Multiple Kalman filters model with shaping filter GPS real-time deformation analysis

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Abstract: In order to detect the deformation in real-time of the GPS time series and improve its reliability, the multiple Kalman filters model with shaping filter was proposed. Two problems were solved: firstly, because the GPS real-time deformation series with a high sampling rate contain coloured noise, the multiple Kalman filter model requires the white noise, and the multiple Kalman filters model is augmented by a shaping filter in order to reduce the colored noise; secondly, the multiple Kalman filters model with shaping filter can detect the deformation epoch in real-time and improve the quality of GPS measurements for the real-time deformation applications. Based on the comparisons of the applications in different GPS time series with different models, the advantages of the proposed model were illustrated. The proposed model can reduce the colored noise, detect the smaller changes, and improve the precision of the detected deformation epoch.

Key words: multiple Kalman filters model; Kalman filter; shaping filter; deformation detection

1 Introduction

Long-term mining causes mine cracks, derrick displacements, surface subsidence and the settlement of constructions which induced the potential safety hazard for the mine, thus mine deformation monitoring is an important issue. With the development of the global navigation satellite systems (GNSS), it is a necessity that the GNSS ground-based real-time deformation monitoring system is applied to the mine deformation monitoring. The time series provided by GNSS must be processed in real-time in order to provide the early warning in time. The Kalman filter is one of the optimal methods to process the time series in real time [1-3]. The elements of the state vector in the Kalman filter are the unknowns of the kinematic/dynamic system. These are typically the position of the object, or also the variation of the position. The property of the Kalman filter is important for the analysis of the behavior of deformations [4].

The detection of the deformation epoch in time and the improvement of the reliability of detecting deformation epoch are key issues for the deformation analysis of GNSS time series. Numerous methods have been used to detect the deformation of the time series, such as, the multiple hypothesis filter [5], the generalized likelihood ratio test by the Kalman filter innovation [5,6], the cumulative sum test [7,8] and the multiple Kalman filters model [9]. Among all the methods, based on MDL criterion, the multiple Kalman filters model has the advantage of deformation detection in time and improving the reliability of the detection epoch. But the Kalman filters model is used under the assumption that only white noise exists in this algorithm. However, the GPS real-time series include coloured noise due to the time-correlated multipath effect, signal propagation, etc. When a real-time GPS series is processed by the multiple Kalman filters model, its noise properties cannot satisfy the Kalman filter’s assumption. Thus, the multiple Kalman filters model should be improved in the application. Usually there are two approaches to reduce the coloured noise: one uses differencing of measurement to remove the coloured noise [10]; the other approach augments the state vector with correlated noise [2,11-13]. It is better to develop an appropriate noise model to augment the multiple Kalman filters model. A shaping filter can be used to describe the

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GPS coloured noise distribution based on the determination of the stochastic model of the GPS coordinate time series [14]. The state vector component of the multiple Kalman filters model can be augmented by a shaping filter. Thus, the multiple Kalman filters model with shaping filter is proposed to process the GPS real-time series with coloured noise and detect the deformation epochs. The results from a GPS experiment are analyzed and its efficiency of the proposed model is verified based on the comparisons of different models.

2 Methodology

The principles of Kalman filter and shaping filter are introduced. And it is followed by the principle of the proposed multiple Kalman filters model with shaping filter.

2.1 Principle of Kalman filter
Kalman filter is an important tool for deformation analysis combining information on object behaviour and measurement quantities. System and measurement equation are combined in a well-known algorithm for estimating an optimal state vector \( \hat{X} \), containing parameters describing deformation behaviour. The principle of the Kalman filter is described as

\[
X_k = \Phi_{k,k-1}X_{k-1} + \omega_k
\]

(1)

where \( X_k \), \( X_{k-1} \) are the state vectors at different epochs; \( \Phi_{k,k-1} \) is the system transition matrix; \( \omega_k \) is the system noise.

The measurement equation is given by

\[
L_k = H_k X_k + \epsilon_k
\]

(2)

where \( L_k \) is the measurement vector at epoch \( k \); \( H_k \) is the observation transition matrix; \( \epsilon_k \) is the measurement noise.

The Kalman filter recursively evaluates an optimal estimator of the state of a linear system [11,15–17]. The Kalman filter process consists of two sub-processes: the time update process and the measurement update process [17].

The time update process are

\[
\bar{X}_k = \Phi_{k,k-1}\hat{X}_{k-1}
\]

(3)

\[
P_{\bar{X}k} = \Phi_{k,k-1}P_{\hat{X}k-1}\Phi_{k,k-1}^T + Q_k
\]

(4)

where \( \bar{X}_k \) is the predicted value of the state vector; \( \hat{X}_{k-1} \) is the optimal estimator of the state vector at the previous epoch \( k-1 \); \( P_{\bar{X}k} \) is the covariance of \( \bar{X}_k \); and \( P_{\hat{X}k-1} \) is the covariance of \( \hat{X}_{k-1} \); \( Q_k \) is the variance of the system noise \( \omega_k \).

The measurement update equations are

\[
G_k = P_{\bar{X}k}H_k^T (H_kP_{\bar{X}k}H_k^T + R)^{-1}
\]

(5)

\[
V_k = L_k - H_k \bar{X}_k
\]

(6)

\[
Q_{\epsilon_k} = H_kP_{\bar{X}k}H_k^T + R
\]

(7)

\[
\hat{X}_k = \bar{X}_{k-1} + G_k V_k
\]

(8)

\[
P_{\hat{X}k} = (I - G_k H_k)P_{\bar{X}k}
\]

(9)

where \( G_k \) is the gain matrix; \( R \) is the covariance of the observation noise; \( V_k \) is the innovation; \( Q_{\epsilon_k} \) is the covariance of \( \epsilon_k \).

In the measurement update process, the newest updated state estimate \( \hat{X}_k \) is computed by the predicted value of the state vector \( \bar{X}_k \) and the newest observations. The generalized likelihood ratio test using the Kalman filter innovation \( V_k \) is one of the approaches to detect the changes in the time series [5,6].

2.2 Principle of shaping filter
In the case of the standard Kalman filter, the noises in the system model and measurement model are white Gaussian noises which are not correlated with any other random variables. But in many real cases, it may not be justified to assume that all noises are white Gaussian noise processes. It is useful to generate an autocorrelation function from real-time series and then develop an appropriate noise model using differential or difference equations. These models are called shaping filters [2]. If non-white noise exists in the system model or measurement model, the state vector can be augmented by appending the shaping filter to the state vector components of Kalman filter.

In the case the GPS, receiver provides the real-time measurements with the high sampling rate, and the time-correlated measurement noise exists in the measurement equation. The measurement equation includes not only the white noise but also the coloured noise. The variable \( x_{sf} \) is used to describe the long-term movement of correlating measurement deviations. The new augmented state vector is defined as

\[
X_k = \begin{bmatrix} x_i(k) \\ x_{sf}(k) \end{bmatrix}
\]

(10)

where \( x_i(k) \) denotes the state vector of GPS coordinates.

The measurement equation can be augmented as

\[
L(k) = H(k)x_i(k) + x_{sf}(k) + \epsilon(k)
\]

(11)

Let the coloured noise of the measurements be modeled by the difference equations,

\[
x_{sf}(k) = \Phi_{sf}(k,k-1)x_{sf}(k-1) + B_{sf}(k)\omega_{sf}(k)
\]

(12)

where \( \Phi_{sf}(k,k-1) \) and \( B_{sf}(k) \) are the coefficient matrices; \( \omega_{sf}(k) \) denotes the zero-mean white noise.

The system equation can be augmented by the shaping filter.
2.3 Multiple Kalman filters model with shaping filter

The multiple Kalman filters model has been used to detect the deformation in GPS real-time series [9]. But it can only be used when there is only white noise in GPS real-time series. The multiple Kalman filters model cannot be applied to many real cases, because colored noise exists. Therefore, the multiple Kalman filters model is improved in this work.

The main principle of the improved model is that each state vector in the Kalman filter takes three former continuous position coordinates into account; another deformation variable \( d \) between the coordinates at two different epochs is added into the state vector. \( d \) should be described with two indexes \( k \) and \( j \). The index \( k \) means the current epoch \( k \), and \( j \) indicates the time shift between the current epoch \( k \) and the deformation epoch. The colored noises exist in the GPS time series and follow an exponential function [14]. In order to reduce the affection of the colored noise in the GPS time series, the state vector in the multiple Kalman filters model is augmented by a shaping filter \( x_{sf}(k) \) (Eq. (12)) which can describe the long movement of colored noise. Therefore, the augmented state vector can be described as

\[
X_k = \begin{bmatrix}
    x_k \\
    x_{k-1} \\
    x_{k-2} \\
    x_{k-3} \\
    d_{k,j} \\
    x_{sf}(k)
\end{bmatrix}
\]

It is supposed that there are four different deformation possibilities among the four neighboring epochs position coordinates. Therefore, four different Kalman filters should be selected. The measurement equations in these Kalman filters are the same. Because the observations contain colored noise and white noise, the measurement equation is defined as

\[
L_k = [1 0 0 0 0 0] + \varepsilon_k
\]

The main difference is the system equations in the different Kalman filters, namely different system transformation matrixes in the Kalman filters’ system equations. The measurement equations and the system equations are discussed in detail in each case.

**Case 1**

There is no deformation among the four neighboring epochs’ position coordinates, which means \( x_{k-3} = x_{k-2} = x_{k-1} = x_k \) (see Fig. 1). There is no deformation and no deformation time shift between epoch \( k \) and the former epochs. That is the deformation \( d_{k,j} = 0, j = 0 \).

![Fig. 1 Case 1](image)

The system equation in case 1 is described as

\[
\begin{bmatrix}
    x_k \\
    x_{k-1} \\
    x_{k-2} \\
    x_{k-3} \\
    d_{k,0} \\
    x_{sf}(k)
\end{bmatrix} =
\begin{bmatrix}
    0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 0 & 0 & 1 \\
    0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
    x_{k-1} \\
    x_{k-2} \\
    x_{k-3} \\
    x_{k-4} \\
    d_{k,0} \\
    x_{sf}(k-1)
\end{bmatrix}
\]

**Case 2**

There is a deformation between \( x_k \) and \( x_{k-3} \) among these four neighboring epochs’ position coordinates. This indicates the time shift \( j = 1 \), \( x_{k-3} = x_{k-2} + d_{k,1} \), and \( x_{k-1} = x_{k-2} + x_{k-3} \) (see Fig. 2).

The system equation in case 2 is described as

\[
\begin{bmatrix}
    x_k \\
    x_{k-1} \\
    x_{k-2} \\
    x_{k-3} \\
    d_{k,1} \\
    x_{sf}(k)
\end{bmatrix} =
\begin{bmatrix}
    0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 0 & 0 & 1 \\
    0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
    x_{k-1} \\
    x_{k-2} \\
    x_{k-3} \\
    x_{k-4} \\
    d_{k,1} \\
    x_{sf}(k-1)
\end{bmatrix}
\]

\[
X_k = \begin{bmatrix}
    x_k \\
    x_{k-1} \\
    x_{k-2} \\
    x_{k-3} \\
    d_{k,j} \\
    x_{sf}(k)
\end{bmatrix}
\]

\[
L_k = [1 0 0 0 0 0] + \varepsilon_k
\]
There is a deformation between \( x_{k-2} \) and \( x_{k-3} \) among these four neighboring epochs’ position coordinates. This indicates the time shift \( j=3 \) when the deformation epoch is compared with the present epoch \( k \). The relationship among these four neighboring epochs can be obtained as \( x_i = x_{k-1} = x_{k-2} = x_{k-3} = d_{k,3} \) (see Fig. 4).

The system equation in case 4 is described as

\[
\begin{bmatrix}
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_k \\
x_{k-1} \\
x_{k-2} \\
x_{k-3} \\
d_{k,2} \\
x_{sf}(k) \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\begin{bmatrix}
o_{k-1} \\
w_{k-1} \\
o_{k-1} \\
w_{k-1} \\
o_{k-1} \\
w_{k-1} \\
\end{bmatrix}
\begin{bmatrix}
e^{-\alpha \Delta t} \\
e^{-\alpha \Delta t} \\
e^{-\alpha \Delta t} \\
e^{-\alpha \Delta t} \\
e^{-\alpha \Delta t} \\
e^{-\alpha \Delta t} \\
\end{bmatrix}
\begin{bmatrix}
x_{k-1} \\
x_{k-2} \\
x_{k-3} \\
x_{k-4} \\
d_{k-1,2} \\
x_{sf}(k-1) \\
\end{bmatrix}
\]

(18)

The system equation and the observation equation of each case have been given.

2.4 Model selection

The MDL criterion [18] has been selected to determine the optimal Kalman filter model at each epoch in order to describe the deformation process correctly and identify the deformation epoch.

The MDL criterion in each case can be obtained as

\[
\zeta_{Ris} = \frac{1}{2} V_{k} \text{inv}(Q_{i}) \text{W}_{k} + \log_{2}\pi Q_{i} + i \log_{2}N
\]

(19)

where \( N \) is the number of the observations used in each Kalman filter model from the first epoch to the current epoch \( k \), and \( i \) is the number of the unknowns in each Kalman filter method.

Based on the MDL criterion, the minimization of the \( \zeta_{Ris} \) will find the model with the maximum
probability.

Given any estimated models, the model with the lower value of the MDL criterion (19) is to be preferred. Lower value of the criterion implies that the model better fits the time series. When the model is the correct one, the value of the statistical criterion should be smaller compared with the other models. The deformation detection is reflected in the process compared with the statistical criterion. When the smallest criterion is chosen, the corresponding Kalman filter and the deformation epoch can be determined. For example, if the criterion of the first case is the smallest, no deformation is detected.

3 Experimental

Different GPS time series have been chosen from the GPS experiment. The sites of the GPS experiment were selected on the roofs of the Institute of Geodesy and Geoinformation (IGG) and the Max-Planck Institute (MPI) in Bonn, Germany. The baseline was about 1.2 km. The GPS equipment consisted of Trimble 5700 receivers and Zephyr antennas. A cut-off angle of 10° was chosen and the sampling rate \( \Delta t \) was 1 s.

During the kinematic GPS measurements, the height of the rover station, which was on the roof of the Institute of Geodesy and Geoinformation, was changed with a crank every 30 min in step of 25 mm. The kinematic measurement lasted for 3.5 h (12600 s) (see Fig. 5). In the test the true deformation epochs are the periods between epochs 1800 and 1801, between epochs 3600 and 3601, between epochs 5400 and 5401, between epochs 7200 and 7201, between epochs 9000 and 9001, between epochs 10800 and 10801.

4 Results analysis

The analysis of the noise in GPS time series and the determination of the stochastic model of GPS time series in detail can be seen in Ref. [14]. Different GPS time series (see Figs. 5 and 6) are used to check the efficiency of the multiple Kalman filters model with shaping filter.

![Fig. 6 GPS static height time series](image)

4.1 Application in GPS time series of stepwise deformations at magnitude of 25 mm

Table 1 shows that during the detection of the deformation epochs, all the true deformation epochs are detected; but two false deformation epochs are detected as deformation epochs. Figure 7 demonstrates that the colored noises in the GPS time series have been reduced by a shaping filter.

The standard deviation of the processed time series is 4.0 mm. Compared with the standard deviation of the observations, the precision of the processed time series has been improved.

Compared with the results of the multiple Kalman filters model [9], in the case of the stepwise deformation of 25 mm, all the true deformation epochs can be detected. But the multiple Kalman filters model with

<table>
<thead>
<tr>
<th>True deformation epoch</th>
<th>Detected deformation epoch ((t_{k}, t_{k+1}))</th>
<th>Time delay/s</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>1801</td>
<td>(1801, 1802)</td>
<td>0</td>
<td>Two false deformation epochs (7315,7316) and (10138,10139)</td>
</tr>
<tr>
<td>3601</td>
<td>(3601, 3602)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>5401</td>
<td>(5403, 5404)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>7201</td>
<td>(7207, 7208)</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>9001</td>
<td>(9005, 9006)</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>10801</td>
<td>(10802, 10803)</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
shaping filter can reduce the affection of colored noise and the precision of the processed time series is higher than that of the processed time series by the multiple Kalman filters model.

The standard deviation of the processed time series by Kalman filter model is computed roughly to be 5.8 mm. Compared with Kalman filter model, the multiple Kalman filters model with shaping filter can get precise results because it can reduce the colored noise in the time series. However, the colored noise still exists in the processed time series by the Kalman filter model.

Compared with the Kalman filter model with shaping filter, the multiple Kalman filters model with shaping filter still can get the most precise processed results. It can detect the deformation epoch with short time delay, thus it can modify the new state vector in time when there is a deformation. That is why the better results are obtained by the multiple Kalman filters model with shaping filter.

### 4.2 Application in GPS static deformation time series

As we know no deformation epochs existing in the static time series (Fig. 6), Table 2 illustrates the false detected deformation epochs obtained by multiple Kalman filters model with shaping filter.

<table>
<thead>
<tr>
<th>True deformation epoch</th>
<th>Detected deformation epoch $(t_{j-1}, t_{j-1}+1)$</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>No deformation epoch</td>
<td>(25346, 25347), (25401, 25405), (26586, 26587)</td>
<td>The detected epochs are during the period when the satellite geometry is poor</td>
</tr>
</tbody>
</table>

As analyzed above, during the epochs 25339 to 27623, the satellite geometry is very poor. The accuracy of the results is very low. Some results are not reliable so that the difference between the coordinates becomes large. In that case it can be detected wrongly as a deformation. That is the reason why some epochs are detected as the deformation epochs. The standard deviation of the processed time series (see Fig. 8) is 3.0 mm. The precision has been improved.

Compared with the multiple Kalman filters model, the same conclusion is that all the detected false deformation epochs are during the period when the satellite geometry is poor. The difference is that multiple Kalman filters model with shaping filter can reduce the affection of the colored noise. The precision has been improved.

Compared with Kalman filter model, it has the same problem of the multiple Kalman filters model. Because the colored noise is still in the processed time series by the Kalman filter model.

For the static time series, compared with Kalman filter model with shaping filter, the same precision can be
Conclusions

5 Conclusions

1) The multiple Kalman filters model can only be used under the assumption of the white noise. However, the real-time GPS time series contain colored noise. The multiple Kalman filters model is augmented with a shaping filter which can describe the long term movement of the correlated measurement deviations. Therefore, the multiple Kalman filters model with shaping filter is proposed to detect such small changes and improve the reliability of deformation detection in the GPS real-time series.

2) The proposed model is compared with the Kalman filter model in different GPS time series. The advantage of the proposed model is obvious. The proposed model is better than the Kalman filter model in reducing the colored noise and the detection of the deformation epochs.

3) The proposed model is compared with Kalman filter model with shaping filter. The proposed model is better in detection of the deformation epochs. The proposed model can modify the optimal state vector in time. But for the GPS static time series, the same precision is obtained. Both can reduce the colored noise in the GPS static time series.

4) When the proposed model is compared with the multiple Kalman filters model, the proposed multiple Kalman filters model with shaping filter can reduce the colored noise affections. Both methods can detect the deformation epoch in the kinematic GPS time series.

5) The proposed model can reduce the colored noise, detect smaller changes, and improve the precision of the detected deformation epoch.

6) The proposed model can be used to detect and predict stepwise changes and provide early warning in the mining GNSS real-time deformation system. In the future much work should be done. For example, in many events, the deformation process is nonlinear, nonlinear Kalman filter model should be considered in the model.

References

基于整形多卡尔曼滤波模型的
GPS 实时变形分析

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摘 要: 提出的整形多卡尔曼滤波模型可进行形变的实时检测并提高其可靠性。多卡尔曼滤波模型的扩展主要针对 GNSS 时间形变序列含有有色噪声的情况，而多卡尔曼滤波模型只能应用于白噪声的情况下，故采用整形滤波扩展多卡尔曼滤波模型去除有色噪声的影响。提出的模型可以提高结果的精度且实时进行变形监测，可用于灾害预警系统。通过对比该模型与其他模型(卡尔曼滤波模型，整形卡尔曼滤波模型，多卡尔曼滤波模型)发现，所改进的模型在检测形变的实时性和可靠性方面优于其他模型。

关键词: 多卡尔曼滤波模型；卡尔曼滤波；整形滤波；变形检测

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