table 1 are used, except for the parameter whose COV is varied over the range of

0.01–1.0. Similar to the previous analyses, four parameters are also considered here.

Figures 7 and 8 show failure probability of laterally loaded piles in sloping ground versus coefficients of variation (COV) under the maximum bending moment failure mode and the displacement failure mode, respectively. The failure probability of the laterally loaded pile under the displacement failure mode is highly influenced by the coefficients of variation of H , K_i , P_i and $E_p I_p$. Similar to the case under the maximum bending moment failure mode, the failure probability of the laterally loaded pile is also sensitive to the coefficients of variation of H , K_i , P_i and $E_p I_p$. In contrast, the coefficient of variation of $E_p I_p$ does not significantly affect the failure probability of the pile under the maximum bending moment failure mode. The results show that the greater the scatter in these parameters is, the higher the failure probabilities of the pile are. This means that the accurate determination of the distribution of such parameters is very important in obtaining reliable probabilistic results.

**Fig. 7** Failure probability of pile versus COV under maximum bending moment failure mode**Fig. 8** Failure probability of pile versus COV under displacement failure mode

6 Parametric study

6.1 Effect of coefficient of variation of random variables

The aim of this section is to study the effects of the statistical characteristics of the random variables (the coefficient of variation, the correlation coefficient and the type of the probability density function) on the pile head displacement of laterally loaded pile. The effects of COV of the random variables are studied and presented in Fig. 9.

From Fig. 9, one can note that the probability density function (PDF) of pile head displacement is

sensitive to the coefficients of variation of N_{pu} and H , the coefficients of variation of E_{s0} , N_{po} and $E_p I_p$ have some effects on the PDF of pile head displacement; however, parameter λ has no effect on the variability of the PDF of pile head displacement.

6.2 Effect of correlation coefficient of random variables

For this work, it is assumed that the correlation coefficient between N_{po} and λ is interrelated, N_{pu} and λ are generally regarded as correlated. The effect of correlation coefficient of the random variables (N_{po} , N_{pu} and λ) is studied and presented in Figs. 10 and 11. Figure 10 presents the influence of the correlation coefficient

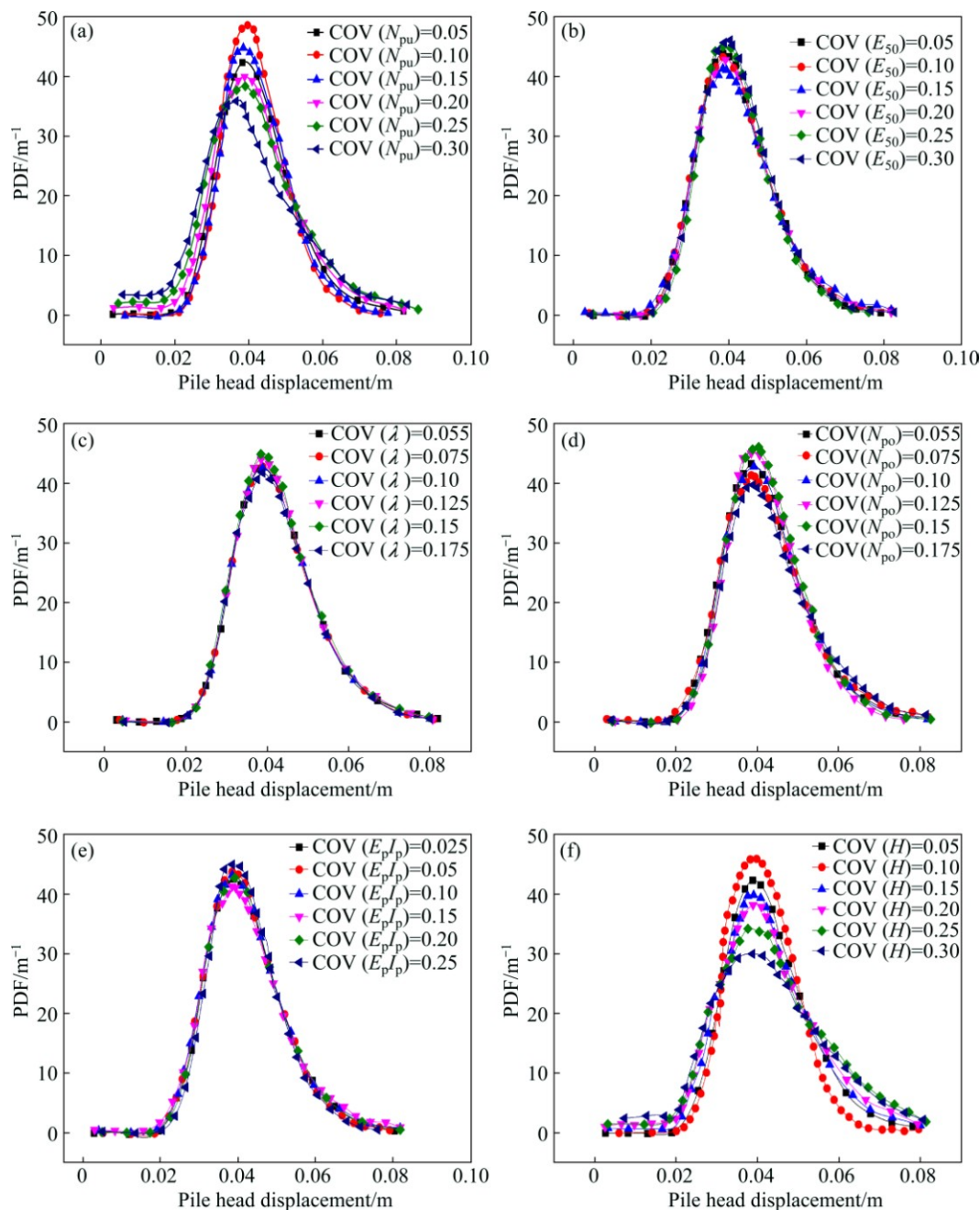


Fig. 9 Influence of coefficients of variation of input random variables on PDF of pile head displacement

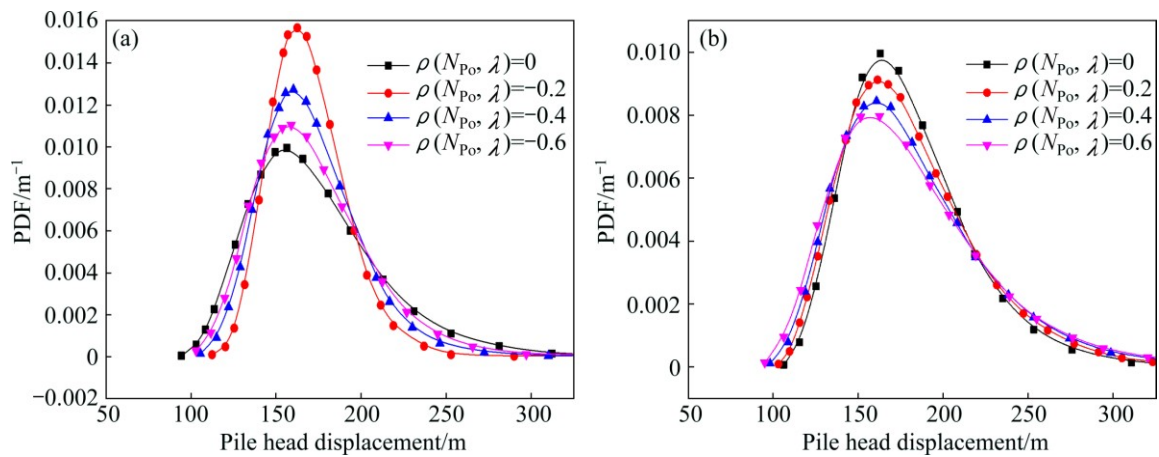


Fig. 10 Influence of correlation coefficient $\rho(N_{po}, \lambda)$ on PDF of pile head displacement: (a) Negative correlation; (b) Positive correlation

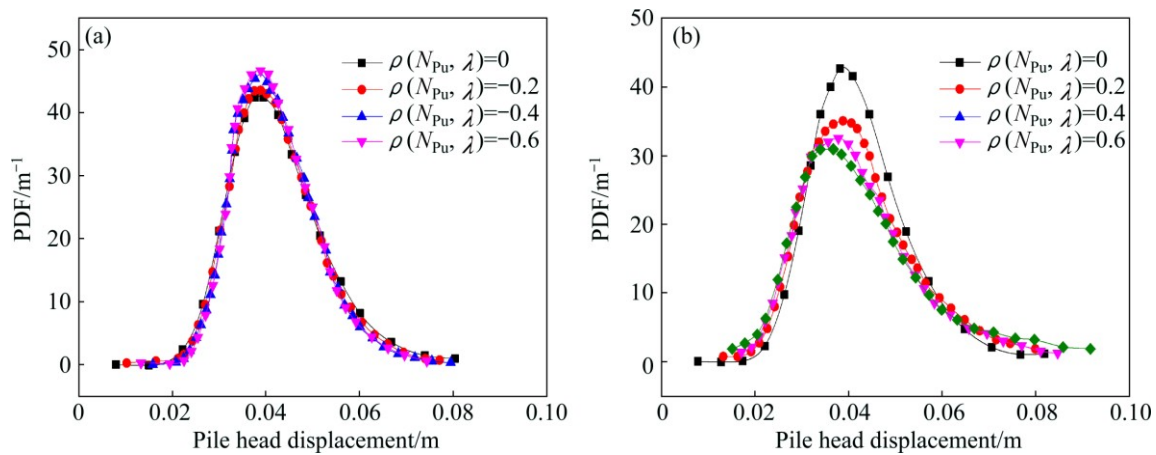


Fig. 11 Influence of correlation coefficient $\rho(N_{pu}, \lambda)$ on PDF of pile head displacement: (a) Negative correlation; (b) Positive correlation

$\rho(N_{po}, \lambda)$ on the PDF of pile head displacement. Figure 10(a) shows that the PDF of pile head displacement is more spread out in the case of a negative correlation between the random variables N_{po} and λ . On the other hand, one can see from Fig. 10(b) that the case of a positive correlation (where both parameters increase or decrease together) leads to an important variation (i.e., variability) in the PDF of pile head displacement. It is obvious that both negative and positive correlations between the random variables N_{po} and λ will lead to an important variation in the PDF of pile head displacement, and a negative correlation will obviously increase the variability of the response as seen from Fig. 10. This is because the increase of one parameter value implies a decrease in the other parameter.

Figure 11 presents the PDF of pile head displacement for different values of the correlation coefficient $\rho(N_{pu}, \lambda)$. Figure 11(b) shows that the case of a positive correlation leads to an important variation (i.e., variability) in the PDF of the pile head displacement, while the PDF of pile head displacement is

less spread out in the case of a negative correlation between the random variables N_{pu} and λ as shown in Fig. 11(a).

6.3 Effect of type of probability density function

The effect of the type of the probability density function of the input random variables on the PDF of pile head displacement is studied. In this study, two cases of normal and log-normal random variables combined with two configurations of COVs are considered. The “standard COVs” corresponds to the reference case presented in Table 1 ($\text{COV}(N_{pu})=0.2$, $\text{COV}(N_{po})=0.1$, $\text{COV}(\lambda)=0.1$, $\text{COV}(E_{50})=0.2$, $\text{COV}(H)=0.15$, $\text{COV}(E_p I_p)=0.15\%$), while the “high COVs” corresponds to the values increased by 20% ($\text{COV}(N_{pu})=0.24$, $\text{COV}(N_{po})=0.12$, $\text{COV}(\lambda)=0.12$, $\text{COV}(E_{50})=0.24$, $\text{COV}(H)=0.18$, $\text{COV}(E_p I_p)=0.18\%$). The log-normal of the input random variables has a significant influence on the shape of the PDF of pile head displacement as can be seen from Fig. 12 for both cases of “standard COVs” and “high COVs”.

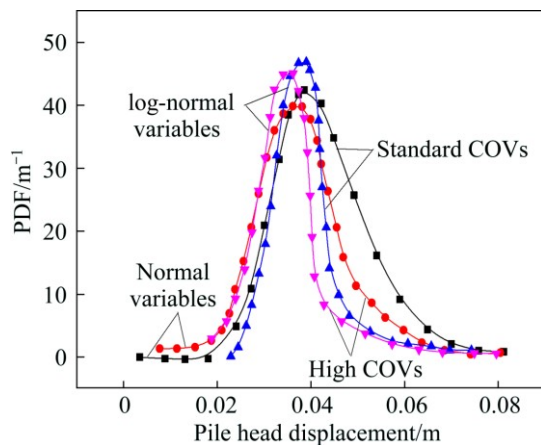


Fig. 12 Influence of type of probability density function of input random variables for two different sets of COVs on PDF of pile head displacement

7 Conclusions

1) The reliability analyses under the pile head displacement and the maximum bending moment failure modes show that the failure probabilities obtained from the response surface methods are slightly different from those obtained from the Monte Carlo simulation. Under the pile head displacement failure mode, the response surface methods provide a failure probability between 1.63% and 1.64% while the Monte Carlo simulation gives the failure probability of 1.75%. For the failure mode specified in terms of the maximum bending moment, the failure probability of the response surface methods is between 0.61% and 0.65% while the failure probability of the Monte Carlo simulation is 0.72%. These above differences are caused by the assumption of hyper plane.

2) The parametric studies show that the failure probability of the laterally loaded pile in sloping ground under the displacement failure mode is highly influenced by the coefficients of variation of H , K_i , P_i and $E_p I_p$. Similar to the case under the maximum bending moment failure mode, the failure probability of the laterally-loaded pile is also sensitive to the coefficients of variation of H , K_i , P_i and $E_p I_p$. The results show that the greater the scatter in these parameters is, the higher the failure probabilities of the pile are. This means that the accurate determination of the distribution of such parameters is very important for obtaining reliable probabilistic results. In contrast, the coefficients of variation of $E_p I_p$ do not significantly affect the failure probability of the pile under the maximum bending moment failure mode.

3) It is shown that the variability of the PDF of pile head displacement increases with the increase in the coefficients of variation of N_{pu} , E_{50} and H ; N_{pu} and H

have a greater effect. The investigation on the effect of correlation coefficient of the random variables shows that the PDF of pile head displacement is less spread out when there is a negative correlation between N_{pu} and λ ; conversely, an important variation will appear with a positive correlation between them. As for N_{po} and λ , both negative and positive correlation between them give an important variation in the PDF of pile head displacement, and a negative correlation will obviously increase the variability of the response.

4) The influence of the probability density function of the input random variables on the shape of the PDF of pile head displacement is performed. The results indicate that the log-normality of the input random variables has a significant influence on the shape of the PDF of pile head displacement.

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横向荷载作用下斜坡桩的可靠性分析

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摘 要: 利用响应面方法开展横向荷载下斜坡桩的可靠性研究。引入一种考虑斜坡效应的新型载荷-位移(p - y)曲线方法模拟斜坡上横向受荷桩桩-土作用系统, 采用斜坡桩桩顶位移和桩身弯矩作为功能准则, 建立桩顶位移失效和桩身最大弯矩两种失效模式下横向荷载作用斜坡桩基的可靠性分析方法, 利用蒙特卡罗法验证了斜坡地面上横向受荷桩的可靠性分析方法。讨论横向荷载作用下斜坡桩基可靠性的影响因素。结果表明, 桩顶位移的变异性随着极限承载因数(N_{pu})、50%水平的弹性模量(E_{50})和水平荷载(H)变异系数的增加而增加, N_{pu} 和 无量纲系数(λ) 的负相关关系使桩顶位移概率密度函数(PDF)值的分散性降低; 相反, N_{pu} 和 λ 的正相关关系使桩顶位移的 PDF 值产生较大变化。对于地面水平极限承载因数(N_{po})和 λ , N_{po} 和 λ 之间的正相关关系和负相关关系均使桩头位移的 PDF 值产生较大变化, 并且 N_{po} 和 λ 之间的负相关关系会明显增加这种变异的响应。

关键词: 侧向受荷桩; 响应面法; 概率; 可靠性分析

(Edited by Wei-ping CHEN)