Constitutive relationship for high temperature deformation of Al–3Cu–0.5Sc alloy

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Abstract: The high temperature compressive deformation behavior of Al–3Cu–0.5Sc alloy was investigated at temperatures from 350 to 500 °C, and strain rates from 0.01 to 10 s⁻¹ with the Gleeble–1500 thermo-mechanical simulator machine. The flow curves after corrections of the friction and temperature compensations were employed to develop constitutive equations. The effects of temperature and strain rate on deformation behaviors were represented by Zener-Hollomon parameter in an exponent type equation. The influence of true strain was incorporated in the constitutive equation by considering the effect of true strain on material constants. A four-order polynomial is found to be suitable to represent the influence of strain on the constitute equations.

Key words: Al–3Cu–0.5Sc alloy; constitutive relationship; high temperature deformation

1 Introduction

The research on scandium (Sc) addition in aluminum (Al) alloys has been received increasing attention over the last decade because of their interesting benefits. Most of the benefits are related to the formation of Al₃Sc particles, including Al₃Sc dispersoids and Al₃Sc precipitates [1]. The addition of Sc in pure aluminum or non-heat-treatable Al alloys has been extensively investigated [2,3]. In comparison, there are much fewer studies on the Sc addition in heat-treatable Al alloys [4]. But previous studies have shown that Sc can improve the strength of Al–3Cu alloy [5], however, as a deformed aluminum alloy, the deformation behavior is as important as the heat treatment process.

The flow stress of metals during hot deformation processes can be significantly influenced by several metallurgical phenomena such as working hardening, dynamic recovery and dynamic recrystallization. Therefore, the understanding of flow stress is of great importance in metal forming processes and the hot deformability can be improved by optimizing the process parameters [6,7]. So far, many researches have been done on the hot deformation behaviors and microstructure evolution of Al alloys [8–10]. However, many constitutive models are based on the Arrhenius type of equation, which assume that the influence of strain on high temperature deformation behavior is insignificant. In fact, the flow stress is changed with the increase of the true strain, which is important for the high temperature deformation behavior. On the other hand, the effects of the friction and the temperature raise on the stress–strain curves are usually ignored. Since a strain-dependent parameter for the sine hyperbolic constitutive equation was introduced by SLOOFF et al [11], a revised sine hyperbolic constitutive equation was adopted by the incorporation of the strain to predict the elevated temperature flow behaviors for steel [12,13], pure titanium [14] and P/M TiAl based alloy [15].

In this work, a comprehensive constitutive model for describing the relationship among the flow stress, strain rate and temperature was proposed with the compensation of the strain, and it was used to predict the high temperature flow behaviors of the Al–3Cu–0.5Sc alloy.

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2 Experimental

The composition (mass fraction) of the alloy used in the test was as follows: 3.0% Cu, 0.5% Sc and balance Al. The ingot was homogenized at 500 °C for 18 h. The specimens were cut from the ingot with dimensions of d 8 mm×12 mm. The specimens were compression deformed in the temperature range from 350 to 500 °C and the strain rate range from 0.01 to 10 s$^{-1}$ on the Gleeble−1500 thermo-mechanical simulator. All specimens were deformed to the total true strain of about 0.7. The specimens were induction-heated to deformation temperatures within 1 min and held for 3 min in order to obtain a stable and uniform temperature prior to the deformation.

3 Results and discussion

3.1 Friction correction

It is well known that the interfacial friction between the specimen and dies will affect the symmetrical deformation of the specimens [16]. In this work, although lubricants were used to minimize the interfacial friction, the interfacial friction becomes more and more evident with the increase of deformation. Thus, the deformation is more and more heterogeneous, leading to the drum-like shape of specimens, as shown in Fig. 1. ROEBUCK et al [17] developed a criterion for evaluating the effect of friction by a barreling coefficient, which is expressed as

$$B = \frac{hR_M^2}{h_0R_0^2}$$

(1)

where $B$ is the barreling coefficient; $h$ is the height of deformed specimens; $R_M$ is the maximum radius of deformed specimens; $h_0$ and $R_0$ are the initial height and radius of specimens, respectively. When $1 < B \leq 1.1$, the difference between the measured flow stress and the true flow stress is small, the measured flow stress curves do not need to be corrected; when $B \geq 1.1$, the measured flow stress curves must be corrected.

![Fig. 1 Schematic plot of sample before (a) and after (b) compression](image)

Based on the above criterion, the sizes of deformed specimens under various deformation conditions were measured, and the values of $B$ were calculated, as shown in Table 1. From Table 1, it can be observed that all the values of $B$ are greater than 1.1, so the measured flow stresses under all the deformation conditions must be corrected.

<table>
<thead>
<tr>
<th>Strain rate/s$^{-1}$</th>
<th>Value of $B$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>350 °C</td>
</tr>
<tr>
<td>0.01</td>
<td>1.16628</td>
</tr>
<tr>
<td>0.1</td>
<td>1.11878</td>
</tr>
<tr>
<td>1</td>
<td>1.16793</td>
</tr>
<tr>
<td>10</td>
<td>1.15967</td>
</tr>
</tbody>
</table>

Based on the upper-bound theory, a simple theoretical analysis of the compression test for the determination of the constant friction factor ($m$) was proposed [18]. The base equation is shown as follows:

$$\frac{p}{\sigma} = \frac{8bR}{H} \left( \frac{1}{12} + \left( \frac{H}{Rb} \right)^{3/2} \right) - \left( \frac{H}{Rb} \right)^{3} - \frac{me^{-b/2}}{24\sqrt{3}(e^{-b/2} - 1)}$$

(2)

where $\sigma$ is the corrected true stress; $p$ is the external pressure applied to specimens in compression (uncorrected stress); $b$ is the barrel parameter; $m$ is the constant friction factor; $R$ and $H$ are radius and height of samples, respectively, $R = R_0e^{-b/2}$ and $H = h_0e^{-\varepsilon}$. $m$ and $b$ can be evaluated by the following equations:

$$m = \frac{R_t}{h} \frac{3\sqrt{3}b}{12 - 2b}$$

(3)

$$b = 4 \frac{R_M - R_t}{R_t} \frac{h}{h_0 - h}$$

(4)

where $R_t$ is the average radius of samples after the deformation; $R_t$ is the top radius of deformed samples.

$$R_t = R_0 \sqrt{\frac{h_0}{h}}$$

(5)

$$R_t = \sqrt[3]{\frac{h_0^2}{h} \cdot R_0^2 - 2R_M^2}$$

(6)

Therefore, by this method, the corrected true stress can be calculated only by measuring the maximum radius and the height of samples after the deformation.

The true stress—true strain curves modified by considering the effects of interfacial friction are shown in Fig. 2. It can be easily found that the measured flow stress is greatly larger than the corrected ones. Meanwhile, the effect of the friction is obvious with the increase of the strain rate and the decrease of the deformation temperature.
3.2 Temperature correction

Flow softening is a common characteristic of stress—strain curves at high temperatures. The softening can be caused by the deformation heat or/and microstructural changes. In this compression tests, the temperature was measured and controlled by the thermocouple and computer, respectively. As the response time of the thermocouple is limited [19], the instantaneous temperature change cannot be measured while the strain rate is large enough. At the moment the deformation process is not completely isothermal, thus, the flow stress data must be corrected for the temperature rising induced by the deformation. At a strain rate of 0.001 s⁻¹, the amount of heat generated is usually very small and will be transmitted away, so the temperature corrections are conducted for high strain rate situations, with the following equation [20]:

\[
\Delta T = \frac{(0.9 - 0.95) \eta \int_0^\varepsilon \sigma d\varepsilon}{\rho c_p} \tag{7}
\]

where \(\Delta T\) is the change of temperature; \(\eta\) is the adiabatic correction factor; \(\int_0^\varepsilon \sigma d\varepsilon\) is the area under the uncorrected stress—strain curve (in this work, the area is under the friction-corrected stress—strain curve); \(\rho\) is the density (2.75 g/cm³ for Al–3Cu–0.5Sc alloy); \(c_p\) is the specific heat capacity (0.88 J/(g·K) for Al–3Cu–0.5Sc alloy); the factor of (0.9–0.95) is the fraction of mechanical work transformed to heat (0.9 for Al–3Cu–0.5Sc alloy). The adiabatic correction factor \(\eta\) is used under the isothermal condition at strain rates ≤0.001 s⁻¹, \(\eta=0\), at strain rates ≥10 s⁻¹; \(\eta=1\), at strain rates between 0.001 s⁻¹ and 10 s⁻¹. GOETZ and SEMIATIN [20] found that \(\eta\) was a complex function of the strain rate, temperature and strain, the thermal properties of workspaces and tooling, and the heat transfer coefficient at the interfaces. DADRAS and THOMAS [21] found that in the Ti–6Al–2Sn–4Zr–2Mo–0.1Si alloy, \(\eta\) was typically taken to vary linearly with \(\lg \dot{\varepsilon}\), i.e., equaling 0.25, 0.50 and 0.75 at strain rates of 0.01, 0.1 and 1 s⁻¹, respectively.

Figure 3 shows the calculated temperature changes during the compression test at temperatures of 350, 400, 450 and 500 °C and strain rates of 0.01, 0.1, 1 and 10 s⁻¹, respectively. It can be seen that, at strain rates of 0.01 s⁻¹ and 0.1 s⁻¹, the temperature change is small, however, at a strain rate of 1 s⁻¹ and 10 s⁻¹, the temperature increase is obvious. Due to the temperature rise, especially at a strain rate of 1 s⁻¹ and 10 s⁻¹, the measured flow stress curves must be corrected, which takes one of the following forms [22–24]:
Fig. 3 Calculated temperature changes of specimens during compression test at different pre-set temperatures and strain rates

\[ Z = A_1 \sigma^{n_1} = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) \quad \text{(for low stress level)} \quad (8) \]

\[ Z = A_2 \exp(\beta \sigma) = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) \quad \text{(for high stress level)} \]

\[ Z = A[\sinh(\alpha \sigma)]^n = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) \quad \text{(for all stress level)} \quad (9) \]

where \( Z \) is the Zener-Hollomon parameter; \( Q \) is the activation energy (kJ/mol); \( R \) is the universal gas constant (8.314 J/(mol·K)); \( T \) is the deformation temperature (K); \( A_1, A_2, A, \beta, n_1, n \) and \( \alpha \) are materials constant, \( \alpha = \beta / n_1 \).

In Eq. (10), \( \alpha \) is an adjustable constant. An optimum \( \alpha \) value can be found when the constant temperature curves in the \( \ln[\sinh(\alpha \sigma)] \) against \( \ln \dot{\varepsilon} \) plots are almost linear and parallel with each other. In the present work, however, an optimum \( \alpha \) value cannot be directly obtained from the uncorrected flow stress data because the specimen temperature varies at high strain rates, and the constant temperature curves cannot be plotted. In this case, the flow stress should be corrected using Eqs. (8) and (9) for low stress and high stress, respectively. The correction of flow stress for deformation heating was accomplished by plotting \( \ln \sigma - T^{-1} \) at the low strain rate of 1 s\(^{-1}\) and plotting \( \sigma - T^{-1} \) at the high strain rate of 10 s\(^{-1}\), as shown in Figs. 4 and 5, respectively. Because there is no correlation of the flow stress with the strain in Eqs. (8) and (9), the correction should be made for each selected strain value.

3.3 Flow stress

Figure 6 shows the temperature corrected and friction-corrected flow stress curves of Al–3Cu–0.5Sc alloy under different experimental conditions. It can be seen that at strain rates of 1 s\(^{-1}\) and 10 s\(^{-1}\), however, the differences are obvious, especially at low temperatures. From Fig. 6, it can also be seen that the deformation temperature and the strain rate have a significant effect on the flow stress. At the same strain, the flow stress decreases with increasing the temperature, and rises with increasing the strain rates.

Fig. 4 Plot of ln \( \sigma - T^{-1} \) at strain of 0.2 and strain rate of 1 s\(^{-1}\)

Fig. 5 Plot of \( \sigma - T^{-1} \) at strain of 0.2 and strain rate of 10 s\(^{-1}\)

3.4 Determination of material constants for constitutive equation

The effect of strain on the stress is not considered in Eqs. (8)–(10), which may be reflected by the material constants in the constitutive equation. Taking the strain of 0.2 as an example, the determination of the material constants was conducted. From Eqs. (8) and (9), Eqs. (11) and (12) can be obtained:

\[ \ln \dot{\varepsilon} = n_1 \ln \sigma + \ln A_1 - \frac{Q}{RT} \quad \text{(for low stress level)} \quad (11) \]

\[ \ln \dot{\varepsilon} = \beta \sigma + \ln A_2 - \frac{Q}{RT} \quad \text{(for high stress level)} \quad (12) \]

At low stress level, an optimum \( n_1 \) value can be obtained by plotting \( \ln \sigma \) against \( \ln \dot{\varepsilon} \) at constant temperatures, and the slope of the \( \ln \sigma - \ln \dot{\varepsilon} \) plot is
At high stress level, an optimum $\beta$ value can be obtained by plotting $\sigma$ against $\varepsilon \ln$ at constant temperatures, and the slope of the $\sigma$—$\varepsilon \ln$ plot is $\beta$.

Then, substituting the corrected flow stress and corresponding strain rate at the strain of 0.2 into Eqs. (11) and (12), the relationship between the flow stress and strain rate is obtained. Because the slopes of the lines are approximately the same, the average slopes are used for deriving the values of $n_1$ and $\beta$, which are 10.8403975 and 0.154925 MPa$^{-1}$, respectively. Then, $\alpha = \beta / n_1 = 0.01429$ MPa$^{-1}$.

For all stress levels, Eq. (10) can be represented as follows:

$$\dot{\varepsilon} = A[\sinh(\alpha \sigma)]^n \exp(-Q/RT)$$  \hfill (13)

Differentiating Eq. (13) gives:

$$Q = R \left[ \frac{\partial \ln \dot{\varepsilon}}{\partial \ln[\sinh(\alpha \sigma)]} \right]_T \left[ \frac{\partial \ln[\sinh(\alpha \sigma)]}{\partial (1/T)} \right]_{\dot{\varepsilon}}$$  \hfill (14)

From Eq. (14), it can be seen that the $Q$ value can be derived from the slopes of $\ln \dot{\varepsilon} - \ln[\sinh(\alpha \sigma)]$ and $\ln[\sinh(\alpha \sigma)] - 1/T$ plots. Substituting the values of the corrected flow stress, deformation temperature and corresponding strain rate at the strain of 0.2 into Eq. (14), the relationship between $\ln \dot{\varepsilon}$ and $\ln[\sinh(\alpha \sigma)]$ and $\ln[\sinh(\alpha \sigma)] - 1/T$ can be obtained. Because the slopes of the lines are approximately the same, the average slopes are used to derive the activated energy, which is 133.3 kJ/mol. Then, substituting the values of $Q$, $\dot{\varepsilon}$ and $T$ into Eq. (10), the values of $Z$ at different deformation temperatures and strain rates can be obtained.

Taking the natural logarithm of both sides of Eq. (10) gives:

$$\ln Z = \ln A + n \ln[\sinh(\alpha \sigma)]$$  \hfill (15)

The values of $\ln A$ and $n$ are the intercept and slope of $\ln Z - \ln[\sinh(\alpha \sigma)]$ plot, respectively. By substituting the values of $\ln Z$ and corresponding corrected flow stress into Eq. (15), the relationship between $\ln Z$ and $\ln[\sinh(\alpha \sigma)]$ is shown in Fig. 7. Therefore, the values of $\ln A$ and $n$ are determined as 21.1 and 8.102 MPa$^{-1}$, respectively.

### 3.5 Compensation of strain

It is assumed that the influence of the strain on high temperature deformation behavior is insignificant and
thereby it is not considered in Eq. (10). For the compensation of strain, the influence of strain in the constitutive equation is incorporated by assuming that the material constants \(n, \alpha, Q\) and \(A\) are the polynomial function of the strain. The material constants \(n, \alpha, Q\) and \(A\) of the constitutive equations were computed at various strains and an interval of 0.05. Then, these values were employed to fit the polynomial. A four-order polynomial, as shown in Eq. (16), is found to represent the influence of strain on material constants with a very good correlation for Al–3Cu–0.5Sc alloy, as shown in Fig. 7.

\[
\begin{align*}
\alpha &= B_0 + B_1 \varepsilon + B_2 \varepsilon^2 + B_3 \varepsilon^3 + B_4 \varepsilon^4 \\
n &= D_0 + D_1 \varepsilon + D_2 \varepsilon^2 + D_3 \varepsilon^3 + D_4 \varepsilon^4 \\
Q &= E_0 + E_1 \varepsilon + E_2 \varepsilon^2 + E_3 \varepsilon^3 + E_4 \varepsilon^4 \\
\ln A &= F_0 + F_1 \varepsilon + F_2 \varepsilon^2 + F_3 \varepsilon^3 + F_4 \varepsilon^4
\end{align*}
\]

(16)

Once the material constants are evaluated, the flow stress at a particular strain can be predicted. The flow stress can be written as a function of Zener-Hollmon parameters. So, the proposed constitutive model can be summarized as follows:

\[
\sigma = \frac{1}{\alpha} \ln \left( \frac{Z}{A} \right) + \left[ \frac{Z}{A} \right]^{\frac{2}{\alpha}} + 1 \right]^\frac{1}{2}
\]

\[
Z = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right)
\]

\[
\begin{align*}
\alpha &= 0.01772 + 0.00988 \varepsilon - 0.02223 \varepsilon^2 + 0.05739 \varepsilon^3 - 0.05121 \varepsilon^4 \\
n &= 9.8258 - 16.32404 \varepsilon + 58.10011 \varepsilon^2 - 105.49637 \varepsilon^3 + 68.30878 \varepsilon^4 \\
Q &= 112.96793 + 312.64183 \varepsilon - 1294.9349 \varepsilon^2 + 2165.25594 \varepsilon^3 - 1267.76685 \varepsilon^4 \\
\ln A &= 16.72821 + 69.39087 \varepsilon - 308.02186 \varepsilon^2 + 540.30323 \varepsilon^3 - 327.72077 \varepsilon^4
\end{align*}
\]

(17)

4 Conclusions

1) The measured flow stress of Al–3Cu–0.5Sc alloy was modified by the friction and temperature corrections,
and the friction-corrected flow stresses are lower than the measured ones. The effect of temperature on flow stress is obvious at strain rates of 10 and 1 s⁻¹.

2) The influence of strain in the constitutive analysis was incorporated by considering the effect of strain on material constants, and a four-order polynomial was found.

3) The constitutive equation considering the compensation of strain was derived.

References


Al–3Cu–0.5Sc 合金高温变形的本构关系

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摘 要：采用Glueclo–1500 热力模拟试验机研究Al–3Cu–0.5Sc 合金在温度为 350–500 ℃、应变速率为 0.01–10 s⁻¹条件下的高温压缩变形行为，利用经摩擦修正和温度补偿修正后的流变应力曲线建立合金的本构方程，温度和应变速率对变形行为的影响可使用包含Zener-Hollomon参数的指数方程来描述。通过考虑应变对材料常数的影响，建立包含应变的本构方程；其应变对本构方程的影响规律，可通过材料常数的多项式拟合来实现。

关键词：Al–3Cu–0.5Sc 合金；本构关系；高温变形

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