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ScienceDirect
Trans. Nonferrous Met. Soc. China 24(2014) 2612−2617
Transactions of Nonferrous Metals Society of China
www.tnmsc.cn

# Generation of artificial earthquakes for matching target response unsmooth spectrum via wavelet package transform

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Received 18 July 2013; accepted 30 June 2014

**Abstract:** The seismic records of target response spectrum used in the time-history analysis should be allowed to meet the norms. However, the previous fitting methods of target spectrum are mostly for the situations that the target spectrum is a smooth curve. In many cases, it needs to match unsmooth target spectrum for single determined response spectrum. An adjustment of time history via wavelet packet transform was presented, which is able to fit unsmooth target spectrum. It was found that there is a certain bias between the band center frequency of the component of seismic record after wavelet packet decomposition and the peak frequency of response spectra of wavelet packet components. For this reason, five strategies were presented to select iteration points, and the effects of the five strategies were compared with two calculation examples. It was turned out that the peak frequency of the response spectrum of wavelet packet component can lead to good fitting effect when it is selected as the iteration point. In the iteration process, it shows great promise in fitting non-smooth target spectrum and has a trend of converge.

**Key words:** time history acceleration; wavelet packet transform; spectral matching; peak frequency of response spectrum

## 1 Introduction

In the evaluation of seismic behavior of structures, time-history analysis is a relatively mature approach, which is very appropriate for the real situation [1]. It is currently widely used in various structures analysis [1−3]. During the last decade, elastic and inelastic dynamic analyses in the time domain have been made feasible for complex structures with thousands of degrees of freedom, thanks to rapidly increasing computational power and the evolution of engineering software [4].

The selection of time history is an important issue in time-history analysis. At present, principal methods of selections are those based on earthquake magnitude ($M$) and distance ($R$), spectral matching and ground motion intensity measures [4]. In the method of spectral matching, there is a class of commonly used method based on ground motion record. By the use of some special techniques (e.g., wavelet), the ground motion record is adjusted, and the adjusted response spectrum of the record is consistent with a given standard response spectrum in specification (target spectrum). Since this method has much easier access to a large number of seismic records fitting the response spectrum [5], there has been a lot of research. MUKHERJEE and GUPTA [6] gave method direct using wavelet transform to adjust measurement records so as to match the target spectrum. SUÁREZ and MONTEJO [7] proposed a new wavelet based on the impulse response function of an under damped oscillator. AMIRI et al [8] accomplished this purpose by combining stochastic neural networks and wavelet analysis. By the use of genetic algorithms, NAEIM [9] achieved a rapid method for generating time history, and the algorithm was stable. However, all the above methods were based on given (from specification) glossy target spectrum. The commonly used uniform hazard spectrum (UHS) is an envelope of the spectral accelerations at all periods that are exceeded with a specified rate, as computed using probabilistic seismic hazard analysis (PSHA) [10]. In many cases, we hope that the differences of response spectrum should be

**Foundation item:** Projects (41272304, 51374244, 41372278, 51304241) supported by the National Natural Science Foundation of China; Project (2010CB732004) supported by the National Basic Research Program of China
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DOI: 10.1016/S1003-6326(14)63390-2

considered. That is, the response spectrum itself is not smooth, but it fluctuates within a certain range. Then, to find a way to fit this unsmooth response spectrum needs to be studied further.

As mentioned above, a number of artificial time history methods have been proposed by scholars. They applied a variety of instruments [6,7,10−12]. Many of them used the wavelet transform or wavelet packet transform as a tool [6,7,10,11]. In contrast with the continuous decomposition of low frequency component of wavelet transform, wavelet packet method is a generalization of wavelet decomposition that offers a richer range of possibilities for signal analysis [13]. It is therefore more suitable for the synthesis of artificial seismic waves. In this work, fitting a fixed target spectrum is achieved by using wavelet packet transform.

## 2 Wavelet packet transform

Simply put, wavelet packet decomposition is such a process. At the first level, signal is decomposed into distinct two parts in frequency domain. The first part is called the approximation component $A_1$, and the second part is called the detail component $D_1$. The approximation component is the lower frequency in original signal frequency domain, accordingly the detail component is the higher frequency in original signal frequency domain. In the second level of the decomposition, the decomposition process of previously obtained component $A_1$ and component $D_1$ is similar to the first level. This process is repeated in the later third level, the fourth level and even more advanced decomposition. The signals are divided into $2^N$ constituent. These $2^N$ components respectively occupy $1/2^N$ bandwidth of original signal band (Fig. 1). At the top of the decomposition tree, the time resolution of the WP components is good but at an expense of poor frequency resolution, whereas at the bottom with the use of wavelet packet analysis, the frequency resolution of the decomposed component with high frequency content can be increased. As a result, the wavelet packet analysis provides better control of frequency resolution for the decomposition of the signal [14].

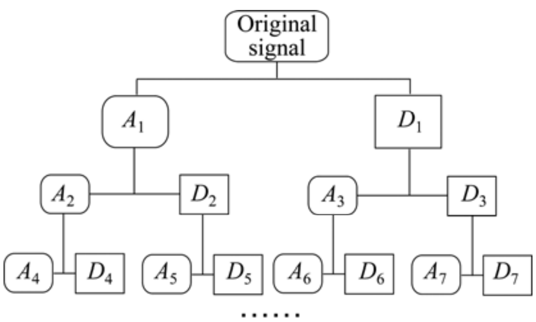


**Fig. 1** Wavelet packet decomposition tree

The function $\psi$ is used to represent a wavelet packet.

$$\psi^{i}{}_{j,k}(t)=2^{-j/2}\psi^{i}(2^{-j}t-k) \tag{1}$$

In the formula, $i$=1, 2, ⋯, $2^n$ is called modulation parameter, $n$ is the level of the decomposition on wavelet packet tree. Let $j$ be dilation parameter, and $k$ be translation parameter. The wavelet packet decomposition of the next level can be obtained by the iteration relationship.

$$\begin{cases}\psi^{2i}(t)=\dfrac{1}{\sqrt{2}}\sum\limits_{k=-\infty}^{\infty}h(k)\psi^{i}\left(\dfrac{t}{2}-k\right)\\ \psi^{2i+1}(t)=\dfrac{1}{\sqrt{2}}\sum\limits_{k=-\infty}^{\infty}g(k)\psi^{i}\left(\dfrac{t}{2}-k\right)\end{cases} \tag{2}$$

where $\psi^i(t)$ is called a mother wavelet, and $h(k)$ and $g(k)$ are quadrature mirror filters associated with the scaling function and the mother wavelet function [8]. These two filter, $h(k)$ and $g(k)$, are also called group conjugated orthogonal filters [15].

The corresponding wavelet packet coefficients of signal can be calculated using the following formula:

$$C^{i}{}_{j,k}=\int_{-\infty}^{\infty}f(t)\psi^{i}{}_{j,k}(t)\mathrm{d}t \tag{3}$$

The component of each wavelet packet on wavelet packet tree can be expressed as

$$f_j^{\,i}(t)=\sum_{k=-\infty}^{\infty}C^{i}{}_{j,k}\psi^{i}{}_{j,k}(t) \tag{4}$$

The superposition of the $j$ layer of wavelet packet components can restore the original signal. And this process will result in the next formula using the reconstruction formula:

$$f(t)=\sum_{i}^{2^j}f_j^{\,i}(t) \tag{5}$$

For applications, we generally use discrete wavelet packet transform [7,16]. The actual seismic waves tend to be treated in the form of discrete digital signals. According to different purposes, the wavelet packet coefficients in Eq. (4) need to be adjusted before the reconstruction process of Eq. (5) in practice [7]. This is exactly the principle of this work: wavelet packet transform used to adjust measured time history for fitting target spectrum.

## 3 Five strategies for fitting target spectrum

The response spectrum is a curve of maximum response versus $T$, where the maximum response of one degree of freedom oscillator could be calculated when the natural period of $T$ is changed. When the maximum response is an absolute acceleration of one degree of freedom oscillator, the response spectrum is the familiar pseudo-spectral acceleration, PSA ($g$), which often appears in seismic codes of states.

In general, the process of the use of wavelet transform or wavelet packet transform in adjusting the measured time history for fitting target response spectrum is roughly the same. The measured signal is first decomposed to obtain the wavelet coefficient (or wavelet packet coefficients). Then different strategies are used to adjust these coefficients, and these adjusted wavelet coefficients are reused to reconstruct. The reconstructed signal repeats the process as original signal until desirable earthquake process is obtained.

The following is the details of a common adjustment strategy. For each decomposition wavelet coefficients component $i$ is multiplied by an adjustment factor $\gamma_i$.

$$\gamma_i = \frac{[g(T_i)]_{\text{target}}}{[g(T_i)]_{\text{reconstructed}}} \tag{6}$$

where $T_i$ is a constant periodic points according to the spectral characteristics of wavelet components. For instance, the band that the time history of a sampling interval of $\Delta t$ can represent is (0, $1/2\Delta t$). This band is evenly divided into $2^J$ parts by a wavelet packet transform scale of $J$. Then the band that the number $i$ wavelet packet component can represent is $([(i-1)]/[2^{J+1}\Delta t], i/[2^{J+1}\Delta t])$. $T_i$ can take the cycle of the midpoint of this bandwidth, that is, $T_i = (2^{J+2}\Delta t)/(2i-1)$. In general, $T_i$ has been given at the outset. In order to facilitate subsequent narrative, we call such strategy used in selecting $T_j$ is Strategy 1 (S1). In previous studies, the target spectrum is generally a smooth curve given in seismic codes [6,7]. The strategy one is simple and effective when the target spectrum is smooth. However, the target spectrum studied in this work is not smooth, so there will be some drawbacks in Strategy 1 (it will be mentioned later). Therefore, we need to find some better strategies that can match the target spectrum better.

In the response spectra calculations of wavelet packet component, it is found that there is such a phenomenon that the calculated response spectrum of wavelet packet component is significantly large. However, there is a certain bias between the frequency of response spectrum peak and the bandwidth center frequency of wavelet packet components.

Figure 2 shows the separately calculated response spectrum of the fourth component and seventh component of the decomposed Northridge wave using wavelet packet. The peak frequency of the band of wavelet packet component 4# is 5.47 Hz, while the peak of the response spectrum appears at 5.00 Hz. There is a bias of 0.47 Hz. The center frequency of the band of wavelet packet component 7# is 10.16 Hz. The peak of the response spectrum appears at 9.51 Hz. There is a bias of 0.65 Hz. Given the above observation, although there is no strict proof, but we believe that the corresponding frequency of response spectrum peak that $T_i$ selects $i$ component in Eq. (6) is more reasonable than the center frequency of the band in component $i$. When $T_i$ selects the corresponding frequency of response spectrum peak of component $i$, $\gamma_i$ is more accurate in the measure of the gap between target spectrum and count spectrum. When $T_i$ selects the center frequency of the band of component $i$, it seems there will be “overshooting” phenomenon. It requires adjustment in the next iteration.

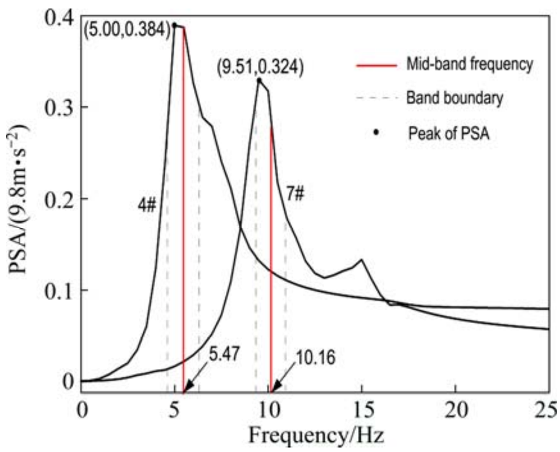


**Fig. 2** Response spectra of 4# and 7# wavelet packet component of Northridge wave (the tenth order Daubechies wavelet, $J$=4)

However, on the other hand, if a peak frequency calculation is carried out at each iteration step, a large number of response spectra calculations will greatly increase the computational time. We don’t like to see this. According to the above descriptions, we give the other four strategies selecting $T_i$. The four policies all follow the principles that $T_i$ can be close to the corresponding frequencies of peak response spectrum of wavelet packet components.

Strategy 2 (S2): Two points are added in the frequency range of component $i$. With the center frequency of the band, there are three points. Let $T_i$ be the biggest response spectrum of these three points, the process is repeated at each iteration step. Thus, $T_i$ is dynamic in each step. In order to ensure the entire frequency evenly divided by these selected points, “Pick” represents the selection process. The expression of $T_i$ is

$$T_i=\text{Pick}\left\{1/\left(\frac{2i-1}{2^{J+2}\Delta t}-\frac{1}{3\times 2^{J+1}\Delta t}\right),\ 1/\left(\frac{2i-1}{2^{J+2}\Delta t}\right),\ 1/\left(\frac{2i-1}{2^{J+2}\Delta t}+\frac{1}{3\times 2^{J+1}\Delta t}\right)\right\} \tag{7}$$

Strategy 3 (S3): Suppose the peak frequency of wavelet packet component at each iteration step is relatively stable, we just need to calculate the peak frequency for once. Subsequent iterations can directly follow the calculated peak frequency.

Strategy 4 (S4): In an iterative process, the peak frequency may change. Therefore, at the peak frequencies of the new iteration, the peak frequency should be recalculated after several times iteration. In this work the peak frequency is recalculated every five times.

Strategy 5 (S5): Let $T_i$ be a random number in the band of wavelet packet component $i$ in each iteration, then the iteration point can change in the band.

## 4 Results and discussion

This work gives numerical examples of two famous strong earthquake records. One is the Northridge wave which lasts for 30 s, and the other is the Kobe wave which lasts for 50 s. The sampling interval $\Delta t$ is 0.02 s. The sampling interval is 0.02 s. With the two strong motion records, advantages and disadvantages are compared on a given four strategies as well as the previously mentioned strategy S1. According to the actual need, the target spectrum is a curve with poor smoothness. In fact, the target spectrum is gained by superposing smooth target spectrum given in the specification on a random quantity. In this work, the response spectrum is represented in the natural frequency domain. The wavelet packet components of wavelet packet transform are equant in the frequency domain. As long as the natural frequency is divided by 2π, the abscissa expressed by frequency domain can be expressed by the cycle. In order to compare the five strategies, we select the tenth order Daubechies wavelet for decomposition and reconstruction in practice. The decomposition scale $J$=6 (it can get better fitting effect when choosing higher decomposition, e.g. $J$=7, 8; however, for comparison, this work only selects the case of $J$=6). Twenty times iterations were carried out. In order to measure the pros and cons of different strategies, we define the Root-Mean-Square of the differences in percent at each of the $N$ frequencies [7,17] to evaluate and calculate the error between response spectrum with the target response spectrum.

$$E=\sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(\frac{[g(\omega_j)]_{\text{reconstructed}}-[g(\omega_j)]_{\text{target}}}{[g(\omega_j)]_{\text{target}}}\right)^2} \tag{8}$$

Figures 3 and 4 depict the error curves of the process of the Northridge wave and the Kobe wave for matching the target spectrum under the five strategies. Figures 5 and 6 compare the response spectrum of the Northridge and Kobe waves before and after adjustment.

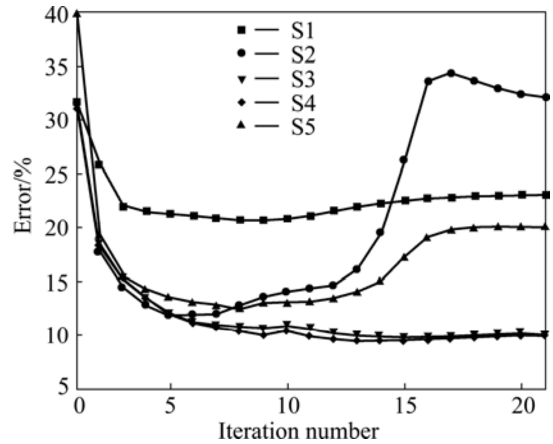


**Fig. 3** Variation of RMS error with iteration step for Northridge earthquake

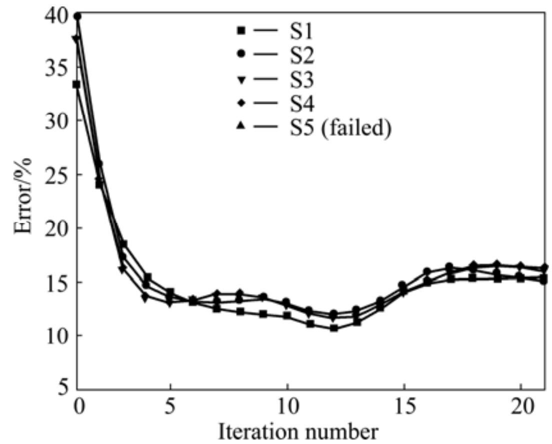


**Fig. 4** Variation of RMS error with iteration step for Kobe earthquake

Selecting Northridge wave as an original wave to adjust the response spectrum (as shown in Fig. 3), it is found that the error of strategy S1 is close to 20% at the end of the third iteration step. The error does not significantly reduce further ever after. The effect is not acceptable and even don't meet the requirements of specification [9,18,19]. The other four strategies have achieved good results in the seventh iteration step, which is far better than that of strategy S1. But the gap emerges in the resulting performance. The error of strategies S2 and S5 becomes larger in the process of further iteration. The final result is unacceptable. However, the error strategy S3 remained close to that of strategy S4, and the error of iteration slowly dropped below 10% level after 7 steps.

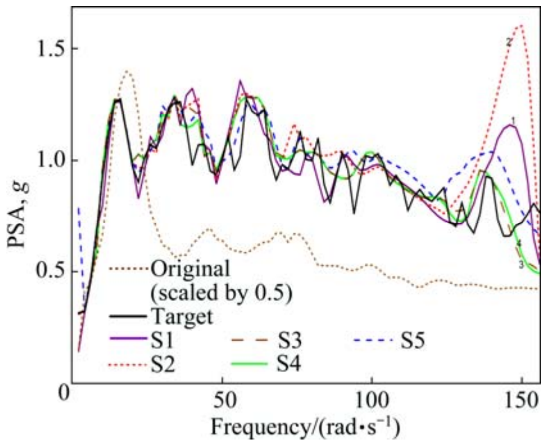


**Fig. 5** Spectra of target and modified record of Northridge earthquake by 5 strategies

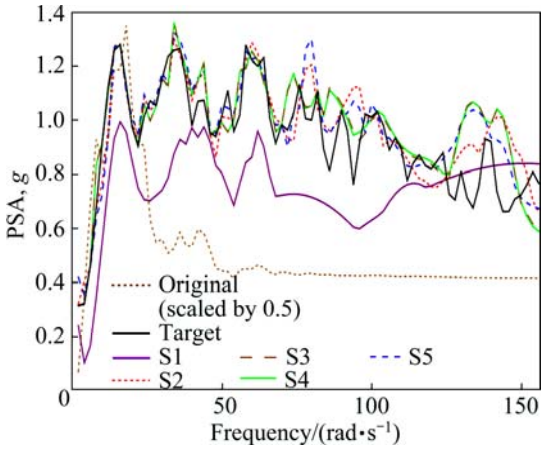


**Fig. 6** Spectra of target and modified record of Kobe earthquake by 5 strategies

Selecting Kobe as an original wave to adjust response spectrum (shown in Fig. 4), it is found that the first four strategies (S1, S2, S3 and S4) have similar performance. In the twelfth iteration step, they achieve their minimum error level (nearly 10%) at the same time. But the strategy S5 fails (the error is still unacceptable after the completion of the previous twenty iterative steps). We believe that this phenomenon is due to the random selection of iteration points in strategy S5. This randomness leads directly to the instability of the strategy. There is also unstable performance in strategy S2. Strategy S2 becomes worse in the process of the adjustment of Northridge wave after the seventh iteration step. By comprehensive consideration of the iterative adjustment process of these two waves, we believe that the strategies S3 and S4 are better. Not only can the small error level reach but also good stability in the further iteration can maintain in these two strategies. On the other hand, the performance of strategy S3 is very close to that of S4. We believe that it is a result of the stable peak frequency in each iteration step. Therefore, we can choose strategy S3. The peak frequency can be determined before the starting of iteration, and it remains the same in subsequent iterations. There is no need to spend a lot of time on repeated calculation of the peak frequency.

## 5 Conclusions

1) In the calculation of wavelet packet component, there is a certain bias between the band center frequency of the component of seismic record after wavelet packet decomposition and the peak frequency of response spectra of wavelet packet components. Five strategies of the selection of iteration point were presented.

2) Instances showed that, the strategy that chooses the peak response frequencies of wavelet packet components as the iterative points matches the target spectrum well and is stable. So, this strategy is recommended as the best choice.

3) The numerical example also confirmed that peak frequency of response spectrum is stable in the process of iteration. Therefore, the peak frequency achieved in the decomposition of signals for the first time can be selected as iteration points for the following iteration, and repeated calculation of the peak frequency could be avoided.

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# 基于小波包变换的拟合非光滑目标谱的地震波生成

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**摘 要：**地震时程分析中需要使用符合规范的目标反应谱的地震记录，而以往的拟合目标谱的方法大多是针对目标谱为光滑曲线的情况的。在许多情况下，用单一的确定的反应谱，往往需要拟合不光滑的目标谱。给出一种基于小波包变换的、能够很好地拟合非光滑目标谱的地震时程调整方法。经过观察，发现地震记录经过小波包分解后的成分的频带中心频率与小波包成分的反应谱的峰值频率存在一定的偏差。基于这个现象给出了 5 个选择迭代点的策略，通过两个计算实例比较了这 5 个策略的效果。结果表明，以小波包成分的反应谱的峰值频率作为迭代点的策略能获得很好的拟合效果，并且在迭代过程中稳定趋向收敛，能很好地适用于不光滑目标谱的拟合。

**关键词：**加速度时程；小波包变换；谱匹配；反应谱峰值频率

**(Edited by Sai-qian YUAN)**