Constitutive modeling of hot deformation behavior of X20Cr13 martensitic stainless steel with strain effect

Fa-cai REN, Jun CHEN, Fei CHEN
Institute of Plasticity Technology, Shanghai Jiao Tong University, Shanghai 200030, China
Received 8 May 2013; accepted 7 December 2013

Abstract: Hot deformation behavior of X20Cr13 martensitic stainless steel was investigated by conducting hot compression tests on Gleeble−1500D thermo-mechanical simulator at the temperature ranging from 1173 to 1423 K and the strain rate ranging from 0.001 to 10 s\(^{-1}\). The material constants of \(\alpha\) and \(n\), activation energy \(Q\) and \(A\) were calculated as a function of strain by a fifth-order polynomial fit. Constitutive models incorporating deformation temperature, strain rate and strain were developed to model the hot deformation behavior of X20Cr13 martensitic stainless steel based on the Arrhenius equation. The predictable efficiency of the developed constitutive models of X20Cr13 martensitic stainless steel was analyzed by correlation coefficient and average absolute relative error which are 0.996 and 3.22%, respectively.

Key words: martensitic stainless steel; hot deformation behavior; flow stress; constitutive modeling

1 Introduction

Stainless steel has many desirable characteristics which can be exploited in a wide range of construction applications. It is corrosion-resistant and long-lasting. The annual consumption of stainless steel has increased at a compound growth rate of 5% over the last 20 years, surpassing the growth rate of other materials. The rate of growth of stainless steel used in construction has been even faster, not least due to rapid development in China [1]. Martensitic stainless steels are usually used for manufacturing components with excellent mechanical properties and moderate corrosion resistance such as turbine blades, steam generators, pressure vessels and medical treatments. They can work under high temperature and erosion medium [2]. In order to improve the properties of stainless steel, the hot working parameters should be designed carefully. The understanding of hot deformation behavior of steel is therefore required. The modeling of dynamic softening mechanisms, such as dynamic recrystallization (DRX) and dynamic recovery (DRV), is of great importance in controlling of the microstructure as well as the flow stress of material during hot deformation [3].

In this regard, several models have been developed to assess the kinetics of DRX and DRV in hot working for stainless steels. SAMANTARAY et al [4] analyzed the high temperature flow behavior of various grades of austenitic stainless steels, such as 304L, 304, 304(as-cast), 316L and alloy D9, using the modified Zerilli−Armstrong(MZA) model. EBRAHIMI et al [5] studied the dynamic recrystallization behavior of austenite and investigated the initiation of dynamic recrystallization of a superaustenitic stainless steel containing 16%Cr and 25%Ni. CABRERA et al [6] analyzed the microstructural changes of EN 1.4462 and EN 1.4410 duplex stainless steels by means of optical and electron microscopy. The characteristics of high temperature plastic flow of both DSSs were interpreted in terms of the classical hyperbolic sine equation. MOMENI and DEHGHANI [7] analyzed the relation between the flow stress and Zener−Hollomon parameter via the hyperbolic sine function and developed the power dissipation and the instability map of AISI 410 martensitic stainless steel. ZENG et al [8] established the mathematical models of peak strain and kinetic equation for DRX of 403b heat-resistant martensitic stainless steel based P-J method. AGHAIE-KHAFRI and ADHAMI [9] evaluated the strain rate sensitivity and developed the
power dissipation map and instability map for hot working of 15–5 PH stainless steel. Martensitic stainless steel has drawn little attention despite large amount of efforts invested into the hot deformation behavior of other grades of stainless steels. With regard to martensitic stainless steel, most of studies focused on heat treatment, corrosion behavior, weldability and surface hardening [10–13]. So it is necessary to investigate the hot deformation behavior in order to optimize the forging process parameters of X20Cr13 martensitic stainless steel.

In this work, hot compression tests of X20Cr13 martensitic stainless steel were conducted at different temperatures and strain rates conditions. The flow stress behavior was analyzed, and the constitutive models incorporating the deformation temperature, strain rate and strain were developed using the experimental data on the basis of Arrhenius-type equation. The developed constitutive models were used to predict the flow stress of X20Cr13 martensitic stainless steel and the validity was evaluated via correlation coefficient and average absolute relative error statistical parameters.

2 Experimental

X20Cr13 martensitic stainless steel with a chemical composition of 0.19C–0.131Cr–0.18Si–0.16Mn–0.02P–0.004S–(bal.)Fe (mass fraction, %) was used in this investigation. Cylindrical specimens for hot compression test with a diameter of 10 mm and a height of 15 mm were machined from the as-received hot forged bar. The hot compression tests were carried out according to the schedule illustrated in Fig. 1 on a Gleeble–1500D thermo-mechanical simulator. The specimens were heated to 1243 K at a heating rate of 10 K/s and held for 3 min and then cooled to the test temperature at the cooling rate of 10 K/s. Then, the specimens were held at the forming temperature for 30 s to get a uniform temperature distribution. The tests were performed at 1173, 1223, 1273, 1323, 1373 and 1423 K and strain rates of 0.001, 0.01, 0.1, 1 and 10 s\(^{-1}\), respectively. The true stress–strain curves were recorded automatically in the isothermal compression process. All specimens were compressed to a true strain of 0.8 and then instantly quenched into cold water in order to preserve the hot deformation microstructure.

3 Results and discussion

3.1 Analysis of hot deformation flow curves

The true stress–strain curves obtained from hot compression tests at different deformation temperatures and strain rates are shown in Fig. 2. It obviously shows that the true stress–strain curves are sensitive to deformation temperature in Fig. 2(a). The flow stress will decrease with the increase of deformation temperature. That is because the increase of deformation temperature increases the rate of the vacancy diffusion, cross-slip of screw dislocations and climb of edge dislocations. It also can be seen that the dynamic softening phenomenon is especially sensitive to the strain rate in Fig. 2(b). At the strain rates of 0.01 s\(^{-1}\) and 0.1 s\(^{-1}\), most of the curves show a single peak

Fig. 1 Schematic illustration of hot compression test

Fig. 2 True stress–strain curves of X20Cr13 martensitic stainless steel at different deformation conditions: (a) Strain rate of 0.1 s\(^{-1}\), (b) Temperature of 1423 K
followed by a decrease of stress and finally reach a plateau, which implies the occurrence of the DRX phenomenon. At the strain rates of 1 s⁻¹ and 10 s⁻¹ for all the deformation temperatures, the true stress–strain curves show dynamic recovery character without a peak stress. It can be explained that at higher strain rates, there is no enough time for the nucleation and growth of DRX grains and dislocation annihilation. At the lower strain rate of 0.001 s⁻¹, the multi-peak behavior occurs. The reason is that the first cycle of DRX completed before the next one starts. Otherwise, if the successive DRX cycles overlap, the single-peak behavior can be expected.

Figure 3 shows the influence of deformation temperature and strain rate on the flow stress at a true strain of 0.5. The flow stress decreases with the increase of deformation temperature and decrease of strain rate.

![Fig. 3 Influence of deformation temperature and strain rate at a true strain of 0.5](image)

### 3.2 Calculation of hot deformation constants

The relationship among the flow stress, the deformation temperature and the strain rate in the plastic deformation process of metallic materials can be expressed by [14]

\[
Z = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) = f(\sigma)
\]  

(1)

where the Zener–Hollomon parameter \(Z\) is the temperature-compensated strain rate; \(\dot{\varepsilon}\) is the strain rate; \(\sigma\) is the stress; \(Q\) is the activation energy of deformation; \(R\) is the gas constant (8.314 J/(mol·K)); \(T\) is the thermodynamic temperature.

It is obvious that the \(Z\) parameter is also considered a function of stress. Based on Eq. (1), the \(Z\) parameter can be related to the flow stress in different ways as follows [15]:

\[
Z = f(\sigma) = A' \sigma^n' \tag{2}
\]

\[
Z = f(\sigma) = A' \exp(\beta \sigma) \tag{3}
\]

\[
Z = f(\sigma) = A \left[ \sinh (\alpha \sigma) \right]^{\eta} \tag{4}
\]

where \(A', A'', n', n, \beta, \alpha\) and \(\alpha\) are the apparent material constants, and \(\alpha = \beta/n'\). The stress multiplier \(\alpha\) is an adjustable constant which brings \(\alpha \sigma\) into the correct range to make constant \(T \) curves in \(\ln \dot{\varepsilon}\) versus \(\ln \sinh(\alpha \sigma)\) plots linear and parallel. The power law description of stress (Eq. (2)) is preferred for relatively low stress while exponential law (Eq. (3)) is suitable for high stresses. However, the hyperbolic sine law (Eq. (4)) can be used in a wide range of temperatures and strain rates.

At a certain deformation temperature, the \(f(\sigma)\) is substituted into Eq. (1) and the logarithm of both sides is taken and rearranged respectively as follows:

\[
\ln \dot{\varepsilon} = n' \ln \sigma + \ln A' - \left[ Q/(RT) \right] \tag{5}
\]

\[
\ln \dot{\varepsilon} = \beta \ln \sigma + \ln A'' - \left[ Q/(RT) \right] \tag{6}
\]

\[
\ln \dot{\varepsilon} = n \ln [\sinh(\alpha \sigma)] + \ln A - \left[ Q/(RT) \right] \tag{7}
\]

The description of flow stress by Eq. (1) is incomplete because of lacking of strain. Characteristic stresses such as peak, steady or the stress corresponding to a specific strain can be used for the calculation of material constants [15]. For the fixed deformation temperature and strain, by differentiating Eqs. (5)–(7), respectively, the value of \(n, \beta \) and \(n\) can be expressed as

\[
n' = \left[ \frac{\partial \ln \dot{\varepsilon}}{\partial \ln \sigma} \right]_T \tag{8}
\]

\[
\beta = \left[ \frac{\partial \ln \dot{\varepsilon}}{\partial \sigma} \right]_T \tag{9}
\]

\[
n = \left[ \frac{\partial \ln \dot{\varepsilon}}{\partial \ln \sinh(\alpha \sigma)} \right]_T \tag{10}
\]

In this study, the relationships between the flow stress and strain rate are obtained by substituting the stress values under the strain rates of 0.01, 0.1, 1 and 10 s⁻¹ and deformation temperatures of 1173 to 1423 K at the interval of 50 K corresponding to the true strain of 0.5 into Eqs. (8)–(10). Multi-peak (i.e. strain rate=0.001 s⁻¹) was not taken into calculation. Then the values of \(\beta \) and \(n'\) can be calculated as the mean slopes of the plot of \(\ln \dot{\varepsilon}\) against \(\sigma\) and \(\ln \dot{\varepsilon}\) against \(\ln \sigma\), respectively, as shown in Figs. 4(a) and (b). The values of \(\beta \) and \(n'\) are 0.0559 MPa⁻¹ and 7.383, respectively. The value of \(\alpha\) is given as

\[
\alpha = \beta / n' = 0.00757 \text{ MPa}^{-1} \tag{11}
\]

According to Eq. (10), the mean slope and intercept of \(\ln \dot{\varepsilon}\) against \(\ln \sinh(\alpha \sigma)\) can be used for calculating the values of \(n\) and \(\ln A\) (Fig. 4(c)), respectively. The \(n\) and \(\ln A\) are calculated as 5.366 and 31.341, respectively. So the value of \(A\) is \(4.06 \times 10^{13}\).

For a constant strain rate, partial differentiation of Eq. (7) is given as
\[ Q = nR \left[ \frac{\partial \ln[\sinh(\alpha \sigma)]}{\partial (1/T)} \right] \varepsilon \]  

So at the true strain of 0.5, the value of activation energy is calculated as the average of slopes of the plot of \( \ln[\sinh(\alpha \sigma)] \) against \( 1/T \) (Fig. 4(d)). The value of \( Q \) is determined as 359.402 kJ/mol.

### 3.3 Constitutive models

In Eq. (1), the influence of strain on the flow stress is not considered. It is significant that the strain has an important effect on the flow stress as shown in Fig. 2. So it is necessary to take the strain term into account in order to predict the flow stress of X20Cr13 martensitic stainless steel precisely. The above method is used to calculate the material constants within the temperature range of 1173–1423 K and strain rate range of 0.01–10 s\(^{-1}\) under the strain range of 0.05–0.75 at the interval of 0.05. These values are fitted by a fifth-order polynomial as shown in Fig. 5. The fifth-order polynomial fit results are expressed as follows:

\[ \alpha = 0.01277 - 0.04834 \varepsilon + 0.18608 \varepsilon^2 - 0.36932 \varepsilon^3 + 0.37353 \varepsilon^4 - 0.15155 \varepsilon^5 \]  

\[ n = 7.19785 - 12.73951 \varepsilon + 57.97832 \varepsilon^2 - 145.77045 \varepsilon^3 + 168.05383 \varepsilon^4 - 71.91728 \varepsilon^5 \]  

\[ Q = 4.54181 \times 10^5 - 3.19488 \times 10^5 \varepsilon - 2.89786 \times 10^5 \varepsilon^2 + 1.77765 \times 10^6 \varepsilon^3 - 3.25728 \times 10^6 \varepsilon^4 + 1.74548 \times 10^7 \varepsilon^5 \]  

\[ \ln A = 40.18281 - 29.51957 \varepsilon + 1.9563 \varepsilon^2 + 134.56396 \varepsilon^3 - 247.06753 \varepsilon^4 + 130.34468 \varepsilon^5 \]  

According to the definition of the hyperbolic law, the flow stress can be written as a function of the Zener–Hollomon parameter \([16,17]\). So the constitutive models of hot deformation behavior of X20Cr13 martensitic stainless steel are expressed as follows:

\[ Z = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) \]  

\[ \sigma = \frac{1}{\alpha} \ln \left( \frac{Z}{A} \right) + \left( \frac{Z}{A} \alpha^2 + 1 \right)^{1/2} \]  

where the values of \( \alpha, n, Q \) and \( A \) can be expressed by Eqs. (13)–(16), respectively.

### 3.4 Verification of developed constitutive models

The experimental data and predicted values are compared to evaluate the accuracy of the developed constitutive models in predicting the hot deformation behavior of X20Cr13 martensitic stainless steel, as shown in Fig. 6. It shows that the plot of predicted flow stress values against those experimentally obtained at

![Fig. 4 Relationships between ln \( \dot{\varepsilon} \) and \( \sigma \) (a), ln \( \dot{\varepsilon} \) and ln \( \sigma \) (b), ln \( \dot{\varepsilon} \) and ln[\( \sinh(\alpha \sigma) \)] (c) and ln[\( \sinh(\alpha \sigma) \)] and 1/\( T \) (d)
Fig. 5 Variation of $\alpha$ (a), $n$ (b), $Q$ (c) and $\ln A$ (d) with $\varepsilon$ and 5th order polynomial fit.

Fig. 6 Comparison of experimental and predicted flow stress values at strain rates: (a) 0.01 s$^{-1}$; (b) 0.1 s$^{-1}$; (c) 1 s$^{-1}$; (d) 10 s$^{-1}$.
strains of 0.05–0.75 (at 0.025 intervals) over the range of strain rates (0.01, 0.1, 1, 10 s⁻¹) and temperatures (1173–1423 K at the interval of 50 K). It can be observed that an agreement between the experimental data and calculated value is satisfactory for most of the experimental conditions in this work.

Standard statistical parameters such as correlation coefficient (R) and average absolute relative error (E) are used to quantify the predictability of the constitutive model. These are expressed as

\[ R = \frac{\sum_{i=1}^{N} (\sigma'_e - \sigma'_e) (\sigma'_p - \sigma'_p)}{\sqrt{\sum_{i=1}^{N} (\sigma'_e - \sigma'_e)^2 (\sigma'_p - \sigma'_p)^2}} \]  

(19)

\[ E = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{\sigma'_e - \sigma'_p}{\sigma'_e} \right| \times 100\% \]  

(20)

where \( \sigma'_e \) is the experimental flow stress; \( \sigma'_p \) is the predicted flow stress; \( \bar{\sigma}_e \) and \( \bar{\sigma}_p \) are the mean values of \( \sigma'_e \) and \( \sigma'_p \), respectively; \( N \) is the number of data used in this investigation. The \( R \) is calculated as 0.996, which indicates a good agreement between predicted and experimental flow stress values, as shown in Fig. 7. The \( E \) is an unbiased statistic for measuring the predictive capability of a model. It is calculated through a term-by-term comparison of the relative error in prediction with respect to the actual value of the variable [18]. The values of \( E \) at different temperatures and strain rates are listed in Table 1. The maximal value is 5.6946% at the strain rate of 10 s⁻¹ and the deformation temperature of 1273 K. The minimum value is only 0.5368% at the strain rate of 0.1 s⁻¹ and the deformation temperature of 1273 K. The mean value of \( E \) for all the predicted values is 3.22%.

Then the developed constitutive models are further used to predict the flow stress and compared with the independent experiments outside of the fit domain at different deformation conditions, as shown in Fig. 8. The values of \( E \) at the strain rates of 0.05 s⁻¹ (1273 and 1373 K) and 5 s⁻¹ (1373 and 1423 K) are 1.1387%, 4.6367%, 2.1692% and 4.2695%, respectively, which further indicate a good predictability of the developed constitutive models.

### 4 Conclusions

1) The flow stress is especially sensitive to strain rate. At the strain rate of 0.001 s⁻¹, multi-peak behavior occurs. At the strain rates of 0.01 and 0.1 s⁻¹, most of the curves show a single peak phenomenon. At the strain rates of 1 and 10 s⁻¹ for all the deformation temperatures, the true stress–strain curves show dynamic recovery character.

2) The material constants (\( \alpha \), \( n \), \( Q \) and \( A \)) are calculated as a function of strain by a fifth order polynomial fit. A new constitutive model coupling the flow stress with the strain rate, deformation temperature and strain is developed based on the Arrhenius equation.

3) The predicted and experimental flow stress values are analyzed by correlation coefficient and average absolute relative error. The values of \( R \) and \( E \) are 0.996 and 3.22%, respectively. The predictable efficiency is also verified by the independent experiments outside of the fit domain. All the values of \( E \) are within 5%.

### References

应变影响下 X20Cr13 马氏体不锈钢热变形的本构模型

任发才, 陈 军, 陈 飞

上海交通大学 塑性研究院, 上海 200030

摘 要: 利用 Gleeble−1500 热模拟机对 X20Cr13 马氏体不锈钢进行等温热压缩实验, 研究此材料在变形温度 1173−1423 K、变形速率 0.001−10 s\(^{-1}\) 条件下的热变形行为。通过五阶多项式拟合计算得出材料参数 Α和 Н, 激活能 \(Q\) 和 \(A\) 作为应变的函数。基于 Arrhenius 方程构建包含变形温度、应变速率和应变的 X20Cr13 马氏体不锈钢热变形的本构模型。通过相关系数和平均相对误差验证建立的 X20Cr13 马氏体不锈钢本构模型的有效性, 其值分别为 0.996 和 3.22%。

关键词: 马氏体不锈钢; 热变形行为; 流动应力; 本构模型

(Edited by Chao WANG)