



Radial mixing and segregation of granular bed bi-dispersed both in particle size and density within horizontal rotating drum

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Abstract: Taking simultaneous variations in both particle volume and density into account, the radial mixing and segregation of binary granular bed in a rotating drum half loaded were investigated by a 3D discrete element method. Then, based on the competition theory of condensation and percolation, radial segregation due to differences in particle volume and/or density was analyzed. The results show that if either percolation effect induced by volume difference or condensation effect induced by density difference dominates in the active layer of moving bed, separation will occur. Controlling the volume ratio or density ratio of the two types of particles can achieve an equilibrium state between percolation and condensation, and then homogenous mixture can be obtained. When the percolation balances with the condensation, the relationship between volume ratio and density ratio presents nearly a power function. Scaling up a rotating drum will not affect the mixing degree of the granular bed so long as the volume ratio and density ratio are predefined.

Key words: binary granular bed; rotating drum; solid particle; radial mixing; segregation; discrete element method

1 Introduction

Mixing granular materials with different properties in a horizontal rotating drum is a commonly used manufacturing technique in the metal smelting, mineral processing, cement manufacturing and other industries. Materials are mostly solid particulates whose mixing effect is directly related to the drying, grinding and thermal efficiency [1–4]. It is required to make a homogenous mixture in most industrial processes. Nevertheless, a distinction in the properties of granular particles can readily result in a sub-optimal mixing process, or even cause a demixion of the granular bed in the drum, with obviously detrimental consequences for the product quality [5]. The practical importance and attractive complexity have made granular mixing the subject of intense research in the last decades.

Binary disperse granular beds of industries usually contain particles which differ in size and/or density [6], therefore, segregation will happen in the transverse

section of rotating drum for the effects of these two granular properties. Studies have focused on the rolling regime, which was considered as the most common movement patterns of granular bed [7]. In the mixing process of particles bi-dispersed in size [8–10], it is well known that the smaller particles will tend to gather into a radial core within the first few revolutions, surrounded by a periphery of larger particles, stretching along the axial length of the drum. The physical mechanism underlying segregation due to size difference is usually referred to as percolation that the smaller particles falling through the temporary voids between larger particles. On the other hand, in the equally sized granules bi-dispersed in density [11,12], the denser particles will located near the central axis of bed, while less dense particles segregate radially to the periphery. This mechanism of segregation is generally attributed to condensation, where less dense particles will float upward in grain flow. In either case, the mixing degree of the bed will be reduced. As suggested by HONG and QUINN [13] from the investigation of Brazil nut problem, the combination

of percolation and condensation may enhance as well as reduce segregation, depending on the simultaneous variations in both particle size and density. However, most studies on mixing behavior of binary materials in rotating drum have focused on particles differing only in particle size or only in particle density [14]. The situation where particles differing both in size and density has received relatively little attention. Moreover, for the discreteness and non-transparency of the solid particles, it has a certain degree of difficulty to make an accurate characterization of the mixing degree experimentally [15–18]. The investigations are commonly restricted to a qualitative description of the mixed patterns such as moon pattern, sun pattern, and wave breaking pattern [19,20]. The number of studies that give precise quantitative definition of the mixability for binary granular materials is comparatively small.

For the purpose to enhance the mixing effect of the binary particles, this work conducted an numerical investigation on the radial mixing behavior of binary granular bed within the rotating drum using the discrete element method (DEM) which is a powerful numerical method for analyzing the microscopic behaviors of discontinuous media [21–24]. As the DEM can offer unparalleled control over the properties of the granular materials, the mixing index was defined and both the variances of particle size and particle density were considered. Then, the results of the DEM simulations were analyzed to quantitatively determine the relationship between particle size and particle density in favor of the mixing of binary granular bed in rotating drum.

2 DEM model description

2.1 Basic model

The modeling of granular flows with DEM involves the trajectories, velocities and positions of all the particles by solving Newton's equation of motion for each of them. As the movement of particles in a rotating drum is a kind of flow of dense particles, the soft-sphere model [16] would be employed. The basic algorithm of DEM is relatively simple with three essential parts [1–3]: firstly, a search grid is used to periodically build a particle near neighbor interaction list; secondly, collisional forces on each of the particles and boundary objects are evaluated efficiently using the near neighbor list and contact force model; thirdly, all the forces on the particles are summed and the resulting equations of motion are integrated with a reasonable time step. Then, several key factors of modeling need to be illuminated, such as contact force model, representation of particle shape, input parameters and time step.

As shown in Fig. 1, the total forces acting on a

spherical particle of mass m_i and rotational inertia I_i include gravity $m_i \mathbf{g}$ and contact force $\mathbf{F}_{ij}$ in normal and tangential directions ($\mathbf{F}_{ij}^n$ and $\mathbf{F}_{ij}^t$), accounting for the particle–particle and particle–drum interactions. $\mathbf{T}_{ij}$ is the corresponding torque of the contact force. Note that the viscous forces such as liquid force and Van der Waals force are excluded in the model for the particles which are cohesionless and millimeter-sized. The effect of air on the flow behavior of the particles in the drum, which may be significant at high rotation speeds [3], is also ignored. Then, the translational and rotational motions of the particle can be described by

$$m_i \ddot{\mathbf{r}}_i(t) = m_i \mathbf{g} + \sum_j (\mathbf{F}_{ij}^n + \mathbf{F}_{ij}^t) \quad (1)$$

$$I_i \ddot{\theta}_i(t) = \sum_j \mathbf{T}_{ij} \quad (2)$$

where $\mathbf{r}_i$ and θ_i are the position vector and angular displacement of the particle, respectively. According to the comparison of performance for different contact models proposed by KRUGGEL-EMDEN et al [21] and DI RENZO and DI MAIO [22], the simplified Hertz–Mindlin model (Hertz–Mindlin no-slip model) can make a better representation of the moving behavior of particle at microscopic scale without increasing too amounts of computations. Therefore, Hertz–Mindlin no-slip model is employed to figuring the contact forces ($\mathbf{F}_{ij}$ and $\mathbf{T}_{ij}$) due to the plastic and elastic deformation of particles. The details about the contact force model can be found in the work of DI RENZO and DI MAIO [22].

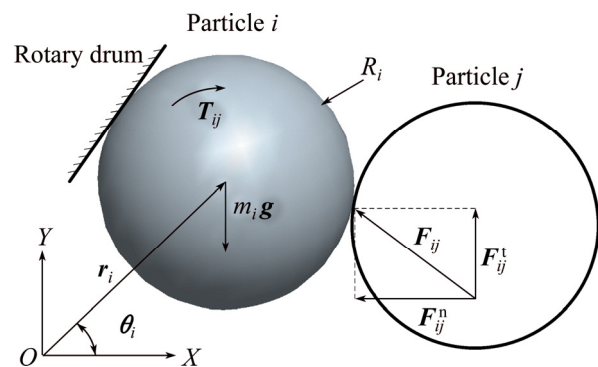


Fig. 1 Schematic of spherical particle in DEM model

The actual shape of the particles commonly used in experimental studies [5,6] is not the ideal sphere. Therefore, using the multi-element method [23], a kind of quasi spherical particle is made in this DEM model. As shown in Fig. 2, the particle is approximately presented by a virtual ball of radius R_i filled with 21 identically sized spherical elements linked each other rigidly. In the multi-element method, by the transformation of the force and mass [23], the contact detection between particles can be sphere-based and any

other procedures for sphere–sphere contact are fully available. Then, the calculation of contact forces between quasi spherical particles can be conducted with a standard discrete element algorithm for spherical particles.

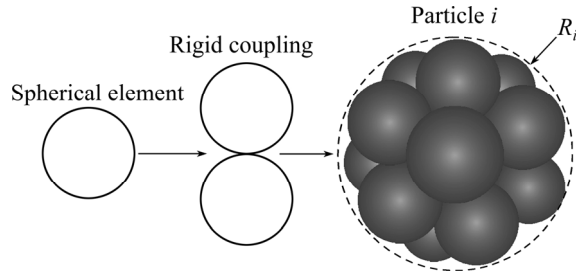


Fig. 2 Particle resulting from multi-element method

In a DEM model, the principle commonly used to determine a time step is that the time step for calculating incremental forces and displacements of particles must be less than Rayleigh critical time step Δt_r , which can be shown as

$$\Delta t_r = \pi \left[\frac{R}{0.163\xi + 0.877} \sqrt{\frac{\rho}{G}} \right]_{\min} \quad (3)$$

where R , ρ , ξ and G are the radius, density, Poisson's ratio and shear modulus of particles in the system, respectively. As CHUNG and OOI [24] reported, the reasonable time step for the simulation of movement of dense particles varies between 20% and 80% of Rayleigh critical time step. Considering both the computation time and computation accuracy, the value of 50% of Δt_r was chosen to solve Eqs. (1) and (2). Then, the simulation was conducted by computer program compiled by language C++, which had been used in our previous work [3].

2.2 Simulation conditions

As shown in Table 1, a rotary drum with laboratory scale was employed in the simulation. The binary granular bed was composed of two kinds of particles, referring to as particles A and particles B. Particle B was supposed to be glass bean, while particle A was supposed to be steel ball except the variables of density and volume. The values for the physical properties such as particle radius R_B , particle density ρ_B , elastic modulus (E_A , E_B and E_d), Poisson's ratio (ξ_A , ξ_B and ξ_d), coefficient of restitution (e_{pp} and e_{pd}) and friction coefficients of particle–particle (μ_{r-pp} and μ_{s-pp}) and particle–drum (μ_{r-pd} and μ_{s-pd}) were obtained directly from the experimental study of JAIN et al [5,6]. Then, variable σ was defined as the volume ratio of particle A to particle B, and variable η was defined as the density ratio of particle A to particle B. As suggested by ARNTZ

et al [14] that the mixing behavior of granular bed was not sensitive to the surface roughness of particles, and the surface roughness of particle A and particle B was assumed to be identical. Therefore, the difference of friction coefficient between two kinds of particles in the model was not considered here. The parameters of the simulations are summarized in Table 1, together with the ranges of parameter variations employed.

Table 1 Parameters used in DEM model

Property	Value
Volume ratio of particle A to particle B, σ	1–3
Density ratio of particle A to particle B, η	1–5
Radius of particle B, R_B / mm	3
Density of particle B, ρ_B / (kg·m ⁻³)	2456
Sliding friction coefficient of particle–particle, μ_{s-pp}	0.63
Rolling friction coefficient of particle–particle, μ_{r-pp}	0.05
Sliding friction coefficient of particle–drum, μ_{s-pd}	0.3
Rolling friction coefficient of particle–drum, μ_{r-pd}	0.85
Elastic modulus of particle A, E_A / GPa	110
Elastic modulus of particle B, E_B / GPa	55
Elastic modulus of drum, E_d / MPa	50
Coefficient of restitution of particle–particle, e_{pp}	0.45
Coefficient of restitution of particle–drum, e_{pd}	0.5
Poisson's ratio of particle A, ξ_A	0.3
Poisson's ratio of particle B, ξ_B	0.25
Poisson's ratio of drum, ξ_d	0.2
Diameter of drum, D /mm	200
Length of drum, L /mm	30
Rotating speed of drum, w /(r·min ⁻¹)	8
Acceleration of gravity, g /(m·s ⁻²)	9.81

Figure 3 shows the initial configuration of binary granular bed. The rotating drum is half filled, where particle A is marked as black, and particle B is marked as white. Particles A and B are separated completely, which are located in the left and right sides of the drum,

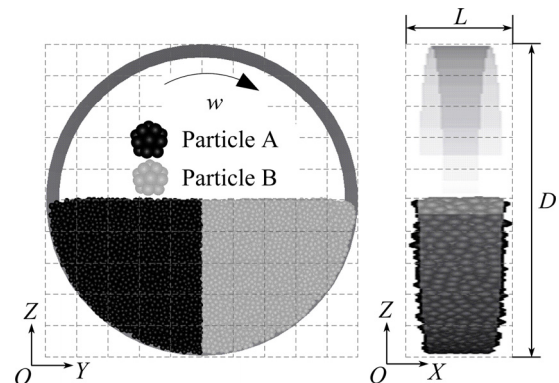


Fig. 3 Initial state of binary granular bed

respectively (when viewed along the X -axis). With the rotation of drum, the particles begin to move and mix with each other. Note that the periodic boundary condition was employed for the simplification of long drum, concerning the end-wall effect [4].

2.3 Characterization of mixing degree

Several methods have been developed to quantify the extent of mixing of a granular bed in the transverse plane [16–18]. Here, the entropy-like quantity [16], which seems to be predominantly concerned with the state of whole mixedness [17], was used to quantify the transverse mixing of the binary granular bed. In this approach, as shown in Fig. 3, a grid with dimensions of 10×10 is defined as the one that overlaps the region of the rotating drum. Then, the mixing index $M(t)$ can be given by

$$M(t) = \frac{\sum_{q=1}^{10} \sum_{p=1}^{10} V_{p,q} (a_{p,q} \ln a_{p,q} + b_{p,q} \ln b_{p,q})}{V_0 (a_0 \ln a_0 + b_0 \ln b_0)} \quad (4)$$

where $V_{p,q}$ is the volume occupied by two kinds of particles in cuboid cell (p, q) , $a_{p,q}$ is the volume fraction of particles A, while $b_{p,q} = 1 - a_{p,q}$ is the volume fraction of particles B. Then, V_0 is the total volume occupied by all the particles in the drum, a_0 is the overall volume fraction of particles A in the system and $b_0 = 1 - a_0$ is the overall volume fraction of particles B in the system.

After solving the DEM model, the evolution of mixing index with the time can be drawn. In the initial state, as shown in Fig. 3, the mixing index $M(0) = 0$, while in the homogeneously mixed state which would be attained as $t \rightarrow \infty$, the mixing index of the bi-dispersed bed is $M(t \rightarrow \infty) = 1$.

3 Results and discussion

3.1 Rolling motion of granular bed

Among the six transverse moving forms of the bed: slipping, slumping, rolling, cascading, cataracting and centrifuging [7], rolling regime is the most common in the industrial conditions [8–12]. Then, the mixing behavior of the binary granular bed was analyzed with respect to the rolling regime. Following the suggestion of MELLMANN [7], the rotational speed of 8 r/min of the drum was adopted by repeated test.

A snapshot for the velocity fields of the binary granular bed in rolling regime is shown in Fig. 4(a). The granular bed is divided by a virtual boundary line into two layers: a thin active layer and a thick passive layer, which has been described experimentally in many previous works [7–10]. In the active layer, the particles flow parallel to the bed surface at a relatively high speed, with numerous random collisions with each other. Then,

in the passive layer, the particles are lifted upwards like a solid bed with the rotating speed of drum, where little relative displacement happens between the particles. For detailed illustration, two particles were sampled randomly from the granular bed, one close to the drum's wall as S_1 and one in the center of the bulk as S_2 , with their trajectory evolution shown in Fig. 4(b). It can be seen that in the passive layer, the motion of each sampled particle produces a series of concentric arcs, without crossing. After entering into the active layer, random jumping will happen to the moving trace of particle for the random collisions, causing the sampled particle to appear in different orbits. This moving law of the sampled particle is qualitatively consistent with the results achieved by PARKER et al [15] using a positron emission tomography imaging method. It is clear that the mixing between particles occurs in the active layer of bed, which provides the foundation stone for the following analysis.

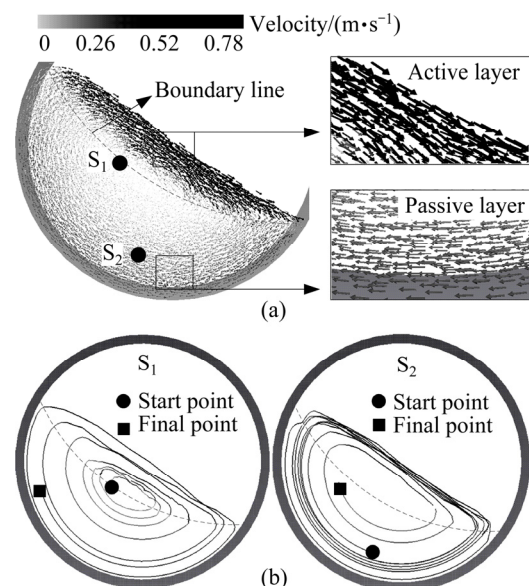


Fig. 4 Schematic of granular bed moving in rolling regime: (a) Velocity fields of particles; (b) Motion paths of sampled particles

The particles in mono-dispersed bed where all the granules have the same physical properties, in general, can be thoroughly mixed in the rotating drum. The mixing curve of granular bed is plotted in Fig. 5 as a function of time when $\sigma = \eta = 1$, with a snapshot show of the bed in stable mixing state. As expected, without density difference or volume difference between particles, segregation will not happen during the mixing process. Then, particles A and particles B can intermingle with each other gradually for the collisions in the active layer. The stable mixing state is achieved after 1 min (nearly 8 rotations), with a mixing index of 1, suggesting that the granular bed was fully mixed.

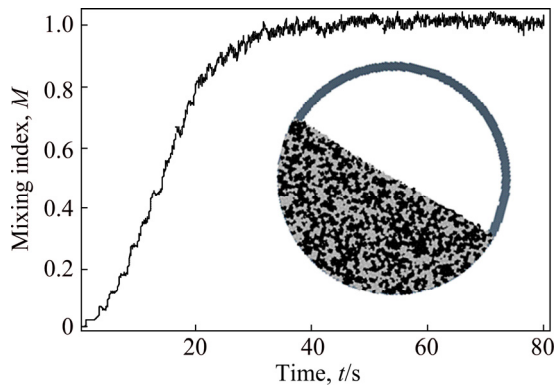


Fig. 5 Mixing curve of granular bed at $\sigma=\eta=1$

3.2 Mechanisms of condensation and percolation

As discussed by HONG and QUINN [13] about the reverse Brazil nut problem in a vibration bed, a density difference in particles induces condensation, causing less-dense particles to go up and denser particles to sink down, as shown in Fig. 6(a). Then, a volume difference in particles leads to percolation, causing smaller particle to fall through the temporary cavities between larger particles, as shown in Fig. 6(b). In the bed dispersed both

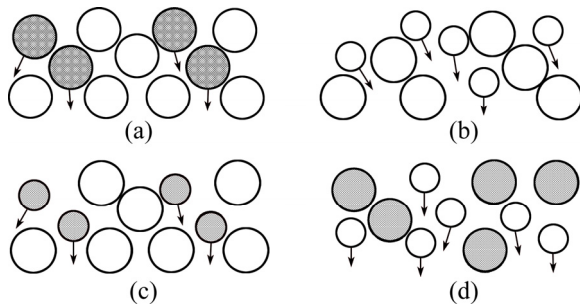


Fig. 6 Segregation in flowing particles: (a) Driven by condensation only; (b) Driven by percolation only; (c) Percolation and condensation reinforcing each other; (d) percolation overriding condensation (Shadow symbol means denser particles)

in particle volume and particle density, the small-heavy particles will naturally sink to lower levels for the combined action of condensation and percolation, as shown in Fig. 6(c). However, when the condensation and percolation compete with each other, reversal will happen to the motion direction. Figure 6(d) illustrates that the big-heavy particles will rise up in the bed when the percolation effect overrides the condensation effect.

When the mechanisms of condensation and percolation operate in the active layer which is responsible for the mixing of binary granular bed (Fig. 4), radial segregation will occur in the rotating drum. A snapshot for the steady mixed state of the bi-dispersed bed is shown in Fig. 7(a) when $\sigma=1$ and $\eta=3$. For the density difference between particle A and particle B, the condensation mechanism will make the denser particles (particles A) fall out of the active layer, move toward the boundary line, and deposit gradually near the center of the bed as a moon-like core. Figure 7(b) shows a steady mixed state of the bed when $\sigma=3$ and $\eta=1$. As expected, under the effect of percolation for volume difference, the smaller particles (particles B) gather in the center of the bed. When the granular bed disperses both in particle density and particle volume, the mixing behavior will be affected by the combination of condensation and percolation. Figure 7(c) illustrates a steady mixed state of the bi-disperse bed when $\sigma=2.2$ and $\eta=3$. Though the size of particles B is smaller, the core of section of the bed is occupied by particles A which are bigger and heavier, for the effect of condensation caused by density difference overrides that of percolation caused by volume difference. This phenomenon of segregation is qualitatively in agreement with the experimental observations of JAIN et al [5,6], where the bi-dispersed bed consists of glass beans and steel balls.

For quantitative analysis, the evolution of mixing index $M(t)$ is plotted in Fig. 8, corresponding to the

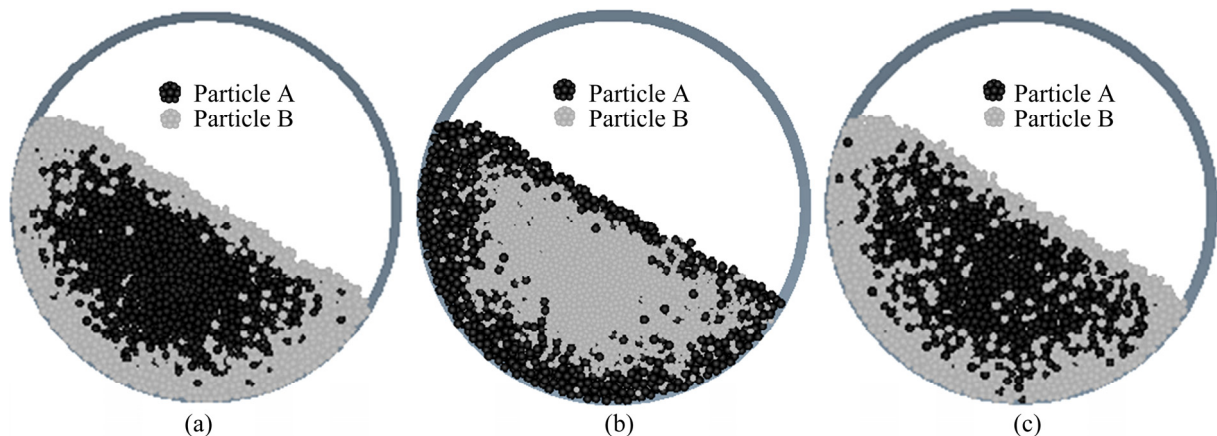


Fig. 7 Snapshots for steady mixed state of binary granular bed in cross section: (a) $\sigma=1$ and $\eta=3$; (b) $\sigma=3$ and $\eta=1$; (c) $\sigma=2.2$ and $\eta=3$

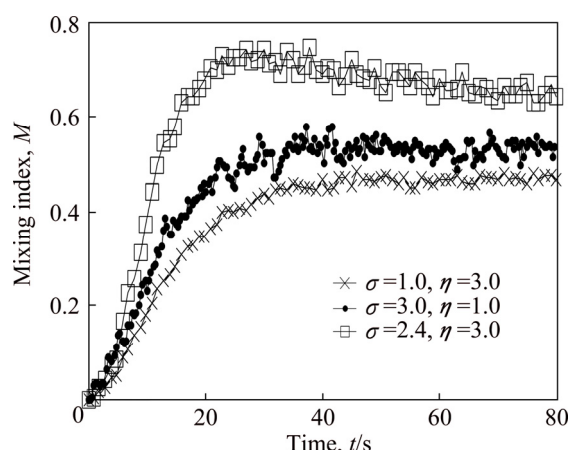


Fig. 8 Evolution of mixing index with time

situations in Fig. 7. As shown Fig. 8, because of the existence of separation, the granular bed in all the situations, as expected, achieves a steady mixed state with a mixing index far less than 1, meaning that the effect of mixing between particles A and particles B is very poor.

3.3 Effects of volume ratio and density ratio

Based on the principle of competition between the mechanisms of percolation and condensation, the equilibrium between these two mechanisms may be made in the granular bed through adjusting the volume ratio (σ) or density ratio (η) appropriately, thus avoiding the separation of particles and achieving a homogenous mixture. The criterion for complete mixing employed here is that the mixing index reaches a value more than 0.95 suggested by SCHUTYSER et al [16].

Figure 9 shows the evolution of mixing index for various volume ratios when the density ratio is 2. As shown in Fig. 9, for $\sigma=1.0$, due to the only condensation effect caused by density difference, serious separation occurs in the granular bed, and then the mixing index of steady state is just about 0.5. For $\sigma=1.4$ and 1.7, the percolation effect induced by volume difference competes with the condensation effect, and then the mixing index of steady state increases to about 0.6 and 0.75, respectively, meaning that the separation in the bed is weakened, though the condensation is still dominant. When σ is about 2.2, the mixing index of steady state is over 0.95, suggesting that the condensation is balanced by percolation, and the two kinds of particles in the bed can be evenly mixed. However, as σ increases to 2.5 and 3.3, the mixing index of steady state decreases to 0.85 and 0.6, respectively. This is because the function of percolation is so strong that it overwhelms the condensation effect, and then the separation caused by percolation happens. Therefore, it comes to the conclusion that the binary granular bed can be mixed

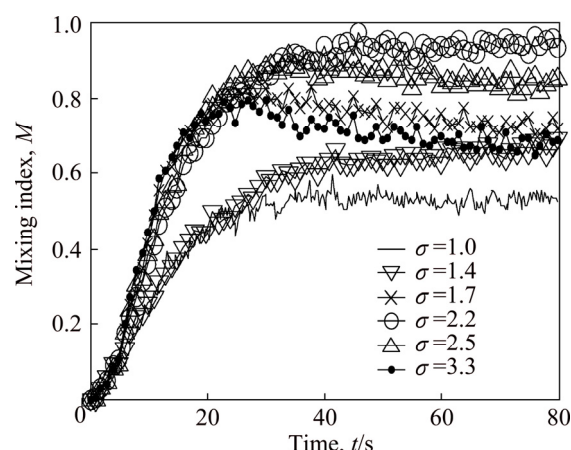


Fig. 9 Evolution of mixing index at $\eta=2$

fully with a configuration of $\sigma=2.2$ when $\eta=2$.

Similarly, the mixing degree of a binary granular bed with a certain volume ratio can be enhanced by adjusting the density ratio. Figure 10 illustrates the evolution of mixing index for various density ratios when $\sigma=3$. As shown Fig. 10, when η is relatively small (1.0, 1.5 and 3.6), the percolation effect plays the dominant role and the separation occurs. When η increases to about 4.4, the percolation is balanced by the condensation function, and then a homogeneous mixture can be made. However, for $\eta=5.2$ and 6.0, the segregation between the two kinds of particles happens again as the excessive condensation overrides the percolation effect. Therefore, the binary granular bed can be mixed fully with a configuration of $\eta=4.4$ when $\sigma=3$.

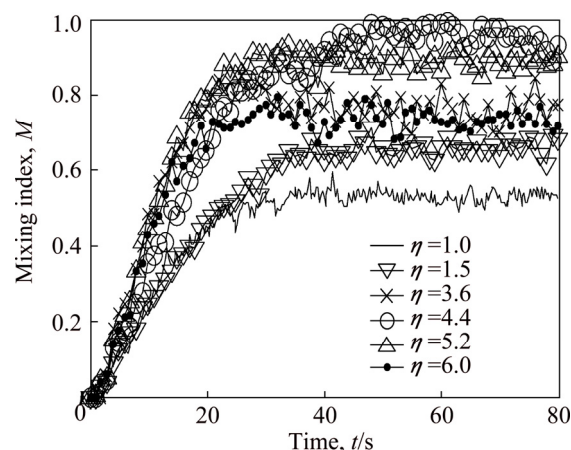


Fig. 10 Evolution of mixing index at $\sigma=3$

Figure 11 shows the phase diagram indicating how a configuration of volume ratio and density ratio should be built for the mixing of binary granular bed. When the point (η, σ) is located on the dividing curve, the equilibrium between the functions of percolation and condensation can be obtained, and then an uniform mixing state of the bi-dispersed bed will be achieved.

The relationship between volume ratio (σ) and density ratio (η) presents nearly a power function, which is more sensitive to the volume ratio, meaning that it is convenient to make the equilibrium between percolation and condensation by adjusting the volume of particles. Actually, there is more feasibility in changing the size of particles to realize the binary granular bed mixed evenly.

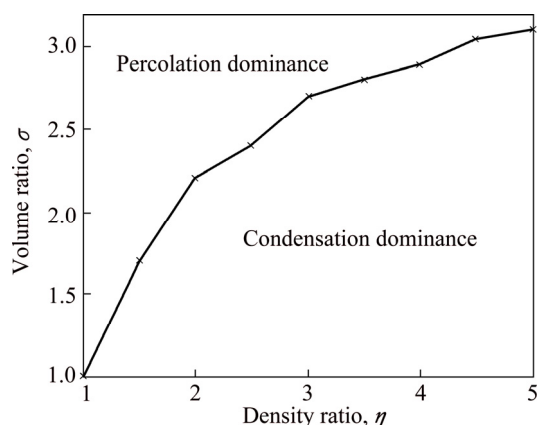


Fig. 11 Phase diagram for equilibrium between percolation and condensation: critical volume ratio in terms of density ratio

3.4 Effect of scale-up on mixing performance

The lack of knowledge about designing and scaling up rotating drums is one of the main reasons for the limited application of research findings in industry. As is well known that scaling-up effect has a serious impact on the solids motion in rotating drums, it is of utmost importance to understand whether it will influence the mixing process of bi-dispersed bed or not. Therefore, the effect of scaling up a rotating drum on the mixing performance was analyzed, as shown in Fig. 12. Three different sizes of half-filled rotating drum were evaluated (diameters of 280, 340, and 400 mm), with a rolling regime and a density ratio of 2. In addition, the grid size for calculation of the mixing index sketched in Fig. 3 was varied proportionally to the diameter of rotating drum in order to keep the sample size constant ($p=q=10$), thus the mixing index can be obtained by Eq. (4).

Figure 12 shows the evolution of mixing index of the binary granular bed in the three rotating drums when $\eta=2$. For the granular bed with a given σ and η , as shown in Fig. 12, the transient evolution of mixing index within the three rotating drums is different from one another due to the variation of drum diameter. However, the focus is given to the steady values of the mixing index, which represents the final mixing degree of the bed. As shown in Fig. 12, when $\sigma=2.2$, the binary granular bed in all of the three rotating drums can achieve a mixed state with a mixing index greater than 0.95 (steady value), meaning that the homogenous mixture is obtained although the diameter of the drum varies. It can be concluded that the mixing degree of binary granular bed in steady state will

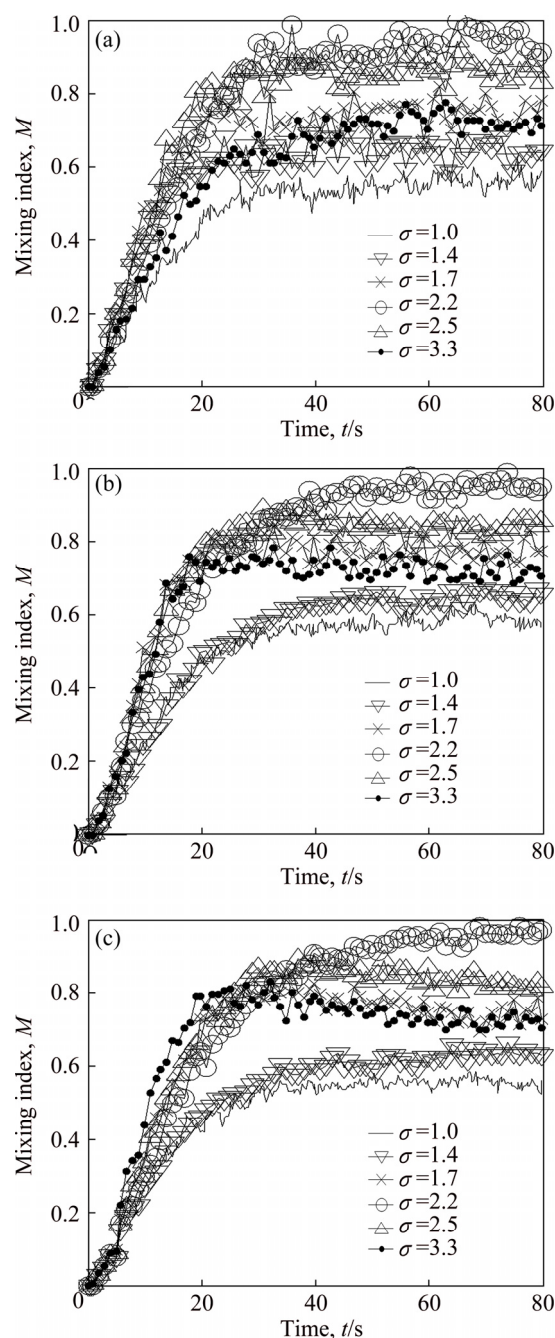


Fig. 12 Evolution of mixing index of binary granular bed in rotating drums with different diameters: (a) $D=280$ mm; (b) $D=340$ mm; (c) $D=400$ mm (particle density ratio is 2)

not be affected by the size of rotating drum so long as the volume ratio and density ratio are predetermined. Note that the moving behavior of particles in all the rotating drums is in the rolling regime. In fact, as in the Brazil nut problem [13], the mechanisms of percolation and condensation which are caused by physical differences between particles will not be influenced by the size of granular system. In the same way, scaling up a rotating drum, which alters the size scale of granular bed, does not affect the percolation and condensation mechanisms within the bed, and then the mixability

of the binary granular bed in steady state is almost constant.

4 Conclusions

1) Either the percolation effect induced by volume difference in particles or the condensation effect induced by density difference in particles will operate in the active layer and produce radial separation. Controlling the volume ratio or density ratio of the two types of particles can make an equilibrium state between the mechanisms of percolation and condensation. Then, segregation is inhibited and homogenous mixture of the binary granular bed can be made.

2) When the percolation balances with the condensation, the relationship between volume ratio and density ratio presents nearly a power function which is more sensitive to the volume ratio.

3) When the granular bed moves in the rolling regime, scaling up a rotating drum will not affect the final mixing degree of binary granular bed where the volume ratio and density ratio are predefined. The results can be up-scaled to industrial conditions.

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“S+D”型二元颗粒床在水平转筒内的径向混合与分离

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摘 要: 同时考虑颗粒体积差异和密度差异, 采用三维离散单元法模拟填充率为 50% 时水平转筒内二元颗粒床的径向混合与分离过程。结合渗流与凝聚竞争机理, 对密度差和体积差导致的颗粒床分离现象进行分析。结果表明: 颗粒体积差引起的渗流机理或密度差引起的凝聚机理均会作用于颗粒床的活动层, 导致颗粒分离; 调整颗粒体积比和密度比可使渗流和凝聚机理相互平衡, 实现物料在转筒内的均匀混合; 当渗流与凝聚平衡时, 颗粒的体积比与密度比基本呈幂函数关系; 当颗粒体积比和密度比确定后, 二元颗粒床的混合度不受转筒尺寸的影响。

关键词: 二元颗粒床; 转筒; 固体颗粒; 径向混合; 分离; 离散单元法

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