Available online at www.sciencedirect.com
ScienceDirect
Trans. Nonferrous Met. Soc. China 23(2013) 2807−2816
Transactions of Nonferrous Metals Society of China
www.tnmsc.cn

# Decomposition laws of tungsten prices fluctuation since 1900 and its applications

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Received 10 May 2012; accepted 19 August 2013

**Abstract:** Tungsten current price was transformed yearly to its constant price since 1900, which is roughly decomposed into four components as trend, cycle, impact and random. The core prices, consisting of the trend and the cycle, present regularities that a long-run cycle is embedded within two major cycles, and major cycle is composed of low-price period and high-price period, along with the rapid rise into a tower, and along with deep down into next trough; three sharply upward shocks occur by the events in a tower. Fluctuations in prices trend to slow cycles and expand the bands. It can be expected that tungsten price will highly stand over 17 a, and is is a advice that reducing production and restricting export maybe maintain a high price level.
**Key words:** tungsten; price; cycle; trend; impact; fixed price; strategy; management

## 1 Introduction

Generally, commodity prices are shown as the interaction between supply and demand in the market mechanism, and influence effective supply and demand, the internal structure of providers and demanders, market environment conditions, marketing strategies of all parties, speculations and operations, and many other factors. According to these, people try to seek the rules hidden in prices on the basis of the price itself or the decisive factors of the price, etc.

On the aspect of the time variation of prices, DANIEL et al [1] strongly confirmed the existence of price cycles of the metal such as steel and molybdenum. LABYS et al [2] statistically confirmed the evidence of cyclical behavior for a number of metals using the Weibull test for duration dependence as well as the structural time series method. DAVUTYAN and ROBERTS [3] statistically proved the existence of the price cycles and characteristics of minerals [3]. HEAP [4,5] framed the super long cycle caused by Chinese factors. BOOLEY and LENIHAN [6] found that commodity prices presented the characteristics such as trends, cycles and gradual increasing volatilities.

On the aspect of the associated factors of metal prices, LABYS et al [7] employed the Geweke Dynamic Factor Model in their empirical study on five metals and found that metal price cycles are related to business cycles and are affected by macroeconomic variables such as industrial productions, consumer prices, interest rates, stock prices and exchange rates. CHEN [8] found that metal prices are affected by world's macro factors and move jointly with them, by investigating the long-term fluctuations of 21 metals based on the data from 1900 to 2007. GEDGAGOV et al [9] studied the competitive advantages generated by the industrial concentration of tungsten.

However, not all empirical researches show regularities. For instance, ROBERTS [10] identified the peaks and troughs in the inflation adjusted prices of 14 metals using monthly average data from January 1947 to December 2007. It was found that the long-term trends and amplitudes of metal prices had little regularity. This suggests that trends, cycles, and other single factors only are not enough to summarize the regularities of the long-term metal price fluctuations, and the reliability of the prediction of metal prices varies according to the

**Foundation item:** Project (2013ZK2001) supported by the Major Soft Science Program of Hunan Provice, China; Projects (1382ZD024, 13BGL105) supported by the National Social Science Foundation of China
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DOI: 10.1016/S1003-6326(13)62801-0

selection of those periods or objects. Meanwhile, the difference rules of many metals will offset the similarities in the study of categories, and the specific regularities of particular metals will not be neglected.

As an example, exports of Chinese tungsten account for 85% of the world, but China still lacks pricing power. It is difficult to explain this issue with the short-term rules of tungsten prices, and we need to explore the long-term rules of tungsten prices fluctuations. KONG [11] attempted to analyze the long-term periodicity rules of tungsten prices, and marked off three fluctuation cycles by the demarcation of extra-low price. However, these three cycles do not show significant repeatability. ZHU and GAO [12] conjectured that he long-term price of tungsten is changed under the influence of cyclical factors, and also affected by other factors. The price structure is improved by adding some factors as economy mechanism, external shocks and randomness [12]. In addition, ZHU et al [13−15] replaced short-term prediction with price forecast for a long period of time, increased the economic explanation content for the price, and got careful market analysis based on the supply and the demand of its specific production characteristics. In this work, we attempt to separate the deterministic components from long-term tungsten prices after eliminating the effect of currency changes, to analyze the statistical rules of the uncertain part, explore the long-term rules of tungsten price fluctuations and measure the amplitudes and risks that actual prices deviate from benchmark price by way of structural decompositions.

## 2 Theory basis and price model

### 2.1 Theoretical framework

Because factors affecting price are various and their relationships are complex, it is always a difficult problem to investigate the forming cause of commodity prices. However, factors affecting the short-term price often trigger price fluctuations rather than decide price trends, and market mechanisms make many factors interact and offset, or lead them to adjusting spontaneously by complementary regression in a feedback adaptive way. Therefore, long-term price rules can be presented. For instance, a rise in price caused by supply shortages can promote an increase of supplies and bring down the price. In addition, economic cycles will drive price fluctuations, significant events can trigger price mutations, and a numerous small factors will disturb price randomly. Thus, the price formation can be classified into four categories.

1) Mechanism factor $S$

In the time dimension, the change of tungsten prices should be stable. The change in posture is decided by market mechanisms and deterministic characteristics. When market mechanisms are changeless, the rules of prices will be changeless. In fact, market mechanisms are usually very stable. So, price changes generally show linear features under the determination of market mechanisms.

2) Cyclical factor $T$

It is called the cyclical factor that causes tungsten price periodically fluctuations according to time-series. The rule of the price cycle is determined by the economic cycle. It is certain foreseeable but not controllable. Understanding the rule of price cycles is helpful for the price prediction and management.

3) Impact factor $I$

The characteristics of impact factors are of intensity, short term and unilateral variation. We call the factor with these functional characteristics as the impact factor. The impacts of tungsten price are usually triggered by world major events including wars, disasters, the economic crisis, social changes, etc. The impact factors are random in time and strength, but they present certain regularities in distribution for the long run. If we can master the distribution rules and the strength of the impacts, it will be easier to predict the rapid changes of tungsten prices in the middle or short term and to handle the business opportunities and risk preventions.

4) Random factor $\varepsilon$

After the factors of trends, cycles and impacts are excluded, the characteristics of the surplus influence factors on the tungsten price are various factors and are independent; each factor influences the price a little and each factor is uncertain to militate. By synthesizing, the feature of these factors is randomness. We assume that these factors obey the normal distribution, that is $\varepsilon \sim N(0,\sigma^2)$ . Random factors which cause price fluctuations are measurable, but not controllable. Mastering the rules of random price fluctuations can effectively prevent business risks.

### 2.2 Data selection and preprocess

2.2.1 Collecting consistent sample

The value of tungsten international trade is more than ten billion dollars. The importance of tungsten does not depend on its trade volume, but mainly depends on the “industry tooth” function and the growing applications in high and new technologies. Though tungsten is one of the most important strategic resources, it has never been run as strategic resources since 1900 except during the war, and its price is mainly the result of market performances rather than strategic controls. Therefore, tungsten market is consistent for a long time. The sample range can be extended to 1900. The tungsten current price ($P_i$) sequence can be written as $\{(i,P_i) \mid i=0,1,\cdots,n\}$ . We assume that it is in dollar terms.

2.2.2 Unify price measure

Price can normally be understood as the measurement of commodity values under a certain price measure. In reality, this price measure is money. High or low money values will affect the price measure of the same commodity, and changes in money values can also affect people's expectations on commodity prices. The interlaced interaction between money values and people's expectations causes complex changes of commodity prices. Therefore, unifying the price measure over the whole period and making the standard of people's expectations are the same principles that we should follow when doing the long-term price research. In the study of the long-term rules of commodity price changes, only by eliminating the influence of changes in the money value, can we unify the price measure and make the commodity prices comparable between every two periods. Let constant price $(\overline{P}_i)$ be

$$\overline{P}_i = P_i / k_i \tag{1}$$

where $\kappa_i$ is the convert coefficient of the money value from the benchmark year to the $i$th year; $\{(i,\overline{P}_i) \mid i = 0,1,\cdots,n\}$ is therefore the constant price sequence which is measured on the money value of the benchmark year. In this case the constant prices can be compared directly because the price measure is consistent.

Let $r_i$ be the convert coefficients of the money value of the $i$th year relative to that of the next year, where $r_n$=0, then $\kappa_i$ can be calculated as follows:

$$\kappa_i = 1/\prod_{j=i}^{n}(1+r_j)\ (i=0,\ 1,\ 2,\ \cdots,\ n) \tag{2}$$

where the final year is the benchmark year.

Generally speaking, money values tend to depreciate so commodity prices tend to rise. For the fact that money values devaluate in per unit time, the secular variation of the commodity current prices rise exponentially on the whole according to the principles of mathematics. If the influence of currency devaluation is not eliminated, price measure cannot be consistent and the long-term price rule will seriously be distorted. The fine regularities of commodity prices may be also submerged in the exponential function and cannot show up. Eliminating the influence of money values can not only unify the pricing measurement in the whole study period, but also eliminate the exponential rise of people's expectations caused by the currency. Thus, the laws of the commodity price itself will show up.

### 2.3 Price structure model

The factors, such as mechanisms, cycles, impacts and random, which influence the constant price in the tungsten market, are coordinate, complementary and summable. The structural relationship of the constant price is shown as (3). Meanwhile, the relationship between the monetary factors and these four factors is cumulative, and they are multiplicative in calculation. The specific relationship is

$$\overline{P} = S + T + I + \varepsilon \tag{3}$$

According to Eq. (1) and Eq. (3), we can get the decomposition equation of the current price written as Eq. (4) where the convert coefficient $\kappa$ is known.

$$P = \kappa(S + T + I + \varepsilon) \tag{4}$$

## 3 Decomposition of constant price

### 3.1 Difference chart of current price and constant price

The key to convert the current price sequence $\{(i,P_i) \mid i = 0,\ 1,\ \cdots,n\}$ into the constant price sequence is determining the convert coefficient $\kappa_i$. We use the terminal as the benchmark time. The convert coefficient $\kappa_i$ can be estimated on the basis of the statistical caliber, the depreciation rate of US dollars or the CPI index. Experiences show that the results estimated in either way have no significant difference. To simplify the calculation, we calculate the convert coefficient $\kappa_i$ on the basis of CPI, then $\kappa_i$ can be written as $\kappa_i$=CPI$_n$/CPI$_i$. The average value of $\kappa_i$ is 2.43% over the period 1900−1971 and 3.53% over the period 1971−2010. As $\kappa_i$ and $P_i$ are known, the constant price $\overline{P}_i$ can be calculated according to Eq. (1). Therefore, the constant price sequence $\{(i,\overline{P}_i) \mid i = 0,1,\cdots,n\}$ can be obtained. The differences between the current price and the constant price during 1900−2010 are shown in Fig. 1, where benchmark year is 2010.

In Fig. 1, the constant price fluctuation rules are highlighted compared with the current price. As the current price and the constant price have the property of one to one correspondence and isomorphism, we can transform research objects. First, we transform the current price into the constant price. By doing this we can study the properties of the constant price clearly. Second, by the reversible transformation we can obtain the corresponding properties of the current price.

### 3.2 Properties of core price

The tungsten constant price is called the core price when the irregularity of the fluctuation is deducted. Core price is actually the sum of trend and cycle, and it is the part of the price determined by certain factors. Core price is shown in Fig. 2(dotted line)

The rules presented by the fluctuations of tungsten core prices are shown in Fig. 2. The prices in the high-price period and in the low-price period appear alternately, making price fluctuations change periodically.

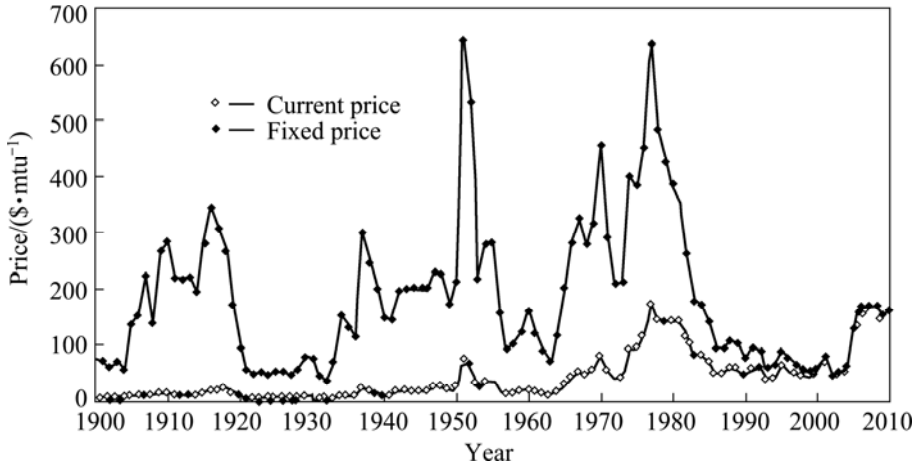


**Fig. 1** Current price during 1900−2010 and constant price in 2010

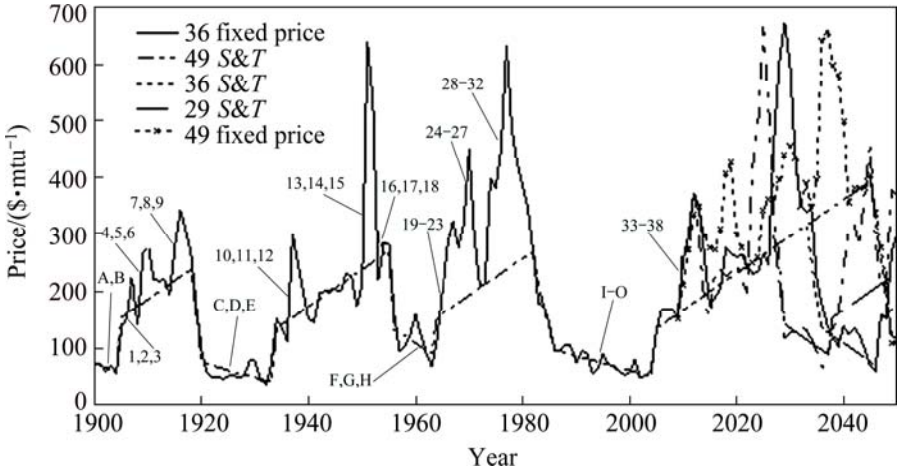


**Fig. 2** Fluctuation laws of world's tungsten constant prices during 1900−2050 (Constant prices during 2010−2050 are simulation-based results predicted in section 4)

The adjacent low-price and high-price periods compose a core price cycle, and two adjacent core price cycles compose a major cycle. The core price in low-price periods rises into high-price periods with an increase of over 60% in a year. Each high-price period includes three interval growing waves and it promotes the trendy rise of the price in the high-price period, until the price declines by no less than 50% within four years and falls into a low price level. Each price summit presents the significant characteristics of broadening amplitudes and short-time impacts. In low-price periods, the price tends to drop and the amplitudes are relatively small.

The corresponding data are as follows.

1) Significant periodicity of price

Two low-price periods front and rear are one long and one short. The short one reaches a length of 5−7 years with an average annual decline of 3−5 dollars per ton unit ($/mtu), while the long one lengthens out to about 14 years with an average annual decline of about 2 $/mtu. The average increase of the prices in high-price periods is about 5 $/(mtu·a). Tungsten price since 1900 can be divided into three cycles: 1900−1920, 1921−1956, 1957−1985, according to the starting points of extra-low price. The lengths of each cycle are 21 years, 36 years and 29 years, respectively. Two price cycles front and rear compose a major cycle, then the second major cycle which began in 1957 and the first cycle was over on 1957−1985, and the second one began in 1986 and its low-price period ended in 2004.

2) Trend of constant price changes little

Observing the three completed cycles, we can mark off two major-cycles. Choosing the key start and end points of the two as the comparative objects, we find that the average annual increase in every possible case is 0.41 $/mtu. In general, we can agree that the tungsten constant price presents a slight linear upward trend in the long term.

3) The core price is the superposition of the periodicity and the tendency. Because the price trend changes very little and the cycle price changes significantly, the core price cycle is synchronized with the periodic price. And because the trend price decides

the core price level, the core price not only reflects the price level but also the price fluctuations. This can be seen in Fig. 2. From the statistical point of view, the core price cycle is extending, and its amplitude is enlarging. The main digital characteristics can be summarized in Table 1 and Table 2.

4) The core price in Fig. 2 can be presented by a polygonal line formed by five points. The polygonal line formed by any five continuous points constitutes a core price cycle. We use five points to represent a typical core cycle, written as

$$[ST]=\{(t_i, p_i) \mid t_i < t_{i+1}(i=0,1,2,3,4), \quad p_0 > p_1 < p_2 < p_3 > p_4\} \tag{5}$$

where $[t_0,t_1)$ refers to the low-price period, $[t_2,t_3)$ refers to the high-price period, and $[t_1,t_2)$ refers to the transition stage from the low-price period to the high price period, and $[t_3,t_4)$ refers to the transition stage from the high-price period to the low-price period. Then we can write the following equation:

$$[ST](t)=p_i+\frac{p_{i+1}-p_i}{t_{i+1}-t_i}(t-t_i)(t\in[t_i,t_{i+1}];\ i=0,1,2,3) \tag{6}$$

### 3.3 Properties of impact prices and random prices

3.3.1 Induced reasons of impact prices

KONG [11] showed that tungsten price impacts are highly related to world events with adding some supplements, and main matchups are given as follows.

The world events which rule in the low-prices are the Anglo−Japanese Alliance (A), the formation of the world colonial system (B), the Paris peace conference (C), the establishment of the Soviet union (D), the world economic depression (E), the Asian flu (E), the African independence non-aligned movement (G), China's reform and opening up (H), the collapse of the Soviet union and the eastern Europe upheaval (I), the diplomatic relations between China and America (J), the North America free trade (K), the establishment of the European Union (L), the Asian financial crisis (M), 9.11 event (N), the wars between Iraq and Afghanistan (O).

The world events which cause the price peak are 1−the Japan and Russia war, 2−the 1st Moroccan Crisis, 3−the San Francisco earthquake, 4−the Revolution of 1911, 5−the 2nd Moroccan Crisis, 6−the Balkan War, 7−war Ⅰ, 8−the October Revolution, 9−the Revolution in European, 10−the Diexi Earthquake, 11−Anti-Japanese War, 12−WarⅡ, 13−The Establishment of PRC, 14−the Korean War, 15−the Warsaw Pact, 16−the Capitalist Economic Gold Period, 17−the 2nd Arab-Israeli Conflict, 18−the Cuban Revolution, 19−The Great Cultural Revolution, 20−the Xingtai Earthquake, 21−the Vietnam War, 22−the 3rd Arab−Israeli Conflict, 23−the 1st Oil Crisis, 24−the Treasured island events, 25−the Soviet invasion of Afghanistan, 26−the India War, 27−the 4th Arab−Israeli Conflict, 28−the Tangshan Earthquake, 29−the Iranian Revolution, 30−the Iran−Iraq War, 31−the Falklands War, 32−the 2nd Oil Crisis, 33−The Sino-Japanese dispute, 34−the Earthquake in China, India, Pakistan and Chile, 35−the Nuclear Dispute, 36−the Disputes of the east and the south China sea, 37−the Japan Earthquake and Nuclear radiation, 38−the Unrests in Middle East.

Figure 2 shows that tungsten price is usually rising by world events such as wars, earthquakes and crises, and by maintaining tungsten low price with epidemics, the economic crisis and world orders smooth.

**Table 1** Main numerical characteristics of core price cycles (Price unit: \$/mtu)

| Major cycle | Core price cycle | Low-price period/a | Ascent stage/a | Low-price period | Price in ascent stage |
|---|---|---|---|---|---|
| Cycle 1 (1900−1956) | 1898−1920 | 5 | 14 | 73→67 | 135→201 |
| | 1921−1956 | 13 | 22 | 50→50 | 120→225 |
| Cycle 2 (1957−2049) | 1957−1985 | 8 | 18 | 125→90 | 155→240 |
| | 1986−2049 | 19 | 40 | 90→54 | 135→335 |
| | 1986−2036 | 19 | 29 | 90→54 | 135→275 |
| | 1986−2029 | 19 | 23 | 90→54 | 135→245 |

Note: Results of the fourth core cycle from 2011 are obtained by predicting.

**Table 2** Characteristics of impacts of prices in high-price periods (Price unit: \$/mtu)

| High-price period | Start growth | | First summit | | Second summit | | Third summit | | Declining termination | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Initial | Rate | Up | Time/a | Up | Time/a | Up | Time/a | Down | Time/a |
| 1905−1920 | 67 | 101% | 70 | 1 | 110 | 5 | 145 | 4 | 151 | 3 |
| 1934−1956 | 50 | 140% | 160 | 3 | 30 | 7 | 420 | 2 | 100 | 3 |
| 1957−1986 | 95 | 63% | 150 | 4 | 260 | 3 | 400 | 8 | 150 | 4 |
| 1987−2050 | 54 | 150% | 200 | 6 | 70 | 6 | 350 | 8 | 195 | 5 |

3.3.2 Basic pattern of impact price

The occurrence time and the range of tungsten impact prices are both random. The impacts occur only in the high-price period. They will not occur in the low-price period. Generally, the width of the first shock wave accounts for about 10% of the total length of the ascent stage, and the widths of the second and third shock waves account for about 13% and 24%, respectively. Usually, the interval between impacts is over 2 years. Let the core price be the benchmark, the amplitudes of the shock waves usually are above 38%. They present a successive increasing and the maximum amplitude is over 150%. The general form of the function of the impact price is as follows:

$$I(t)=\sum_{j=1}^{J} c_j \cdot \chi(t,a_j,b_j) \tag{7}$$

where $\chi(t,a_j,b_j)$ is the impulse function, and $a_j < b_j \le a_{j+1}\left(j=1,2,\cdots,J\right)$.

Actually impact prices are not independent moment, but they are correlated in time series. Due to the correlation of the impact in the time sequence, the impacts have certain types. Three basic types are: for the enhancing type $[I]_e$, the amplitude of the impact increases gradually and finally falls back quickly; for the weakening type $[I]_d$, the amplitude of the impact, decreases gradually after the quick push of the impact prices; for the normal type $[I]_n$, first the amplitudes of impacts increase year by year, and then they reduce year by year after reaching the maximum. Generally, the enhancing and weakening types can be represented by a polygonal line formed by four points, and the impact form of the normal type can be represented by a polygonal line formed by five points. The impact models of these three types can be written as follows:

$$[I]_{\varepsilon}=\{(t_i,p_i)\,|\,t_i<t_{i+1}(i=0,\ 1,\ 2,\ 3),\ p_0=p_3=0<p_1<p_2>p_3\} \tag{8}$$

$$[I]_{\delta}=\{(t_i,p_i)\,|\,t_i<t_{i+1}(i=0,\ 1,\ 2,\ 3),\ p_0=p_3=0<p_1>p_2>p_3\} \tag{9}$$

$$[I]_{n}=\{(t_i,p_i)\,|\,t_i<t_{i+1}(i=0,\ 1,\ 2,\ 3,\ 4),\ p_0=p_4=0<p_1<p_2>p_3>0\} \tag{10}$$

3.3.3 Variance estimation of random price

The random variable $\varepsilon \sim N(0,\sigma^2)$, so we can get $E(\varepsilon)=0$. The properties of the price will be clear if $D(\varepsilon)=\sigma^2$ is estimated. The variance can be estimated as

$$\hat{\sigma}^2=\frac{1}{n}\sum_{i=0}^{n}(\bar{P}_i-[ST]_i-I_i)^2 \tag{11}$$

**3.4 Decomposition algorithm**

3.4.1 Criteria for decomposition algorithm

Generally, the time series of the price can be decomposed obviously into the sum of the trend sequences, periodic sequences, impact sequences and random sequences. However, the decomposition results are not only one. If there exist two types of decompositions, then we can prove that their linear combination still satisfies the conditions. In other words, when the decomposition is not unique, there will be an infinite variety of decompositions. Therefore, we need to increase additional conditions so as to find the pleasing decomposition from infinite possible decompositions. In fact, the price time series have restrictive conditions. When restriction conditions are given, the possible results of the decomposition will be reduced. For the sake of adding conditions in the price decomposition, we usually use the following two algorithm criteria.

1) Better criterion generates more regularity. We want the rules to be more regular rather than more uncertain. So when the factor cannot be attributed in technology, we would rather attribute it into the deterministic rules of the trend and cycle rather than the uncertainty of the impacts and random factors.

2) Better criterion generates more significance. As the trend, cycle and impact are much easier to be observed, so we highlight the role of significant factors and try not to attribute the factor into the influence of the random. In quantitative, this requires the variance of the random variable minimized and the variable to follow the normal distribution.

3.4.2 Key points of decomposition algorithm

1) Due to the characteristics of tungsten constant prices, the trend and periodic prices can be merged, and they constitute the core prices. The function of the core prices can be expressed by a polygonal line formed by five points, and written as (5)

2) A positive $\sigma$ and $\alpha$ are given, where $\alpha>>\sigma$.

3) The impact price is calculated as:

$$I_{\alpha}=\{(t,\bar{P}_t-[ST]_t)\,||\,\bar{P}_t-p_t\,|\ge\alpha,t\in[t_2,t_3]\} \tag{12}$$

4) The variance of the random price is estimated as:

$$\bar{\sigma}^2=\frac{1}{n-1}\sum_{i=1}^{n}[\bar{P}_i-[ST]_i-I_i]^2 \tag{13}$$

5) If $\sigma^2>\hat{\sigma}^2$, then the decomposition meets the requirements; and the calculation of the decomposition is finished, and the results of this decomposition can be obtained, otherwise continue.

6) Within limits, narrow $\alpha$, and turn to 2), otherwise to 7).

7) Fix the trend price and the cycle price, and turn to 1).

According to the algorithms above, putting in numerical values of tungsten prices and do the calculation, we can get $\hat{\sigma}^2=29.437=5.425^2$.

## 4 Simulation-based prediction of tungsten price

According to formulas (1) and (3), a general thought for predicting the tungsten price is to transform the tungsten current price into the constant price; decompose the constant price into the core price, the impact price and the random price; obtain the properties and parameters of the core price, the impact price and the random price by way of statistical methods; predict the core price, the impact price and the random price according to their own properties and parameters; figure out the sum of the core price, the impact price and the random price of the same moment, which can be used as the predicted constant price. According to formulas (1) and (3), the predicted constant price can be transformed into its current price.

### 4.1 Prediction of core price

The key elements to predict the core price are the total length of the cycle, the lengths of the high-price and low-price periods, the start-end values in the high-price and low-price periods, the length of the transition periods from low prices to high prices and from high prices to low prices, etc. The prediction accuracy can be improved by doing rolling forecast. The price cycle which started in 1986 has lasted for 27 a, and it has gone into the high-price period in 2005. The key to predict the residual part of this cycle is to ascertain when the high-price period stops, how long is the transition period from the high-price to the low-price period, and what is the price of the initial point of the next low-price period. In addition, the prediction cannot be contradictory with the facts already happened.

There are three possible predicting results of the new cycle length: 1) Based on the three high-price periods in the price cycles which have already happened, we estimate by the maximum likelihood method and find that the length of the high-price period is approximately 24 a, and the period will end in 2029; 2) When we estimate on the basis of the whole cycle length ratio, the length of the total cycle will be 50 a, and the length of the high-price period will be 32 a, and the period will end in 2036; 3) The two first core price cycles show great similarity, and the low-price periods in the two second core price cycles are also similar. Therefore, we estimate that the length of the high-price period in the new cycle is 45 a, and this period will end in 2049, based on the method of the internal structural proportion in one cycle. These three core prices are respectively symbolized by 29 S&T, 36 S&T, 49 S&T As there are no further bases for quality judgments by the three prediction methods, the predicted period can uniformly be cut off in 2050. The predicted core price in the high-price period can be expressed as follows:

$$p_t = 135 + 5(t - t_i)(t \in [t_2, t_3]) \tag{14}$$

### 4.2 Simulation-based prediction of impact price and random price

4.2.1 Simulation-based prediction of impact price

Impacts occur only in high-price periods, and the key elements to predict the impact price are the starting time $t$, the impact duration length $l$, the biggest impact amplitude $B$ and the impact form. Other elements for the prediction of the impact price are all random. The statistical idea to determine the parameters of the impact shape is to: 1) determine the impact form according to the impact type; 2) standardize the same type of impacts according to the definite proportion of its amplitude and its duration length; 3) determine parameters $a$, $b$, $c$, $b1$, $b2$, $B$ based on the impacts of the same standard; 4) make the impacts layered according to the intensity, and ascertain the numerical characteristics of the maximum impact amplitude by statistically observing the events on the same layer; 5) classify events by the influence time span and ascertain the numerical characteristics of the time span of the same events.

The prediction of the impact price includes the long-term prediction and the temporary prediction. The long-term prediction is based on the previous statistical laws and is used to predict the general price trend. It plays the role in early warning. But the temporary prediction is different. Once a particular event happened, we can initially predict the intensity of the event compared with the standard events. Meanwhile, according to the occurrence and development of the event as well as the superposition of the related events, we can further modify the predicted occurrence time and the biggest range of the impact, and also we can track and predict the terminal and turning points of the impact. The prediction of the impact in this study is the long-term forecast. We use the ideas of simulation to predict impacts one by one. The key point of this prediction gives a high-price period, and determines the possible starting period of the first impact; generates random numbers following the uniform distribution in a given period in order to ascertain the shock moments; gives parameters of the greatest amplitude, the form and the persistent period. The next impact can be predicted in a similar way if the impact is still in the high-price period and the remaining time is long enough after the impact, otherwise, impacts will not occur before the end of the high-price period.

4.2.2 Simulation-based prediction of random price

Because of $\varepsilon \sim N(0, \sigma^2)$, we can use the generated random numbers following the normal distribution as the simulation-based prediction of random

prices. The key to predict the random price is a given $\sigma$. The estimated $\sigma$ value can be obtained by statistical method. A rough estimation of the random price of a full cycle can be obtained using the estimated value of the full process variance, but for a more sophisticated estimation, we can use the estimated values in different stages to estimate the random prices in the corresponding stages. In this study, $N(0,5.5^2)$ is used for the simulation prediction. The true value of the sum of the impact and the random prices before 2010 and the predicted value after 2010 are shown in Fig. 3.

### 4.3 Simulation-based prediction of tungsten current price

Putting the above item-by-item predicting outcomes together, we can calculate the predicted value of the constant price according to formula (3). Let 2010 be the benchmark year, after ascertaining the convert coefficient $\kappa_j$ of the future year: 2010+$j$ A.D., we can estimate the predicted value of the current price according to formula (1) which expresses the relationship between the current price and the constant price. Then the predicted results of the current price can be calculated. To simplify the calculation, we can assume that the depreciation rate of US dollars $r_j$ is constant in the forecast period. According to the statistical results, let $r_j$=3.53%, so

$$\kappa_j = \prod_{k=0}^{j}(1+r_k) = (1+3.53\%)^j \approx \exp(0.0353j) \tag{15}$$

So, the calculation equation for the prediction of the current prices is

$$\hat{P}_t = \exp(0.0353t)([ST]_t + I_t + \varepsilon) \tag{16}$$

The simulated and forecasted value of the current price is shown in Fig. 3. In Fig. 3 the prices over 1900−2010 are the real prices in the history, and the prices over 2011−2049 are the simulated and forecasted values.

The common high-price period of the three above mentioned predictions is [2005, 2027]. The predicted values of the average price during the period 2013−2027 are 290, 295, 325 $/mtu, respectively. During this period, it is easy for China to maintain the high price level. However, after 2027, operators should pay attention to

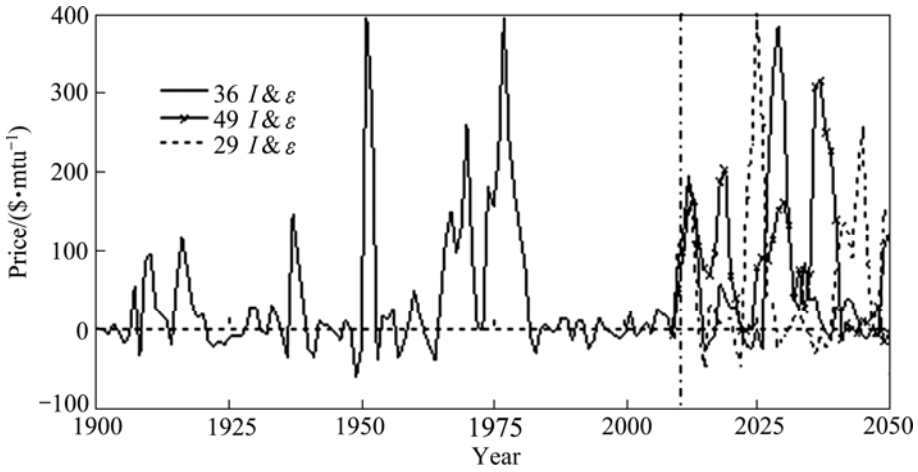


**Fig. 3** Sum of impact and random prices of tungsten over 1900−2049

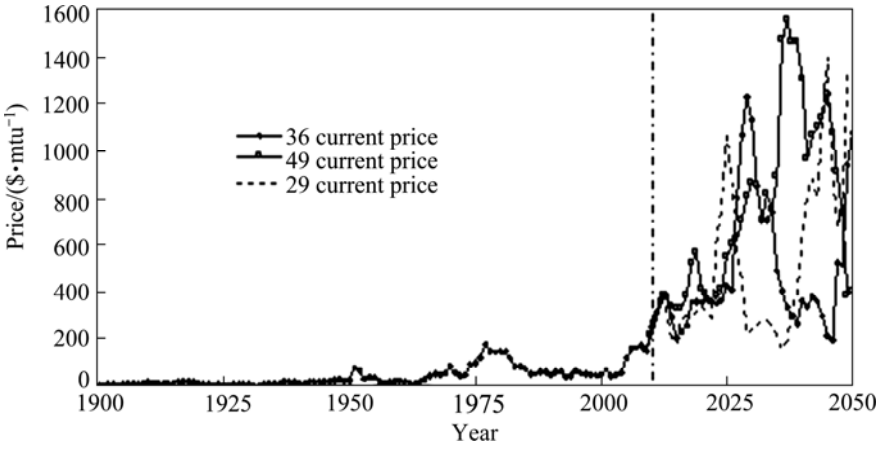


**Fig. 4** Fluctuation rules of tungsten current price in European market during 1900−2050

the possibility of a tungsten price falling and guard against the risk that tungsten prices can possibly fall into the low-price period. Generally, based on the predicted sum of the core price and the impact price, we can define a risk warning index $R$ based on the deviation from the current price to that sum. The risk warning index $R$ can be written as (17). For instance, when $|R_t| > 3\sigma$, the early warning system of the price risk should be started.

$$R_t = P_t - \exp(0.0353t) \times ([ST]_t + I_t) \quad (17)$$

## 5 Conclusions

1) The current price is transformed into a constant price, and the constant price is decomposed into four parts as trend, cycle, impact and random. This makes the economic meaning of the component of price clear. Currency devaluation causes the exponential growth trend of the price on the whole. The overall economic volatility arouses the corresponding price fluctuations. The impacts of great events on the economic trigger price sudden changes. The influence of the integration of a large number of tiny factors showed random price fluctuation. Each part has the concrete form and character on its own change, and the digital features of the cycle price, the impact price and the random price can be obtained by statistical methods.

2) The price can be independently predicted in 4 parts according to the price decomposition structure. After deducting the exponential function determined by CPI, the trend component meets a linear function, and the determination of the cycle part can be transformed into the determination of the five points, and the modal and the parameter of the impact part can be determined according to the type of the modal, and the random part depends on the expectation and variance. These four parts are predicted independently, and the sum of these parts at the same moment is precisely the prediction value of the constant price. Therefore, the current price can be predicted as that the constant price is multiplied by the exponential function value at the same moment. According to such a thinking process, the range of the high-price period of tungsten price is predicted to be 22−40 years in different cases where the prediction can be made, and a consistent prediction is 17 years. The first impact price began in 2011 can be as high as 400 $/mtu.

3) In the high-price period, any small tension would trigger a sharp rise in price. Therefore, the tenet of strategic operating for Chinese tungsten industries should be cutting down exports to make the supply tight and to send the price higher. The specific operational approach is increasing labor cost, employee benefits and tax revenue, increasing environmental protection fees, limiting production and exports and delaying spreading the extraction technology of white tungsten, in order to export less and get more benefits. As long as the market environment with a rising tungsten price is created, market disciplines would strengthen the expectation of a rise in price. Then greater economic benefits can be achieved.

4) In the price structure framework, the long-term price of the trend and the cycle and the short-term price of the impact and the random part split each other but are interdependent at the same time. The impact is often an important condition of forming the inflection point of the short-term price. So, it is worthy for in-depth study. The market conditions such as China's tungsten products exports, overseas tungsten products inventory and the mining cost of tungsten affect the price mechanism and determine the prices trend and the price cycle. This calls for people's mechanism analysis and quantity calculation. In addition, using the method in the study to explore the price rule hidden in other metal resources is an important process for test and improvement. We hope that this study can cast a brick to attract jade.

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# 自 1900 年以来钨精矿价格波动的分解规律及应用

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**摘 要**：将自 1900 年以来钨精矿的现价换算成不变价格后分解为趋势、周期、冲击和随机等部分。研究发现，由趋势和周期构成的核心价格呈现规律，即相邻低谷期和高台期组成一个周期，相邻两周期组成一个长周期。低谷期伴随快速上涨而转入高台期，伴随深度下跌而重回低谷期。一个高台期内与重大事件相伴发生三次大幅向上冲击。价格波动趋于节拍延长、波幅加大。预测钨价将至少高挺 17 年，建议在此期间少产、惜售以维持高价出口。

**关键词**：钨精矿；价格；周期；趋势；冲击；不变价格；战略；经营

**(Edited by Xiang-qun LI)**