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# Indentation of sandwich beams with metal foam core

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**Abstract:** The quasi-static indentation behavior of sandwich beams with a metal foam core was investigated. An analytical model was developed to predict the large deflections of indention of the sandwich beams with a metal foam core subjected to a concentrated loading. The interaction of plastic bending and stretching in the local deformation regions of the face sheet was considered in the analytical model. Moreover, the effects of the shear strength of the foam core on the indentation behavior were discussed in detail. The finite element simulations were preformed to validate the theoretical model. Comparisons between the analytical predictions and finite element results were conducted and good agreement was achieved. The results show that the membrane force dominates indentation behavior of the sandwich beams when the maximum deflection exceeds the thickness of the face sheet. **Key words:** indentation; sandwich beam; energy absorption; metal foam; large deflection

# **1** Introduction

Lightweight sandwich structure has been considered as an alternative to the conventional monolithic solid structure due to the high specific strength and stiffness, high resistance to impact and blast loadings. Metal foam core sandwich panels are typical lightweight structures with a number of advantages, e.g. multi-functionality, easy to shape to curved configurations and to form with integral face sheets. It is well known that there are many failure modes while local indentation is one typical mode of the failures and has received more attention due to the fact that the local indentation may cause a serous reduction in load–carrying capacity of the sandwich structure [1,2].

In earlier analyses of indentation of sandwich beam, FROSTIG et al [3,4] modeled the face sheets and the foam core as elastic beams and an elastic foundation, respectively. TRIANTAFILLOW and GIBSON [5,6] neglected the effect of the strength of face sheets while the indentation load was dictated by plastic strength of foam core. Apparently, this will underestimate the indentation load. SODEN [7] and SHUAEIB and SODEN [8] developed analytical models for indentation of the sandwich beam, in which an elastic beam was resting on a rigid-perfectly plastic foundation and an elastic-perfectly plastic foundation, respectively. Using a simple upper bound calculation, ASHBY et al [2] presented an analytical analysis of the indentation of metal foam core sandwich beams without considering the effect of shear strength of the foam core. Subsequently, KOISSIN et al [9] obtained an analytical solution for the quasi-static indentation of sandwich structure, in which they assumed that the face sheets deflect elastically and the foam core is idealized as elastic-perfectly-plastic foundation without strain rate and shear effects and overall bending. RUBINO et al [10] proposed a new analytical model for the indentation of the metal sandwich beams, in which the shear strength effect of the core on the indentation and compression of the core and bending of the face sheet are considered. However, all the aforementioned analyses are based on the assumption of small deflection.

Recently, RUBINO et al [11] modified the previous analytical model including the contribution of the stretching of the face sheet neglecting interactions between plastic bending and stretching in deformation regions. QIN and WANG [12,13] developed an analytical model for the local indentation of metal foam core

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sandwich beams transversely loaded by a flat punch considering interactions between plastic bending and stretching in deformation regions, and assumed that the deformation region length is fixed. WIERZBICKI and HOO [14] proposed a large deformation model for a plastic string resting on a plastic foundation subjected to a concentrated loading neglecting the effect of plastic bending of the face sheet. Later, this model was applied to predict the local indentation of metal foam core sandwich beam considering the size effects of indenters [15], whilst neglecting the contribution of the shear strength of the core.

The objective of this work is to study the quasi-static large deflections for the indentation of metal foam core sandwich beam subjected to a concentrated loading.

#### 2 Analytical model

Consider a metal foam core sandwich beam, which is supported on a rigid foundation, subjected to a concentrated loading *F*, as shown in Fig. 1. Two identical face sheets with thickness *h* are perfectly bonded to the core with thickness *c*. It is assumed that the face sheets obey rigid-perfectly plastic law with a flow stress  $\sigma_f$ , as shown in Fig. 2(a). The foam core is modeled as a rigid perfectly-plastic locking (r-p-p-l) material with a plateau-stress level of  $\sigma_c$  and shear stress  $\tau_c$ , as shown in Fig. 2(b). The rigid-perfectly plastic model and r-p-p-l model are useful to describe the post-yield deformation behavior of sandwich structures approximately, respectively, if elastic deformation and hardening behavior are neglected in analysis.

Herein, an analytical model is proposed to characterize the local denting deformation of sandwich

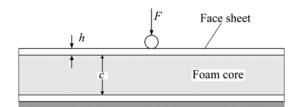


Fig. 1 Sketch of metal foam sandwich beam subjected to concentrated loading

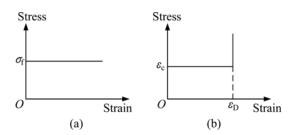


Fig. 2 Idealized strain-stress curves: (a) Face sheet; (b) Foam core

beam as the indentation depth becomes large. Then, the upper face sheet is idealized as a homogenous rigid-perfectly plastic beam, while the foam core is idealized as a perfectly plastic foundation [16]. The assumed deformation mode for the indentation of the metal foam core sandwich beam subjected to a concentrated loading is shown in Fig. 3. The deformation field is then expressed as [11]

$$u(x_1, x_3) = 0 (1)$$

and

$$v(x_1, x_3) = \begin{cases} \frac{\dot{\theta}}{c} (\lambda - x_1) x_3, & 0 \le x_1 \le \lambda \\ \frac{\dot{\theta}}{c} (\lambda + x_1) x_3, & -\lambda \le x_1 \le 0 \end{cases}$$
(2)

where *u* and *v* are material point velocities in the  $x_1$  and  $x_3$  directions, respectively;  $\lambda$  is the half length of the deformation region of the top face sheet that has a rotation rate  $\dot{\theta}$  giving the roller a displacement rate  $\dot{w}_0 = \lambda \dot{\theta}$  at loading point.

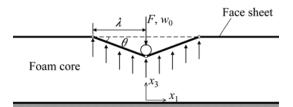


Fig. 3 Assumed deformation mode for indentation of metal foam core sandwich beam

To determine the load-deflection behavior after the collapse, the principle of virtual work is applied during the plastic deformation. The dissipation rate of plastic work is then given by

$$F\lambda\dot{\theta} = 4M\dot{\theta} + 2N\dot{e} + \int_{A}\sigma_{c}\dot{\varepsilon}_{33}dx + \int_{A}\tau_{c}\dot{\gamma}_{13}dx$$
(3)

where *M* and *N* are the plastic bending moment and axial force, respectively;  $\dot{e}$  is the extension rate of the deformation region of the upper face sheet;  $\dot{e}_{33}$  is the strain rate of the foam core in  $x_3$  direction;  $\dot{\gamma}_{13}$  is the shear rate of the foam core in  $x_3$  direction. The following relations also emerge

$$\dot{e} = \frac{1}{\lambda} w_0 \dot{w}_0 \tag{4}$$

where  $w_0$  is the maximum deflection under loading point.

$$\dot{\varepsilon}_{33} = \frac{\partial v}{\partial x_3} = \begin{cases} \frac{\dot{\theta}}{c} (\lambda - x_1), & 0 \le x_1 \le \lambda \\ \frac{\dot{\theta}}{c} (\lambda + x_1), & -\lambda \le x_1 \le 0 \end{cases}$$
(5)

and

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$$\dot{\gamma}_{13} = \frac{\partial v}{\partial x_1} + \frac{\partial u}{\partial x_3} = \begin{cases} -\frac{\theta}{c} x_3, \ 0 \le x_1 \le \lambda \\ \frac{\dot{\theta}}{c} x_3, \ -\lambda \le x_1 \le 0 \end{cases}$$
(6)

Due to the characteristics of the beam-onfoundation model, the axial (membrane) force would be induced by the finite deflections of upper face sheet. Moreover, the axial (membrane) force would dissipate energy and stiffen the deformation portion of upper face sheet. The plastic deformation of the upper face sheet mainly comes from plastic bending and longitudinal stretching. ONAT and PRAGER [17] obtained the yield criterion in space (M, N) for the upper face sheet, i.e.

$$\left|m\right| + n^2 = 1\tag{7}$$

where  $m=M/M_0$  and  $n=N/N_0$ .

Meanwhile, the fully plastic bending moment  $M_0$ and axial force  $N_0$  in the upper face sheet are respectively given by

$$M_0 = \frac{1}{4}\sigma_{\rm f}h^2, \quad N_0 = \sigma_{\rm f}h = \frac{4M_0}{h}$$
 (8)

The yield surface of the face sheet is shown in Fig. 4.

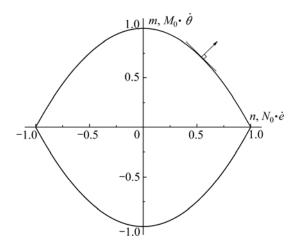


Fig. 4 Yield surface of face sheet of sandwich beam

According to the associated flow rule for yield criterion Eq. (7), there is

$$\frac{\mathrm{d}m}{\mathrm{d}n} = -\frac{2\dot{e}}{h\dot{\theta}} = -2n\tag{9}$$

i.e.

$$\dot{e} = \dot{\theta}hn = \frac{1}{\lambda}hn\dot{w}_0 \tag{10}$$

Substituting Eq. (10) into Eq. (4), and solving for *n* for  $w_0 \le h$ ,

$$n = \frac{w_0}{h} \tag{11}$$

Combining Eqs. (7) and (11) results in

$$|m| = 1 - \left(\frac{w_0}{h}\right)^2 \tag{12}$$

When  $w_0 \ge h$ , we have

$$n=1 \text{ and } m=0$$
 (13)

The load-deflection of the local denting deformation of the sandwich beam can be calculated as follows.

1) When  $w_0 \le h$ , combining Eqs. (3)–(12) yields

$$F = \frac{\sigma_{\rm f} h^2}{\lambda} \left[ 1 + \left(\frac{w_0}{h}\right)^2 \right] + \sigma_{\rm c} \lambda + \tau_{\rm c} c \tag{14}$$

The local deformation region  $\lambda$  can be obtained by minimizing *F* in Eq. (14) with respect to the free parameter  $\lambda$ , i.e.  $\partial F/\partial \lambda = 0$ . That is

$$\lambda = h \sqrt{\frac{\sigma_{\rm f}}{\sigma_{\rm c}}} \left[ 1 + \left(\frac{w_0}{h}\right)^2 \right] \tag{15}$$

When  $w_0=0$ , the initial wavelength  $\lambda_I$  of the local denting deformation of the sandwich beam is expressed as

$$\lambda_{\rm I} = h \sqrt{\frac{\sigma_{\rm f}}{\sigma_{\rm c}}} \tag{16}$$

Substituting Eq. (15) into Eq. (14) yields, the following load-deflection relationship of local denting deformation

$$F = 2h \sqrt{\sigma_{\rm f} \sigma_{\rm c} \left[1 + \left(\frac{w_0}{h}\right)^2\right]} + \tau_{\rm c} c \tag{17}$$

When  $w_0=0$ , the initial loading  $F_1$  of local denting deformation of the sandwich beam is written as

$$F_{\rm I} = 2h\sqrt{\sigma_{\rm f}\sigma_{\rm c}} + \tau_{\rm c}c \tag{18}$$

2) When  $w_0 \ge h$ , the expression for the load-deflection relationship of local denting deformation is obtained from Eqs. (3) and (13). So,

$$F = \frac{2\sigma_{\rm f} h w_0}{\lambda} + \sigma_{\rm c} \lambda + \tau_{\rm c} c \tag{19}$$

Minimization of the loading *F* in Eq. (19) with respect to the free parameter  $\lambda$ , i.e.  $\partial F/\partial \lambda = 0$ , gives a wavelength expression,

$$\lambda = h \sqrt{\frac{2\sigma_{\rm f}}{\sigma_{\rm c}} \left(\frac{w_0}{h}\right)} \tag{20}$$

When  $w_0 = h$ , Eq. (20) is reduced to

$$\lambda_{\rm h} = h \sqrt{\frac{2\sigma_{\rm f}}{\sigma_{\rm c}}} \tag{21}$$

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which is the same as the formula obtained from Eq. (15) by setting  $w_0=h$ . Substituting Eq. (20) into Eq. (19), the expression of the load-deflection relationship of local denting deformation is

$$F = 2h\sqrt{2\sigma_{\rm f}\sigma_{\rm c}\frac{w_0}{h}} + \tau_{\rm c}c$$
<sup>(22)</sup>

If the contribution of the shear strength of the foam core to the load–carrying ability is too small to be neglected in analysis, Eqs. (17) and (22) are reduced to

$$F = 2h \sqrt{\sigma_{\rm f} \sigma_{\rm c} \left[ 1 + \left(\frac{w_0}{h}\right)^2 \right]}, \ w_0 \le h$$
(23)

and

$$F = 2h\sqrt{2\sigma_{\rm f}\sigma_{\rm c}\frac{w_0}{h}}, \ w_0 \ge h \tag{24}$$

respectively, and Eq. (18) is reduced to

$$F_{10} = 2h\sqrt{\sigma_{\rm f}\sigma_{\rm c}} \tag{25}$$

Moreover, it is concluded from Eqs. (15) and (20) that the local denting deformation region increases monotonically with the increase of the deflection of the local denting deformation. In other words, two plastic hinges at a distance  $\lambda_{I}$  from the loading point travel towards in the directions  $x_{1}$  and  $(-x_{1})$  with the increase of the deflection of the local denting deformation.

If  $c\varepsilon_D \leq h$ , the local denting deformation behavior will be determined by Eqs. (15) and (17). The maximum loading  $F_{\text{max}}$  and the local denting deformation region  $\lambda_{\text{max}}$  are given by

$$F_{\rm max} = 2h_{\rm v} \sqrt{\sigma_{\rm f} \sigma_{\rm c} \left[1 + \left(\frac{c\varepsilon_{\rm D}}{h}\right)^2\right]} + \tau_{\rm c} c \qquad (26)$$

and

$$\lambda_{\max} = h \sqrt{\frac{\sigma_{\rm f}}{\sigma_{\rm c}} \left[ 1 + \left(\frac{c\varepsilon_{\rm D}}{h}\right)^2 \right]}$$
(27)

If  $c\varepsilon_D \ge h$ , the local denting deformation behavior will be determined by Eqs. (20) and (22). It is assumed that the upper face sheet deflects until the average compressive strain in the foam core attains the densification strain of the foam core while there is no a sudden loss of load-carrying capacity due to the upper face sheet tearing. The maximum loading  $F_{\text{max}}$  and the local denting deformation region  $\lambda_{\text{max}}$  are obtained as

$$F_{\rm max} = 2h \sqrt{2\sigma_{\rm f}\sigma_{\rm c}} \frac{c\varepsilon_{\rm D}}{h} + \tau_{\rm c}c$$
<sup>(28)</sup>

and

$$\lambda_{\max} = h \sqrt{\frac{2\sigma_{\rm f}}{\sigma_{\rm c}} \left(\frac{c\varepsilon_{\rm D}}{h}\right)}$$
(27)

Upon performing the integration Eq. (28) by using Eqs. (17) and (22), the absorbed plastic energy  $U_i$  induced by the local denting deformation are obtained as

$$U_i = \int_0^{w_0} F(w_0) \, \mathrm{d}w_0 \tag{30}$$

or

$$U_{i} = h^{2} \sqrt{\sigma_{f} \sigma_{c}} \left[ \frac{w_{0}}{h} \sqrt{1 + \left(\frac{w_{0}}{h}\right)^{2}} + \ln \left(\frac{w_{0}}{h} + \sqrt{1 + \left(\frac{w_{0}}{h}\right)^{2}}\right) \right] + \tau_{c} ch \frac{w_{0}}{h}, w_{0} \le h$$
(31)

and

$$U_{i} = h^{2} \sqrt{\sigma_{f} \sigma_{c}} \left[ \frac{4}{3} \frac{w_{0}}{h} \sqrt{\frac{2w_{0}}{h}} + \ln(1 + \sqrt{2}) - \frac{\sqrt{2}}{3} \right] + \tau_{c} ch \frac{w_{0}}{h}, w_{0} \ge h$$

$$(32)$$

If the contribution of the shear strength of the foam core to the energy absorption is too small to be neglected in analysis, Eqs. (31) and (32) then are reduced to

$$U_{i} = h^{2} \sqrt{\sigma_{f} \sigma_{c}} \left[ \frac{w_{0}}{h} \sqrt{1 + \left(\frac{w_{0}}{h}\right)^{2}} + \ln \left(\frac{w_{0}}{h} + \sqrt{1 + \left(\frac{w_{0}}{h}\right)^{2}}\right) \right],$$

$$w_{0} \le h$$
(33)

and

$$U_{i} = h^{2} \sqrt{\sigma_{f} \sigma_{c}} \left[ \frac{4}{3} \frac{w_{0}}{h} \sqrt{\frac{2w_{0}}{h}} + \ln(1 + \sqrt{2}) - \frac{\sqrt{2}}{3} \right], w_{0} \ge h$$
(34)

If  $c\varepsilon_D \leq h$ , the maximum absorbed energy  $U_{i-\max}$  of the local denting deformation is

$$U_{i-\max} = h^2 \sqrt{\sigma_{\rm f} \sigma_{\rm c}} \left[ \frac{c\varepsilon_{\rm D}}{h} \sqrt{1 + \left(\frac{c\varepsilon_{\rm D}}{h}\right)^2} + \ln\left(\frac{c\varepsilon_{\rm D}}{h} + \sqrt{1 + \left(\frac{c\varepsilon_{\rm D}}{h}\right)^2}\right) \right] + \tau_{\rm c} ch \frac{c \cdot \varepsilon_{\rm D}}{h}, w_0 \le h \quad (35)$$

Similarly, if  $c\varepsilon_D \ge h$ , the maximum absorbed energy  $U_{i-\max}$  of the local denting deformation is

$$U_{i-\max} = h^2 \sqrt{\sigma_{\rm f} \sigma_{\rm c}} \left[ \frac{4}{3} \frac{c\varepsilon_{\rm D}}{h} \sqrt{\frac{2c\varepsilon_{\rm D}}{h}} + \ln\left(1 + \sqrt{2}\right) - \frac{\sqrt{2}}{3} \right] + \tau_{\rm c} ch \frac{c\varepsilon_{\rm D}}{h}, \ w_0 \ge h$$
(36)

It is clear that the plastic energy during local indentation deformation will be dissipated by the bending and stretching of the upper face sheet and the Qing-hua QIN, et al/Trans. Nonferrous Met. Soc. China 24(2014) 2440-2446

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compression and shear deformation of the foam core. Actually, there is an implicit assumption in analysis that the deformed region of the upper face sheet during local indentation is much smaller than the span of the metal sandwich beam. It means that the sandwich beam should be long enough to ensure the occurring of local indentation. Otherwise, other failure modes are possible.

#### **3** Finite element analysis

Using ABAQUS code, finite element (FE) calculations were carried out to numerically study the finite deflection behavior of indentation of metal foam core sandwich beam under transversely concentrated loading supported on a rigid foundation. Standard integration solution technique was adopted in calculations. Displacement loading was applied to rigid roller. Contact between the outer surface of the upper face sheet and the rigid roller was modeled through frictionless contact provided by ABAQUS code. Four node-bilinear plane strain quadrilateral elements (type CPE4) with full integration were selected for both foam core and face sheets. All calculations in this section are presented for sandwich beam with the span length 2L=300 mm, thickness of each face sheet h=0.5 mm and 1.0 mm, and thickness of the core c=25 mm. The diameter of the loading roller is d=8 mm. A mesh sensitivity result demonstrates that further mesh refinements do not change apparently the calculation results.

Due to the symmetry of the problem, only half of the sandwich beam was considered in FE calculations. Symmetry boundary conditions were imposed at the midspan of the sandwich beam. The face sheets were assumed to be perfectly bonded to foam core. Appropriate mesh refinement at the location of loading point of the sandwich beam was included. The face sheets were modeled by  $J_2$  flow theory of plasticity. Deshpande-Fleck model [18] was used to model the metal foam core, in which variations of yield surface due to hardening along hydrostatic and deviatoric axes were considered. The yield function for the foam core is

$$\Psi = \hat{\sigma} - \sigma_{\rm c} = 0 \tag{37}$$

where

$$\hat{\sigma}^{2} = \frac{1}{1 + (\gamma/3)^{2}} \left( \sigma_{\rm e}^{2} + \gamma^{2} \sigma_{\rm m}^{2} \right)$$
(38)

where  $\sigma_{\rm e} \equiv \sqrt{3s_{ij}s_{ij}/2}$  is von Mises stress,  $s_{ij}$  is deviatoric stress,  $\sigma_{\rm m} \equiv \sigma_{\rm kk}/3$  is hydrostatic stress and  $\gamma$ is the shape factor of yield surface. The associated plastic flow assumed and plastic Poisson is ratio  $v_{\rm p} = -\dot{\varepsilon}_{22}^p / \dot{\varepsilon}_{11}^p$ for compression uniaxial in one-direction is

$$r_{\rm p} = \frac{1/2 - (\gamma/3)^2}{1 + (\gamma/3)^2}$$
(39)

Considering aluminum face sheets with yield strength  $\sigma_f$ =140 MPa, elastic modulus  $E_f$ =70 GPa, elastic Poisson ratio  $v_{ef}$ =0.3 and linear hardening modulus  $E_{tf}$ =0.01 $E_f$ . The isotropic aluminum foam core has yield strength  $\sigma_c$ =3 MPa, elastic modulus  $E_{cf}$ =1.0 GPa, elastic Poisson ratio  $v_{ec}$ =0.3 and plastic Poisson ratio  $v_p$ =0. The foam core has long plateau stress platform  $\sigma_c$  that continues up to densification strain  $\varepsilon_D$ =0.5, and the stress rises steeply when very large tangent modulus  $E_{cf}$ =0.2 $E_f$ is beyond the densification strain.

## 4 Results and discussion

Comparisons between analytical predictions and FE results of load-deflection and deflection-energy curves of the local indentation of the metal foam core sandwich beams subjected to a concentrated loading are plotted in Figs. 5 and 6, respectively. The mean plastic flow stress

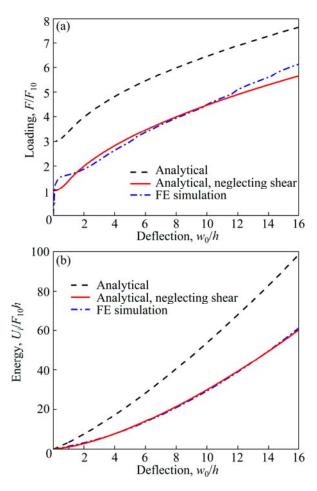


Fig. 5 Indentation of metal foam core sandwich beams subjected to concentrated loading with face sheet thickness h=0.5 mm: (a) Normalized deflection versus normalized loading; (b) Normalized deflection versus normalized energy

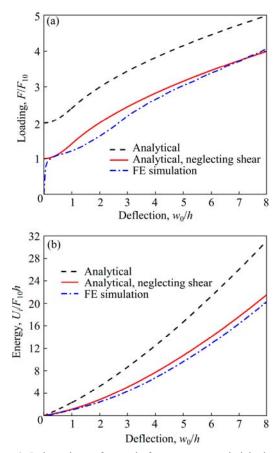
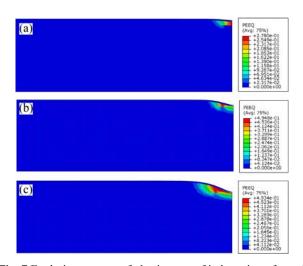


Fig. 6 Indentation of metal foam core sandwich beams subjected to concentrated loading with face sheet thickness h=1.0 mm: (a) Normalized deflection versus normalized loading; (b) Normalized deflection versus normalized energy

 $(\sigma_{\rm f}+\sigma_{\rm u})/2$  (the ultimate tensile strength  $\sigma_{\rm u}=210$  MPa and the rupture strain  $\varepsilon_u=20\%$  for the upper face sheet) is used in analytical model to estimate the effect of strain hardening of the upper face sheet. It is seen readily that the analytical predictions neglecting the effect of foam core shear strength are in excellent agreement with the FE results of deflection-loading and deflection-energy curves, whereas the analytical model considering the effect of core shear strength may overestimate the load-carrying capacity and energy absorption of metal sandwich beam, in which the shear strength of the foam core  $\tau_c$  may be taken as  $\tau_c \approx 2\sigma_c/3$ . The contributions of the shear strength of the foam core can be too small to be neglected due to its low shear strength. However, it may be included in the analytical model when the shear strength of the core significantly exceeds its compressive strength.

Figure 7 shows the evolution process of the plastic zone of the local indentation in the foam core. Contours of the equivalent plastic strain (PEEQ) are included in these deformed profiles also. At small deflections of the upper face sheet, the plastic zone is firstly developed near the indenter and increases with the increase of the deflections of the upper face sheet. Meanwhile, the deformation zone of the upper face sheet travels towards the longitudinal directions. The maximum plastic zone in the foam core is near the indenter while the foam core undergoes almost no deformation in the region far away



**Fig. 7** Evolution process of plastic zone of indentation of metal sandwich beams with face sheet thickness h=0.5 mm deduced from FE simulations with different indentations: (a) 0.5 mm; (b) 2.0 mm; (c) 4.0 mm

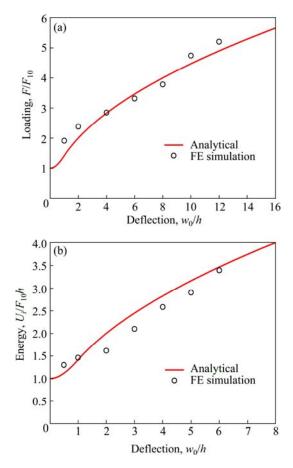


Fig. 8 Deformation zone of indentation of metal foam core sandwich beams subjected to concentrated loading: (a) h=0.5 mm; (b) h=1.0 mm

from the indenter. As can be seen from Fig. 8, the analytical predictions for deformation zone of local indentation of metal sandwich beams agree well with the FE results. Moreover, the deformation zone of local indentation is insensitive to the shear strength of the core.

#### **5** Conclusions

An analytical model was developed to predict the large deflections of indentation of metal foam core sandwich beam subjected to a concentrated loading. In the analysis the upper face sheet was modeled as an infinitely long, rigid plastic bean resting on a rigid plastic foundation which was used to model the constant crushing behavior by the foam core. Moreover, both interaction of plastic bending and stretching in the local deformation regions of the face sheet and contribution of the core shear strength were considered. The closed-form solutions are obtained. Comparisons between the present analytical solutions with the FE results demonstrate that it is necessary to consider the interaction of plastic bending and stretching in the local deformation regions, and the membrane force plays an important role in large deflections of the indentation of the sandwich beams when the maximum deflection exceeds the thickness of the face sheet. The contribution of the shear strength of the foam core with low shear strength may be neglected in analysis.

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# 金属泡沫夹芯梁的压入力学行为

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**摘** 要:研究集中载荷作用下金属泡沫夹芯梁的准静态压入力学行为。考虑到夹芯梁表板弯曲和拉伸的相互作用, 提出了一个金属泡沫夹芯梁局部压入大挠度变形行为的新理论模型,并详细讨论芯材剪切对局部压入行为的影 响。为了验证理论模型的有效性,对夹芯梁的局部压入行为进行有限元数值模拟,理论预测结果与有限元模拟结 果吻合得很好。结果表明:当压入挠度超过夹芯梁表板厚度后,塑性膜力控制着夹芯梁的局部压入行为。 关键词:压入;夹芯梁;能量吸收;金属泡沫;大挠度