

Flow equation and similarity criterion during centrifugal casting in micro-channel

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Abstract: Liquid metal filling flow process in the microscale during the centrifugal casting process was studied by means of similar physical simulation. The research was focused on derived similarity criterion. Based on the traditional flow equations, the flow equation and the Bernoulli's equation for liquid metal flows in micro-scale space were derived, which provides a mathematical model for numerical simulation of micro-scale flow. In the meanwhile, according to the micro-flow equation and the similarity theory, the similarity criterion for the physical simulation of the mold filling behaviors was presented under centrifugal force field, so as to achieve the visual observation and quantitative analysis of micro-flow process.

Key words: micro-channel; micro-flow; centrifugal force field; similarity criterion

1 Introduction

In recent years, research on the micro precision casting technology has attracted much attention globally [1–5]. By means of the centrifugal casting process, BAUMEISTER et al [1,2] successfully produced micro-scale gears and YANG et al [6] produced micro-rod with a minimum diameter of 100 μm and a aspect ratio of 200. In the micro-casting process, many micro-scale effects, which are normally ignored in traditional casting process, such as surface force and size effects, are highlighted. This could lead to different flow behaviors in micro-channels compared with that in the macro-scale channel. The previous studies on the room temperature micro-scale flow principles of the fluids, such as water, oil and gas, have shown that micro-scale fluid flow was different from that at conventional macro-scale [7–9]. The flow behavior of liquid metal has a significant impact on process design and product quality of micro-casting. Therefore, it is of great importance to understand the flow behavior of liquid metal in the micro-scale cavity, which is highly favorable for improving the product quality of the micro-casting

process [10]. At present, numerical simulation method has been widely used in the study of mold filling behavior in macro-scale centrifugal casting [11–14]. However, the numerical simulation can provide only qualitative analysis, but not visual observations. The physical simulation method based on the similarity theory was developed [11,15], to achieve both visualization and quantitative analysis.

Nevertheless, there is a lack of understanding about the liquid metal flow behaviors in the micro-scale cavity during the novel micro-scale casting process. In the present work, based on the flow principles for traditional casting process and the physical condition of micro-casting process, the liquid metal flow equations and the Bernoulli equations for the micro-scale centrifugal casting process are derived. These equations could provide scientific basis for the research on the visualized physical simulation.

2 Liquid flow equation of micro-scale space in centrifugal force

In micro-casting process, the liquid metal flow behavior is equivalent to that of non-isothermal and

non-steady-state Newtonian fluid. Due to the scale effect, the flow behavior shows the following characteristics: more pronounced surface force (surface tension and gas counterpressure, etc.) effects, less mold heat storage capacity, increased cooling rates of liquid metal, increased viscosity of the melt. In this work, gypsum is used as the mold material, and the mold is preheated. The following assumptions are made in the current model:

- 1) The liquid metal flow is continuous in the whole flow process;
- 2) The liquid metal is incompressible;
- 3) The dynamic coefficient of viscosity of the liquid metal is constant;
- 4) The surface tension coefficient does not change with temperature.

2.1 Liquid metal flow equation of macroscopic scale under gravitational field

The Navier–Stokes (N–S) momentum equations are used to model the macro-scale mold filling behavior of the liquid metal under gravity field:

$$\rho \frac{Du}{Dt} = \rho F_M - \nabla p + \mu \nabla^2 u \quad (1)$$

where u is the velocity; ρF_M is the mass force applied on a unit volume of fluid; μ is the dynamic viscosity coefficient; p is the pressure of a point (x , y , and z) in the flow field.

2.2 Liquid metal flow equation of microscopic scale under centrifugal force field

With the effects of surface force and viscosity change taken into consideration, a modified N–S equation for mold filling behavior of metal flow in micro-scale casting was developed in Ref. [16]:

$$\begin{aligned} u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} = & -\frac{1}{\rho} \frac{\partial p}{\partial x} + g_x + \\ & \mu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right) + \frac{2}{\rho} \frac{\partial \mu}{\partial T} \left[\frac{\partial T}{\partial x} \frac{\partial u_x}{\partial x} \right] - \\ & \frac{1}{\rho} \frac{\partial p_a}{\partial x} - \frac{1}{\rho} \frac{\partial p_\sigma}{\partial x} \end{aligned} \quad (2)$$

$$\begin{aligned} u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} = & -\frac{1}{\rho} \frac{\partial p}{\partial y} + g_y + \\ & \mu \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right) + \frac{2}{\rho} \frac{\partial \mu}{\partial T} \left[\frac{\partial T}{\partial y} \frac{\partial u_y}{\partial y} \right] - \\ & \frac{1}{\rho} \frac{\partial p_a}{\partial y} - \frac{1}{\rho} \frac{\partial p_\sigma}{\partial y} \end{aligned} \quad (3)$$

$$\begin{aligned} u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} = & -\frac{1}{\rho} \frac{\partial p}{\partial z} + g_z + \\ & \mu \left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right) + \frac{2}{\rho} \frac{\partial \mu}{\partial T} \left[\frac{\partial T}{\partial z} \frac{\partial u_z}{\partial z} \right] - \\ & \frac{1}{\rho} \frac{\partial p_a}{\partial z} - \frac{1}{\rho} \frac{\partial p_\sigma}{\partial z} \end{aligned} \quad (4)$$

where p_a is the gas counterpressure of the flow front; p_σ is the capillary forces between liquid metal and mold; g_x , g_y , and g_z are the components of the volume force (gravity) in the x -, y -, z -direction, respectively.

The above equations indicate that the influence of the surface tension and the gas counterpressure cannot be neglected in micro-scale casting. Based on Eqs. (2)–(4), the N–S differential equations for liquid metal flow under centrifugal force have been developed.

Due to the assumption of constant viscosity (assumption 3), the 4th term on the right hand side of Eqs. (2)–(4) can be ignored. Also, the high permeability of gypsum has led to the gas counterpressure term (the 5th item on the right hand side of Eqs. (2)–(4)) to be ignored. Therefore, the modified Eqs. (2)–(4) can be obtained:

$$\begin{aligned} u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} = & -\frac{1}{\rho} \frac{\partial p}{\partial x} + g_x + \\ & \mu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial p_\sigma}{\partial x} \end{aligned} \quad (5)$$

$$\begin{aligned} u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} = & -\frac{1}{\rho} \frac{\partial p}{\partial y} + g_y + \\ & \mu \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial p_\sigma}{\partial y} \end{aligned} \quad (6)$$

$$\begin{aligned} u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} = & -\frac{1}{\rho} \frac{\partial p}{\partial z} + g_z + \\ & \mu \left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial p_\sigma}{\partial z} \end{aligned} \quad (7)$$

where g_x , g_y and g_z are the components of the volume force (gravity, centrifugal force and Coriolis force) in the x -, y -, z -direction, respectively, and the other defined as before.

In the micro-scale vertical centrifugal casting process, the liquid particles subject to centrifugal force and thus move outwards from the center; in the meanwhile, the liquid particles subject to the Coriolis force. In the micro-mold cavity, due to the short moving distance on z -axis, less than 500 μm , the effect of gravity field in z -direction on the centrifugal flow can be

neglected. Therefore, in centrifugal casting process, mold filling process is achieved by the combined effects of centrifugal force and the Coriolis force. The force conditions are shown in Fig. 1.

$$F = F_{ce} + F_{co} = m(a_{ce} + a_{co}) = m(\omega^2 R + 2\omega v_r) \quad (8)$$

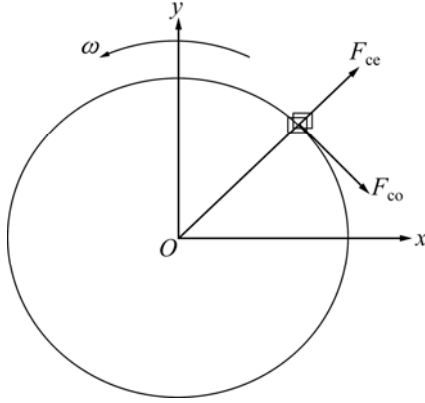


Fig. 1 Force analysis of liquid units in centrifugal field

where F_{ce} and a_{ce} are the centrifugal force, and the centrifugal acceleration, respectively; F_{co} and a_{co} are the Coriolis force, and the Coriolis acceleration, respectively; ω is angular velocity; v_r is the radial movement speed of fluid; R is circular motion radius of fluid; m is the fluid mass.

Through the above analysis, Eqs. (5)–(7) become:

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + g_x + \mu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial p_\sigma}{\partial x} \quad (9)$$

$$u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + g_y + \mu \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial p_\sigma}{\partial y} \quad (10)$$

where g_x , g_y are components of the body forces in the two coordinate axes ($g_x = \omega^2 r_x + 2\omega v_r$, $g_y = \omega^2 r_y - 2\omega v_r$).

Therefore, in the micro-scale centrifugal casting, the equations for the liquid metal flow (N–S equation) were transformed into Eqs. (9)–(10) with the influence of surface tension, centrifugal and Coriolis forces integrated into conventional N–S equations.

2.3 Bernoulli equation in micro-scale space under centrifugal force field

Bernoulli equation for incompressible fluid with constant flow centrifugal force field can be expressed as

$$\frac{p}{\rho g} + \frac{u^2}{2g} + z - \frac{\omega^2 r^2}{2g} = C \quad (11)$$

where C is constant in the each relative streamline.

The mechanical energy per unit weight on a section of the liquid particles in a micro-streamline can be expressed as follows:

$$e = z + \frac{p}{\rho g} + \frac{u^2}{2g} - \frac{\omega^2 r^2}{2g} \quad (12)$$

In the micro-channels of the centrifugal casting process, the potential energy on the vertical direction can be neglected, the fluid flow varies gradually, and the pressure is the hydrostatic pressure. The total fluid energy of the liquid metal micro stream with the weight of $dG = \rho g u dA$ flowing through an effective cross section can be expressed as

$$dE = edG = \left(\frac{p}{\rho g} + \frac{u^2}{2g} - \frac{\omega^2 r^2}{2g} \right) \rho g u dA \quad (13)$$

The total energy of the liquid flowing through the effective cross section in a unit time can be expressed as

$$E = \int_A dE = \int_A \left(\frac{p}{\rho g} + \frac{u^2}{2g} - \frac{\omega^2 r^2}{2g} \right) \rho g u dA \quad (14)$$

When Eq. (14) is divided by $G = \rho g Q$ (the fluid weight of total flow effective section), the average unit energy of specified section can be obtained:

$$\bar{e} = \frac{E}{\rho g Q} = \frac{1}{\rho g Q} \int_A \left(\frac{p}{\rho g} + \frac{u^2}{2g} - \frac{\omega^2 r^2}{2g} \right) \rho g u dA \quad (15)$$

Under centrifugal force field, the pressure distribution on effective section is uneven, thus the pressure on the wall surface near the opposite rotation direction is $p + \Delta p$. Due to the constant rotating speed, the pressure is considered to increase linearly from the front-wall to rear-wall, namely the pressure increases from p to $p + \Delta p$. Thus on the effective section, the effective pressure can be expressed as $p' = p + \Delta p/2$, and Δp can be expressed as

$$\Delta p = 2\rho\omega uL \quad (16)$$

where L is the actual flow width.

Therefore, the pressure term in Eq. (15) becomes $(p + \rho\omega uL)/\rho g$, and centrifugal force term does not pass through the channel cross section. By rewriting Eq. (15), we have

$$\bar{e} = \frac{p}{\rho g Q} \int_A u dA + \frac{\omega L}{g Q} \int_A u^2 dA + \frac{1}{2g Q} \int_A u^3 dA -$$

$$\frac{\omega^2 r^2}{2gQ} \int_A u dA \quad (17)$$

Since the velocity distribution is uneven on an effective section, the average flow speed v can be used to replace the actual flow speed u , the deviation between the actual flow speed and average flow speed is Δu , then

$$u = v + \Delta u \quad (18)$$

By incorporating and rearranging Eqs. (17) and (18), we find

$$\frac{1}{2gQ} \int_A u^3 dA = \frac{1}{2gQ} v^3 A \left(1 + 3 \frac{\int_A \Delta u^2 dA}{v^2 A} \right) \quad (19)$$

$$\frac{\omega L}{gQ} \int_A u^2 dA = \frac{\omega L}{gQ} v^2 A \left(1 + \frac{\int_A \Delta u^2 dA}{v^2 A} \right) \quad (20)$$

We stated that

$$\alpha = 1 + 3 \frac{\int_A \Delta u^2 dA}{v^2 A},$$

$$\beta = 1 + \frac{\int_A \Delta u^2 dA}{v^2 A}.$$

According to Eq.(20), we obtain the specific mechanical energy of a gradually varied flow section in the total flow:

$$\bar{e} = \frac{p}{\rho g} + \frac{\alpha v^2}{2g} + \frac{\beta \omega L v}{2g} - \frac{\omega^2 r^2}{2g} \quad (21)$$

Bernoulli equation for the two flow section is

$$\frac{p_1}{\rho g} + \frac{\alpha_1 v_1^2}{2g} + \frac{\beta_1 \omega L v_1}{2g} - \frac{\omega^2 r^2}{2g} = \frac{p_2}{\rho g} + \frac{\alpha_2 v_2^2}{2g} + \frac{\beta_2 \omega L v_2}{2g} - \frac{\omega^2 r^2}{2g} \quad (22)$$

Equation (22) is the Bernoulli equation for the fluid filling in micro-scale mold filling under the centrifugal force field. It shows that the total energy of the liquid remains constant, namely, the sum of the potential energy, pressure energy, translational kinetic energy and centrifugal rotational kinetic energy keeps constant.

3 Similarity criterion of liquid metal centrifugal filling flow

Due to the small dimension of micro-casting mold, conventional experimental methods cannot be used for the real-time data acquisition during the casting process. Moreover, the mold filling behavior affects the design of micro-casting system and the quality of micro-casting

products. Therefore, details about the mold filling behavior should be studied. Physical simulation techniques can be used to achieve process visualization cost effectively. In order to achieve similarity between the model and the prototype liquid, the similarity theories require not only the mechanics similarity (including geometric, kinematic and dynamic similarity) to be satisfied, but also certain similarity criteria to be fulfilled. The development of similarity criterion for centrifugal filling flow process is presented.

Equations (9) and (10) show the dynamic equations for the metal filling behavior during the micro-centrifugal casting process. According to similarity theories, the following equations should be followed between the prototype and the modelling liquid metal:

$$\frac{\lambda_u^2}{\lambda_l} \left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} \right) = - \frac{\lambda_p}{\lambda_\rho \lambda_l} \frac{1}{\rho} \frac{\partial p}{\partial x} +$$

$$\lambda_\omega^2 \lambda_l \omega^2 r_x^2 + 2 \lambda_\omega \lambda_u \omega v_r +$$

$$\frac{\lambda_\mu \lambda_u}{\lambda_\rho \lambda_l^2} \mu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right) - \frac{\lambda_{p_\sigma}}{\lambda_\rho \lambda_l} \frac{1}{\rho} \frac{\partial p_\sigma}{\partial x} \quad (23)$$

$$\frac{\lambda_u^2}{\lambda_l} \left(u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} \right) = - \frac{\lambda_p}{\lambda_\rho \lambda_l} \frac{1}{\rho} \frac{\partial p}{\partial y} +$$

$$\lambda_\omega^2 \lambda_l \omega^2 r_x^2 - 2 \lambda_\omega \lambda_u \omega v_r +$$

$$\frac{\lambda_\mu \lambda_u}{\lambda_\rho \lambda_l^2} \mu \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right) - \frac{\lambda_{p_\sigma}}{\lambda_\rho \lambda_l} \frac{1}{\rho} \frac{\partial p_\sigma}{\partial y} \quad (24)$$

where λ is the ratio of the physical quantities denoted by subscript, respectively.

According to Eqs. (23) and (24), some similarity criteria are found:

$\frac{\rho u l}{\mu} = Re$ is the Reynolds criterion: the ratio between the inertial and the viscous force;

$\frac{p}{\rho u^2} = Eu$ is the Euler's criterion: the ratio between the pressure and the inertia force;

$\frac{\omega l}{u} = Ce$ is the centrifugal force criterion: the ratio between the centrifugal and the inertia forces;

$\frac{p_\sigma}{\rho u^2} = Ca$ is the capillary force criterion: the ratio between the surface tension and the inertial force.

The centrifugal force criterion and capillary force are unique in micro-scale centrifugal casting. Under high-speed centrifugal force field, the centrifugal force, surface tension and inertia forces are key factors, and the

effects of other forces can be neglected. Therefore, the similarity of prototype and physical model can be achieved, when the centrifugal force and capillary force criteria are satisfied simultaneously. In the capillary force criterion, $p_\sigma = \frac{2\gamma \cos \theta}{r}$, since the wettability between liquid metal and cast mold is very poor, and the wettability between the simulation fluid and PMMA mold is also very poor, thus $\cos \theta \approx 1$. In the meanwhile, the dimensions of model and prototype are the same. In summary, the following relationships should be followed for the physical simulation of liquid metal mold filling process in the micro-scale centrifugal casting process:

$$\omega' = \sqrt{\frac{\sigma' \omega^2 \rho}{\sigma \rho'}} \quad (25)$$

where superscript is model parameters.

In the present work, the physical parameters of the simulate fluid (aqueous solution with the thickening agent) are: $\rho'=1500 \text{ kg/m}^3$, $\sigma'=0.072 \text{ N/m}$. The parameters of ZnAl4 alloy are: $\rho=6600 \text{ kg/m}^3$, $\sigma=0.755 \text{ N/m}$. Substituting parameters into Eq. (25), we obtain $\omega/\omega'=1.54$. Therefore, the similarity between the model and the prototype can be achieved, when the centrifugal speed ratio between the model and the prototype is 1.54.

4 Conclusions

1) The effects of centrifugal and Coriolis forces were integrated into the micro-scale Navier–Stokes differential equations under the gravity field. The N–S differential equation for the micro-scale flow under centrifugal force field was obtained:

$$\begin{aligned} u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + g_x + \\ &\mu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial p_\sigma}{\partial x} \\ u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + g_y + \\ &\mu \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial p_\sigma}{\partial y} \\ u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + g_z + \\ &\mu \left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial p_\sigma}{\partial z} \end{aligned}$$

2) The influence of centrifugal and Coriolis forces was integrated into the Bernoulli equation under the

gravity field, the Bernoulli equation under the centrifugal force was obtained:

$$\bar{e} = \frac{p}{\rho g} + \frac{\alpha v^2}{2g} + \frac{\beta \omega L v}{2g} - \frac{\omega^2 r^2}{2g},$$

where $\alpha = 1 + 3 \frac{\int_A \Delta u^2 dA}{v^2 A}$, $\beta = 1 + \frac{\int_A \Delta u^2 dA}{v^2 A}$.

3) The similarity between the model and the prototype liquid metal can be achieved when the centrifugal speed ratio between the model and the prototype is 1.54.

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微尺度空间内离心铸造过程的流动方程及相似准则

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摘 要: 通过相似物理模拟方法研究离心力场下液态金属在微尺度空间内的流动过程, 相似准则的推导是工作的重点。在传统流动方程的基础上, 充分考虑微精密铸造过程中特殊的物理条件, 推导了离心铸造过程液态金属在微尺度空间内充型流动时满足的流动方程和伯努利方程, 为微尺度空间内液态金属的充型流动过程数值模拟提供了数学模型。同时, 依据微流动方程和相似理论, 获得了离心力场下物理模拟微尺度空间内液态金属流动过程的相似准则, 从而实现微流动过程的可视化观察和定量分析。

关键词: 微空间; 微流动; 离心力场; 相似准则

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